Quantum Algorithm for Path-Edge Sampling

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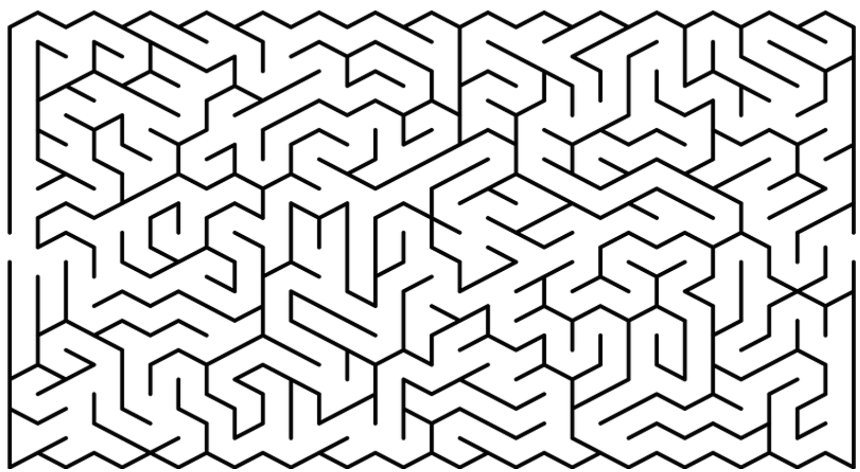
1. CWI Amsterdam
2. Middlebury

Similar problems

- Is there a path from s to t?
- What is the path from s to t?

s  t

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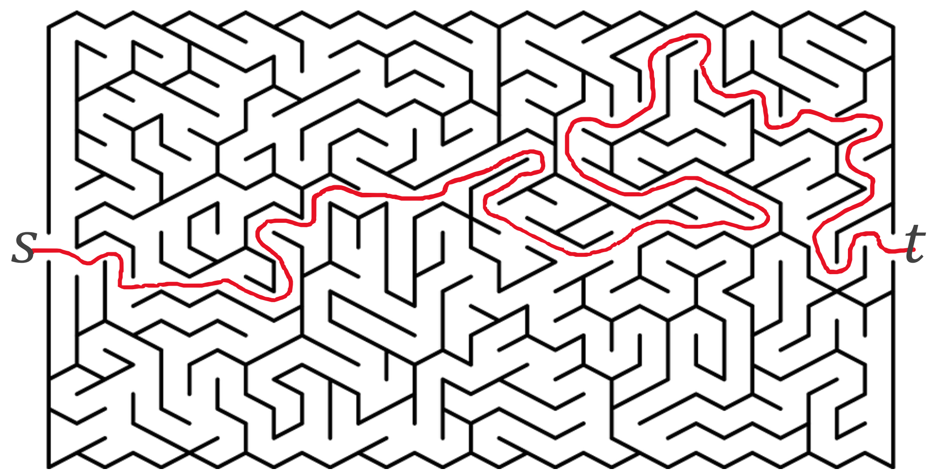

Similar problems - different complexity?

- Is there a path from s to t?
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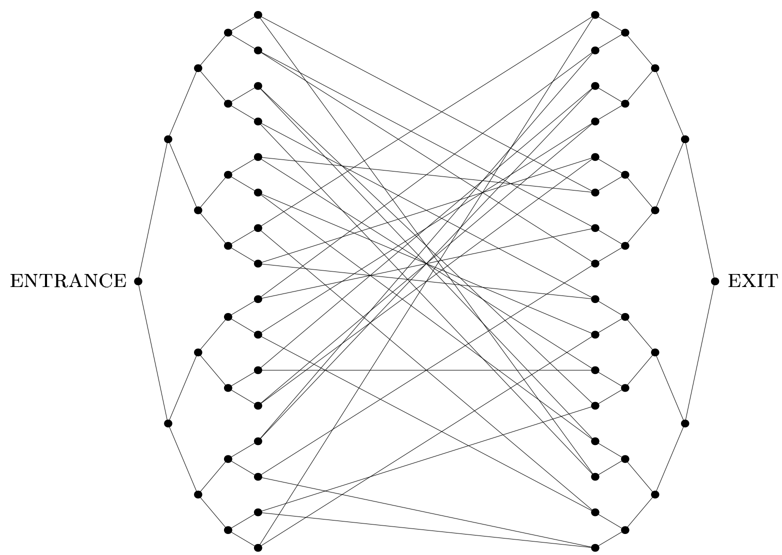

[‘Childs et al., ’03]
[Rosmanis, ’11]
[Childs, Coudron, Gilani, ‘22]
[Aaronson’s top 10, ‘21]

Path-Edge Sampling

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- What is the path from s to t?
- **Find an edge on a path from s to t**

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	Path Detection[1]	**Edge Finding**	**Path Finding**[3]
Av. Query Complexity	$\tilde{O}(n\sqrt{R})$		$\tilde{O}(n^{3/2})$

