Robust Single-Qubit Process Calibration via Robust Phase Estimation

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JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

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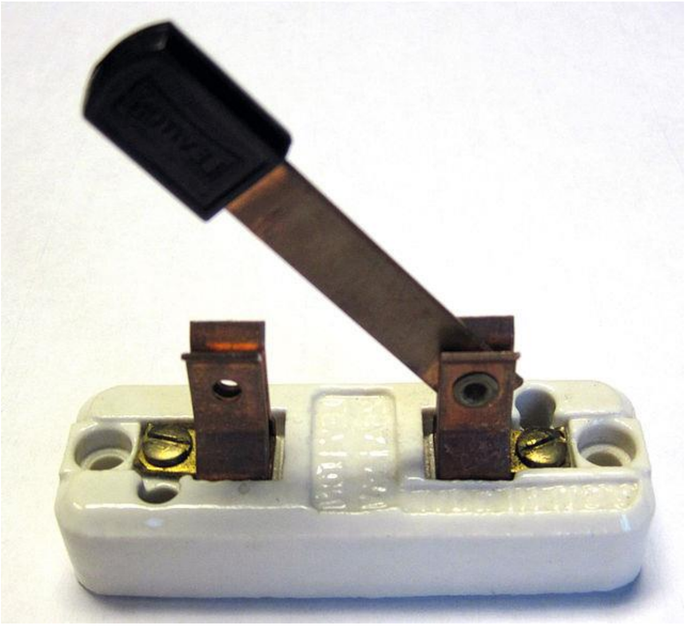
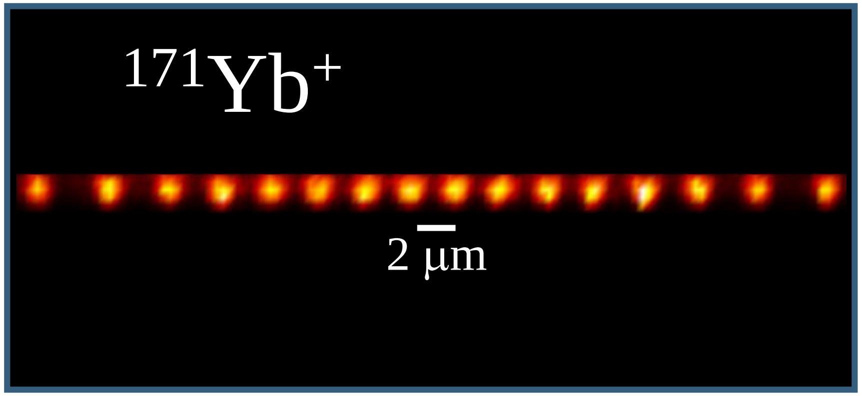
University of Maryland Institute for Advanced Computer Studies

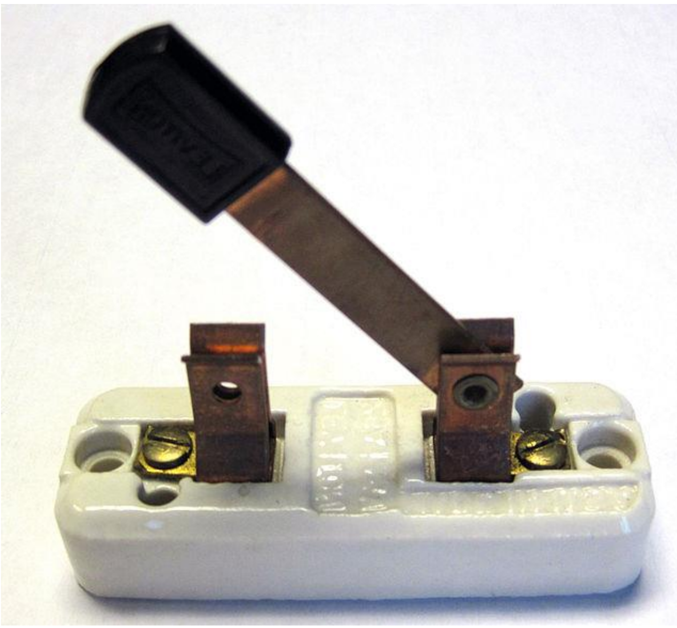
Imagine...

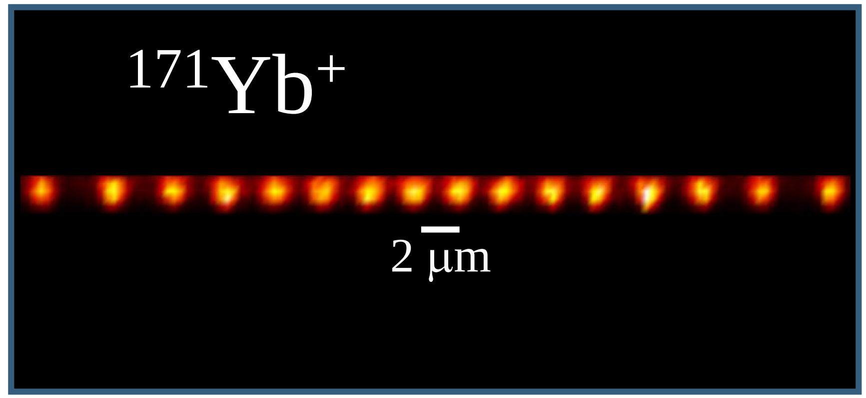




[Monroe Lab]

Imagine…