1: Belovs & Reichardt, ‘12, Anderson et al. ‘23

3: Dürr et al. ‘06

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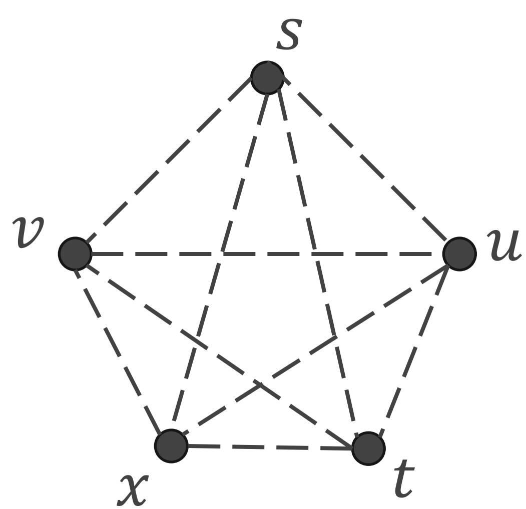
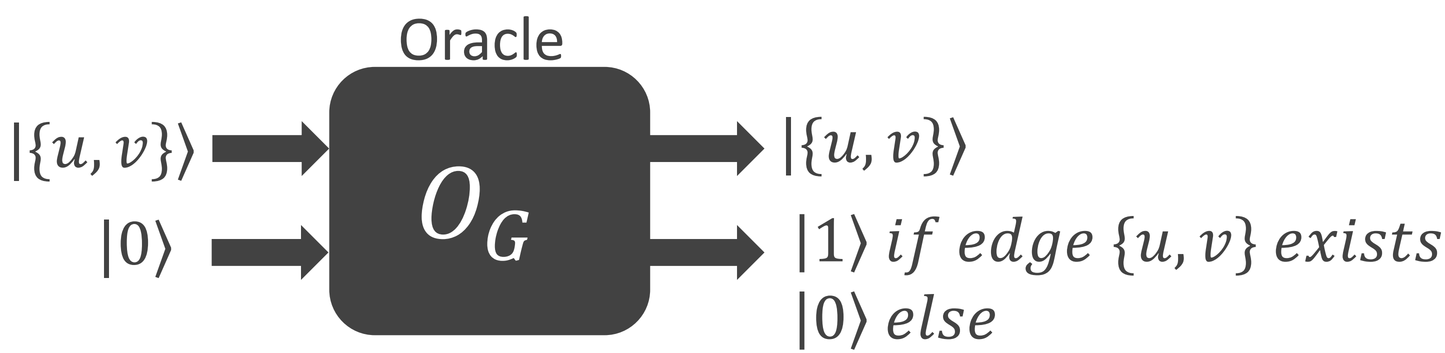
Unique path from s to t

1: Belovs & Reichardt, '12, Anderson et al. '23
2: **This paper**
3: Dürr et al. '06

Problem set-up

G:





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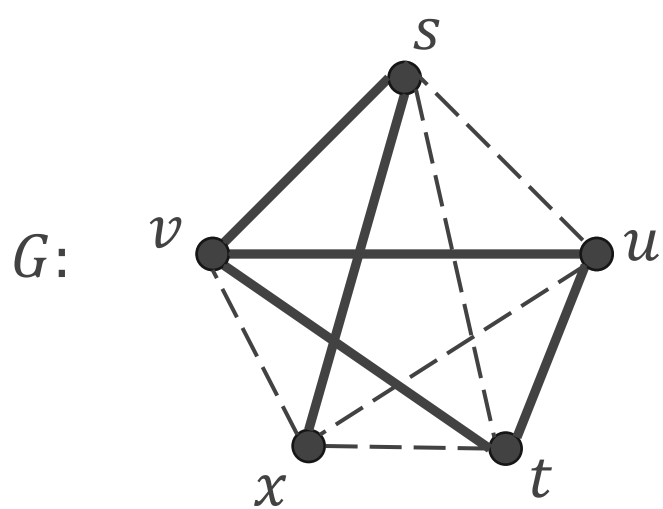

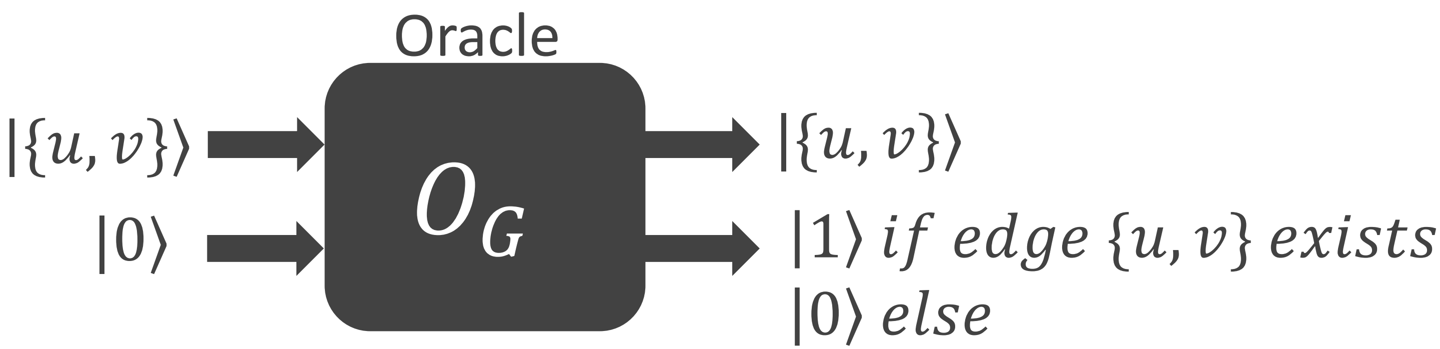

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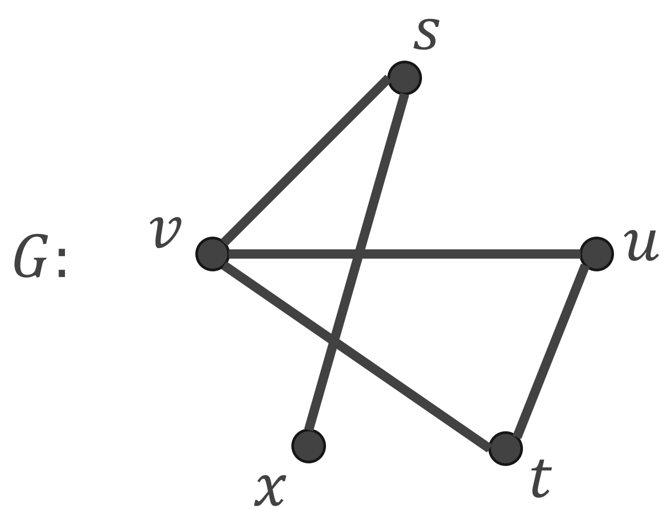
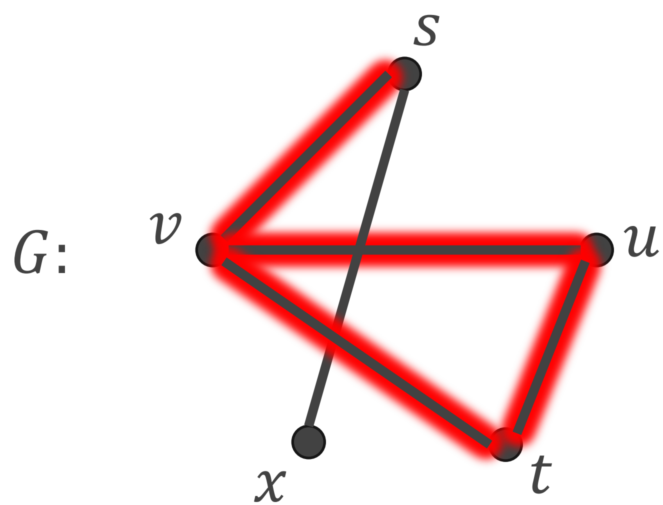


Goal: find from an edge on a path from s to t

Path: sequence of ***distinct*** vertices connected by edges

Problem set-up



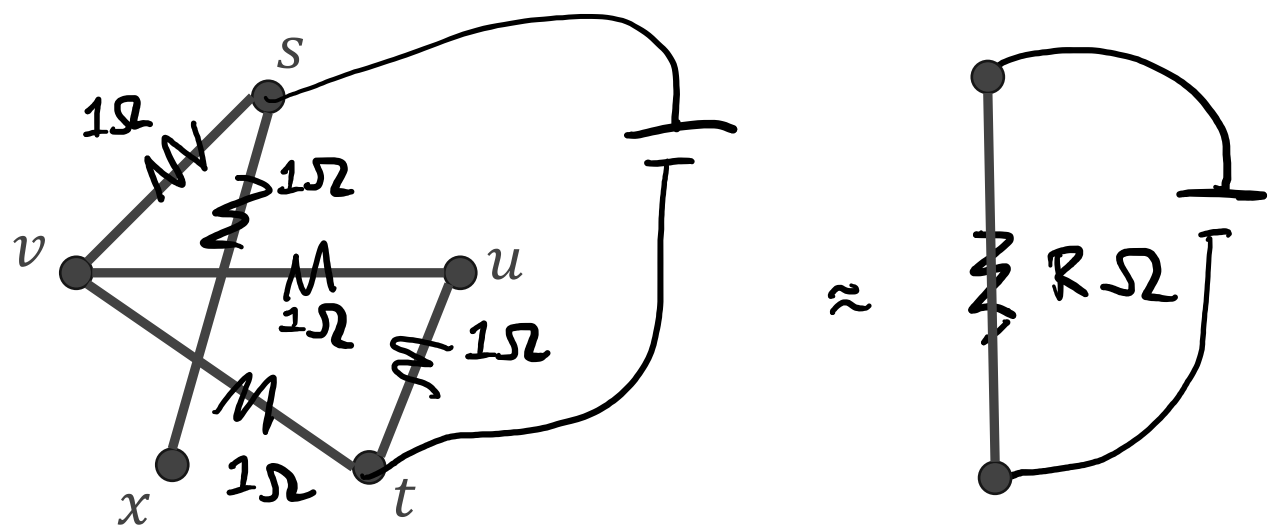
Goal: find an edge on a path from s to t

Average Query Complexity

Average number of oracle uses needed to find an st-path edge of graph G w.h.p.

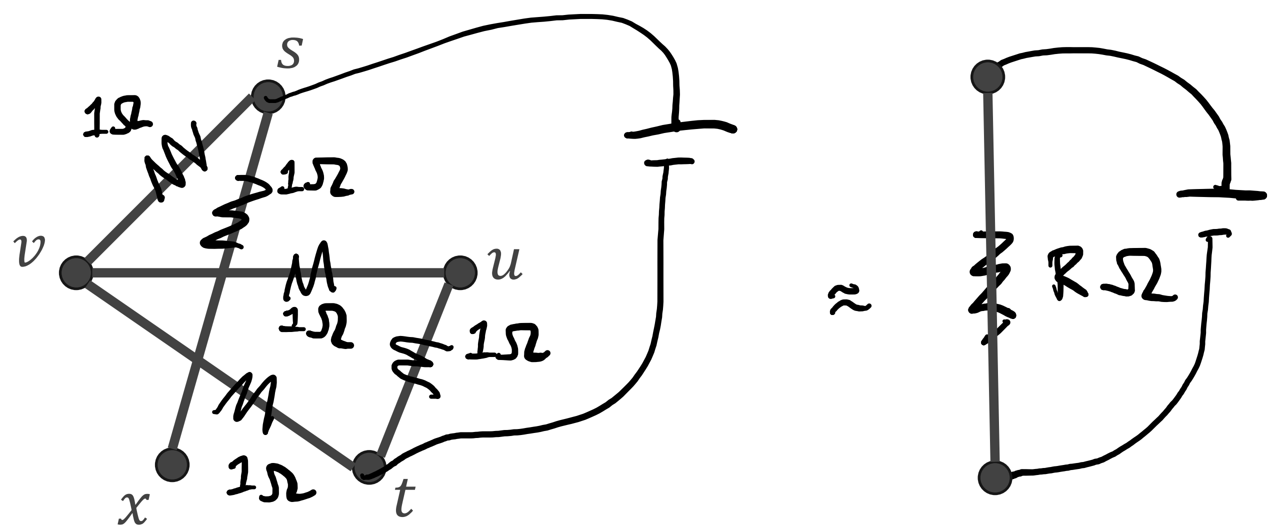
Average Query Complexity

Graph w/ effective resistance R between s and t:



Average Query Complexity

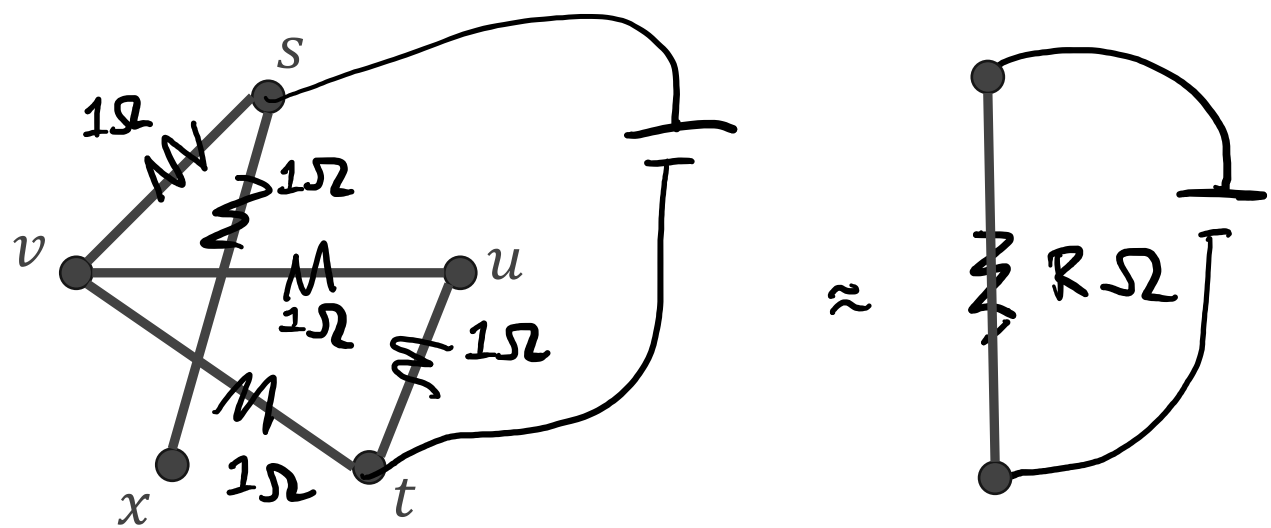
Graph w/ effective resistance R between s and t:
($R \leq$ length of shortest st-path)



Average Query Complexity

Find a path edge in graph w/ effective resistance R b/t s and t:

Av. Quantum QC: $\tilde{O}(n\sqrt{R})$

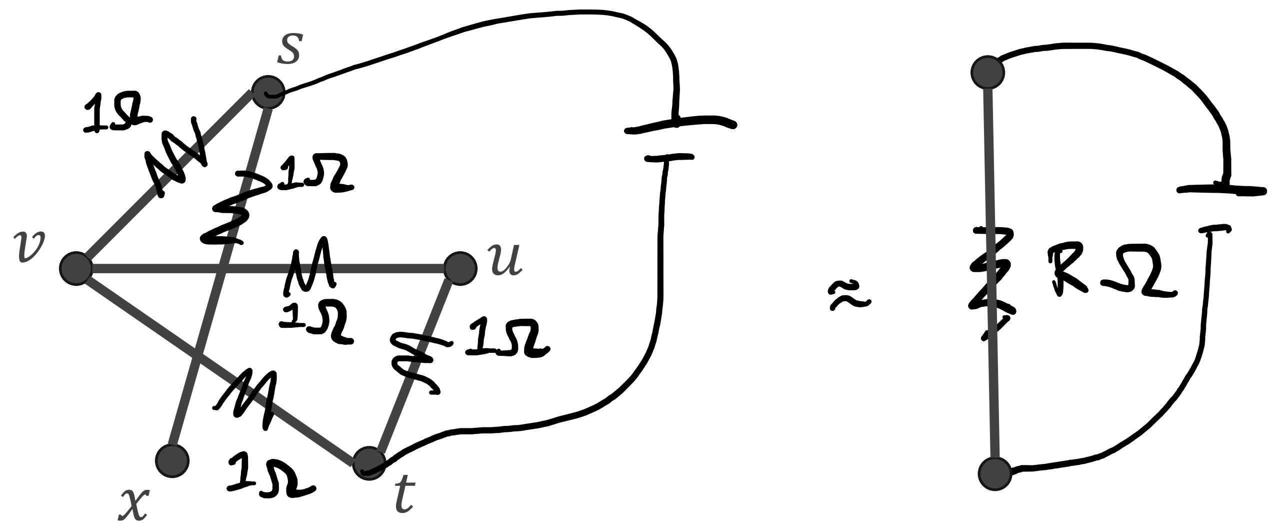


Average Query Complexity

Find a path edge in graph w/ effective resistance R b/t s and t:

Av. Quantum QC: $\tilde{O}(n\sqrt{R})$

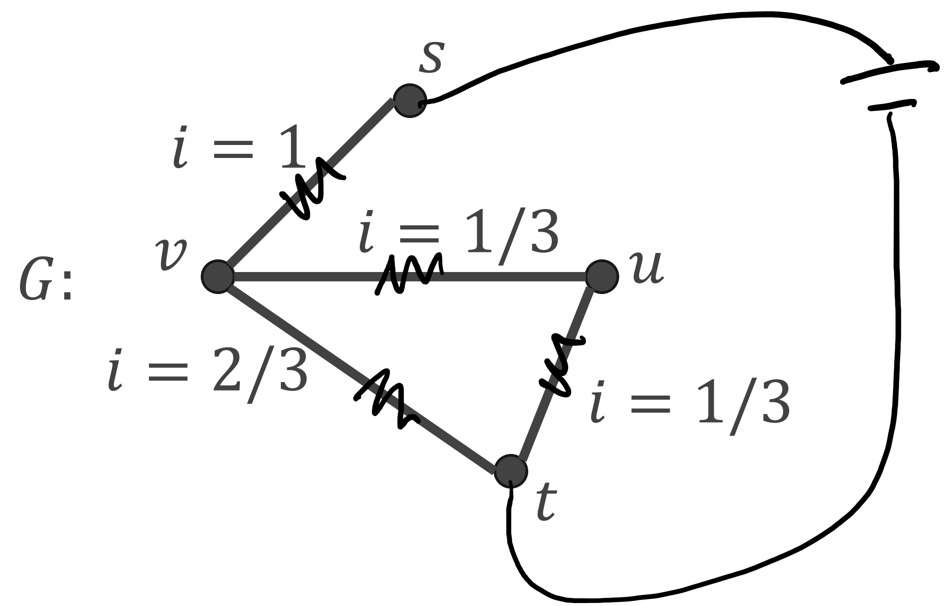
Av. Classical QC: $\Omega(n^2)$ (even for $R = O(1)$)



Max ($R = O(n)$): $\tilde{O}(n^{3/2})$

Path-Edge Sampling

We ***sample*** edges with probability proportional to current flow squared (power dissipated at that edge in resistive circuit)



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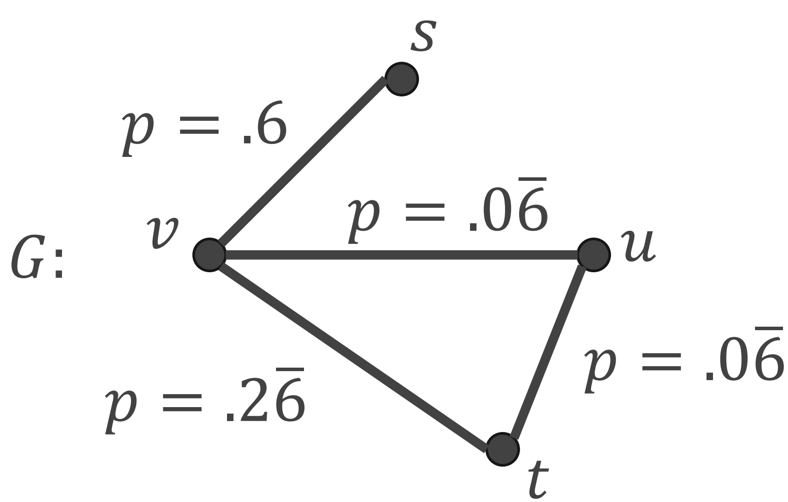
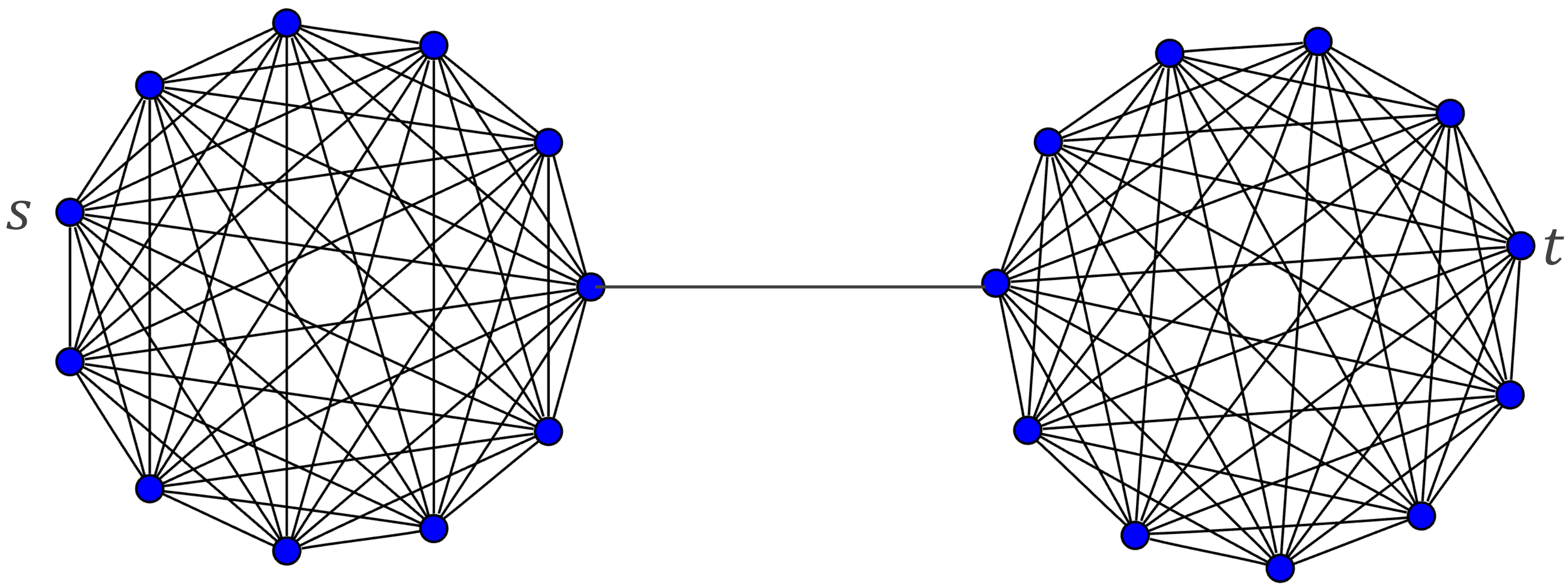


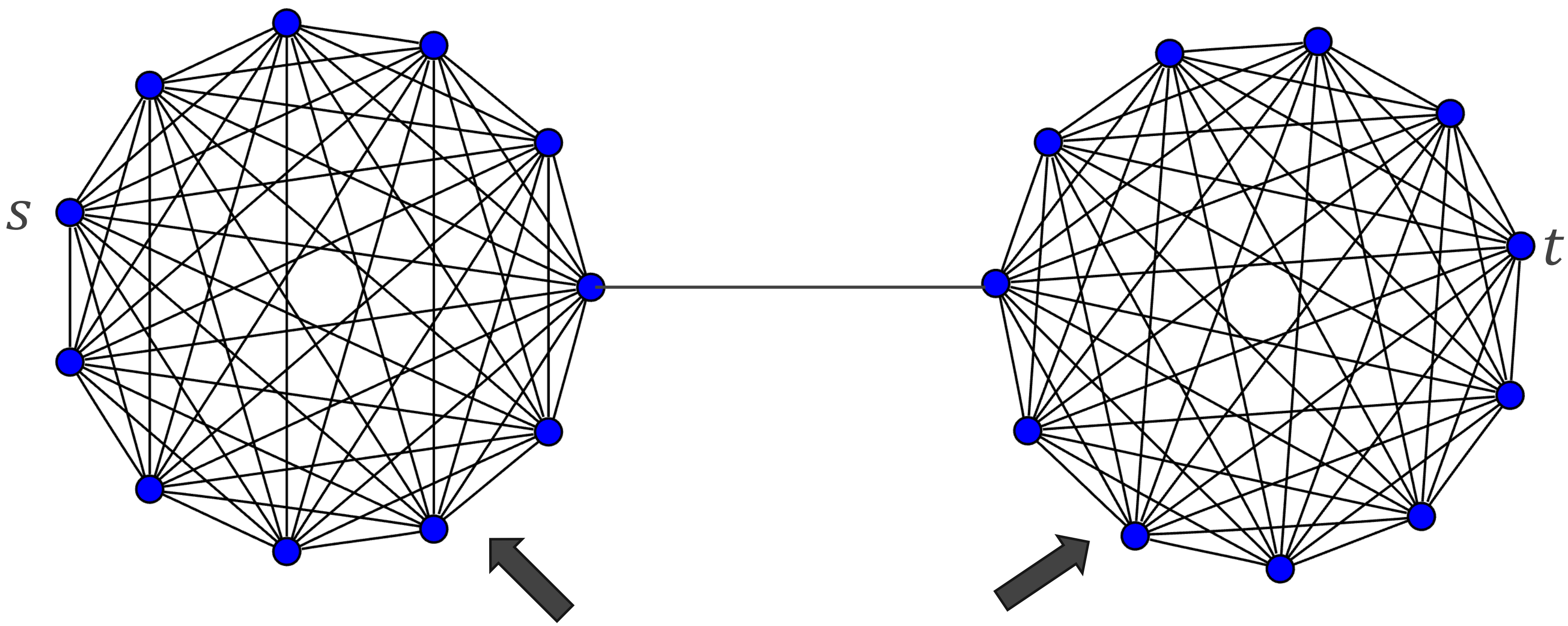
Application 1: Bottlenecks

Want to identify bottleneck edge



Application 1: Bottlenecks

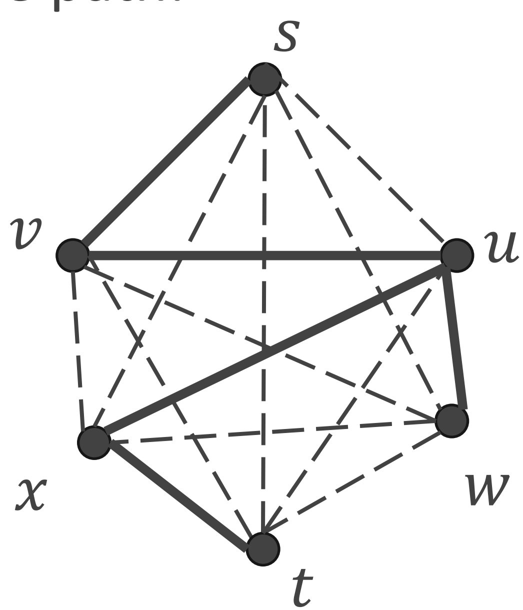
Our algorithm will sample bottleneck edge with constant probability



(when these graphs are expanders)

Application 2: Finding a path

Single path:



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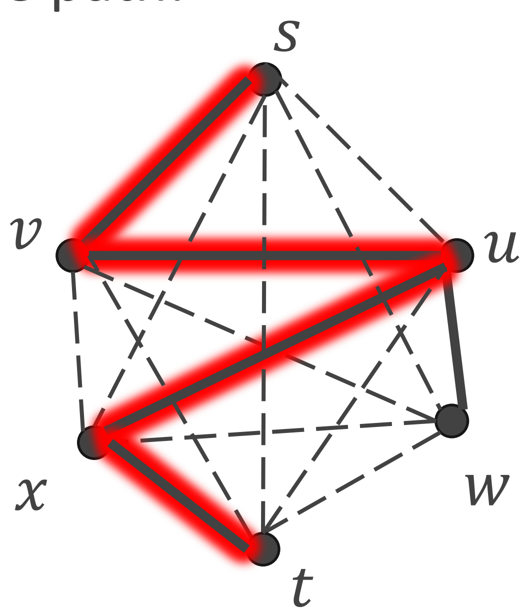
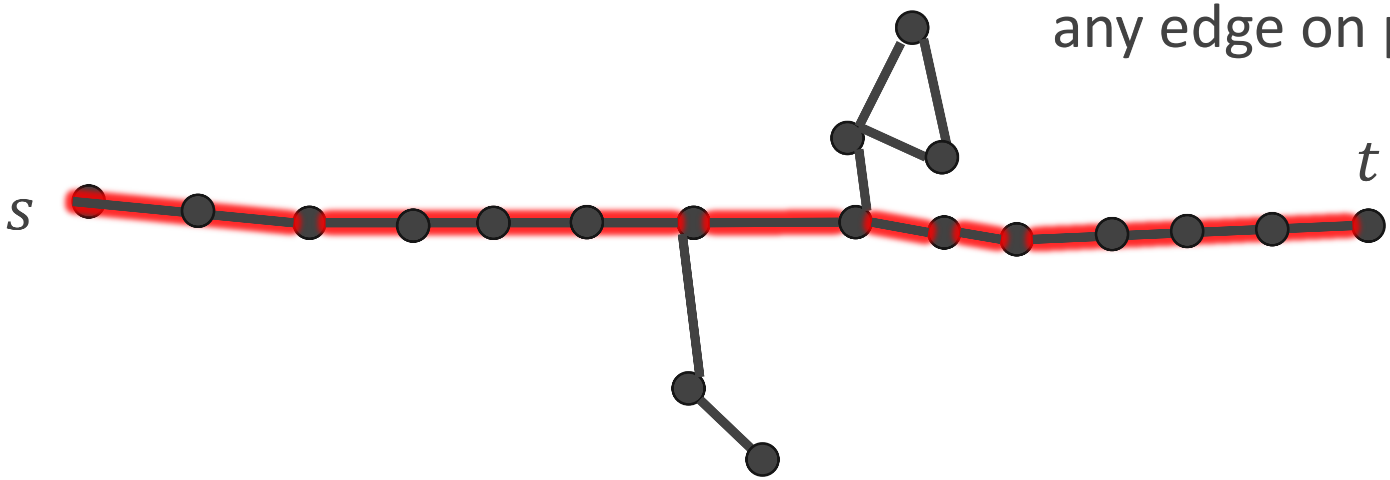


Application 2: Finding a path

Single path:

Equal prob. of sampling any edge on path



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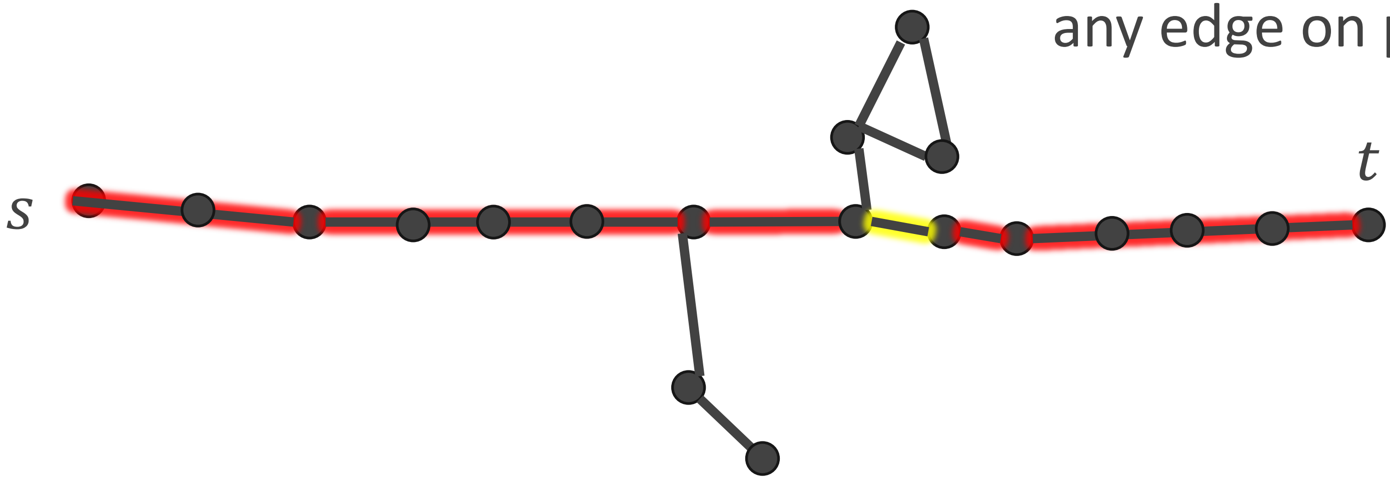
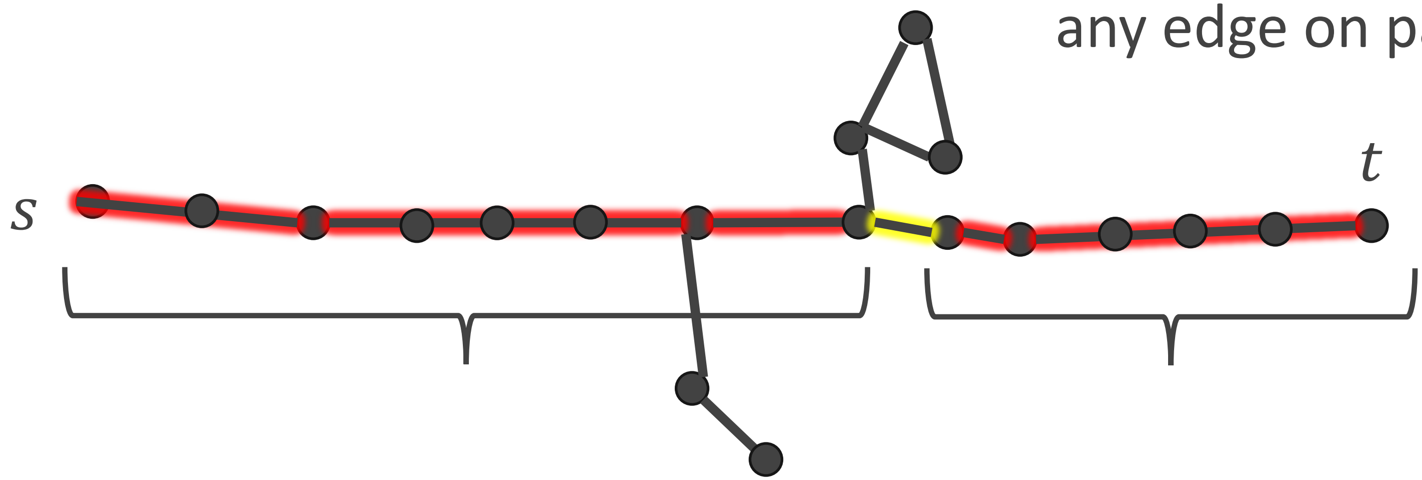


Our algorithm w.h.p. finds one in the middle 9/10ths.

Application 2: Finding a path

Single path:

Equal prob. of sampling any edge on path



Now have two subproblems.

Recurse! Divide and Conquer! (Randomized, like quicksort)

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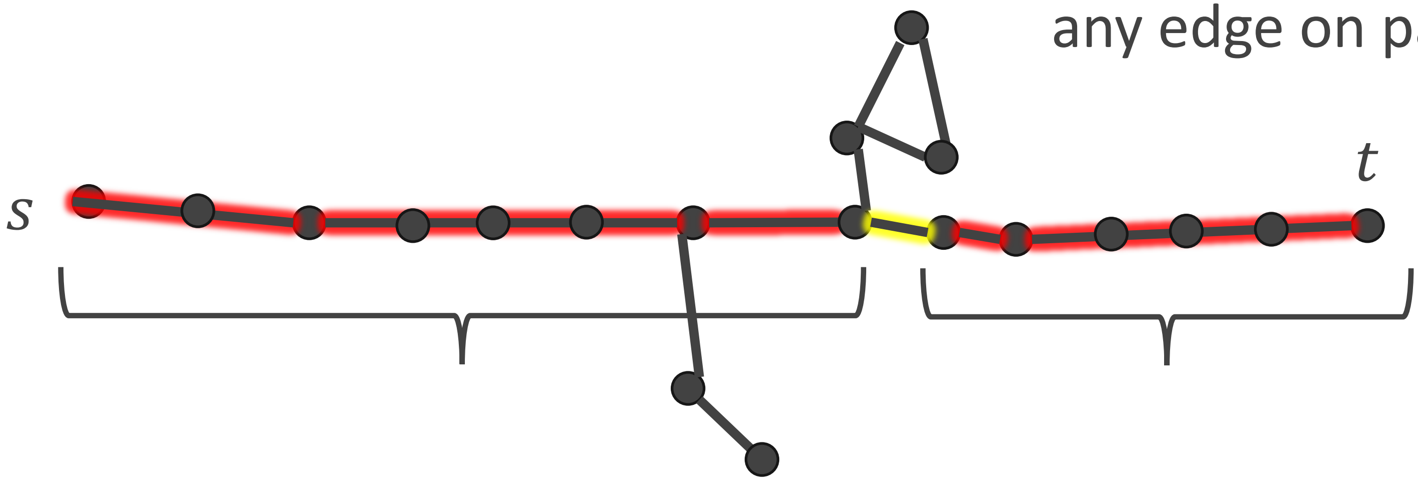


Now have two subproblems.

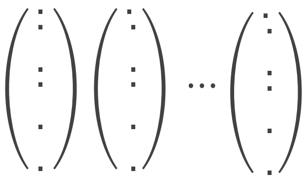
Recurse! Divide and Conquer! (Randomized, like quicksort)

$\tilde{O}\left(nL^{1+o(1)}\right)$ queries.

Outperforms existing best quantum alg for $L = \Omega\left(n^{\frac{1}{2}-o(1)}\right)$

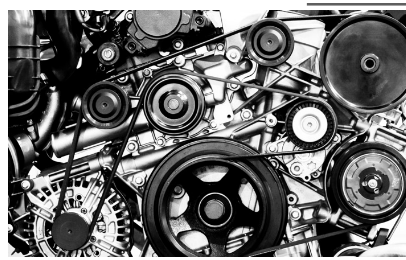
Under the Hood

Span program



Encodes Boolean f on domain X

(See Reichardt 2010 arXiv: 1005.1601)



Quantum Query algorithm to evaluate $f(x)$ for any $x \in X$ (given oracle O_x for x)

Query complexity depends on witness vector w_x (mathematical object used for analysis)

Our algorithm creates a quantum state proportional to w_x

In the case of path detection, w_x is a linear combination of path-edges, weighted according to flow.

Open Questions

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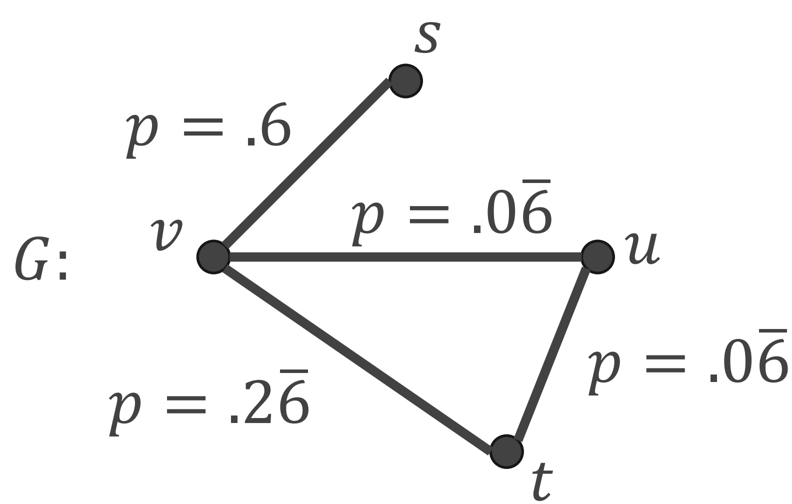

Open Questions

- Other uses of witness vector generation algorithm?
- Use sampling distribution of our algorithm to improve over existing path finding algorithm for more complex graphs (besides single path)
- Path detection vs Path finding

See also: “Elfs, Trees, and Quantum Walks” Apers + Piddock

Thank you!

Funding:

ARO

Collaborators:

Stacey Jeffery

Alvaro Piedrafita

You!

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(For complete parent graphs, we find q. query complexity is equal to time complexity, up to log factors)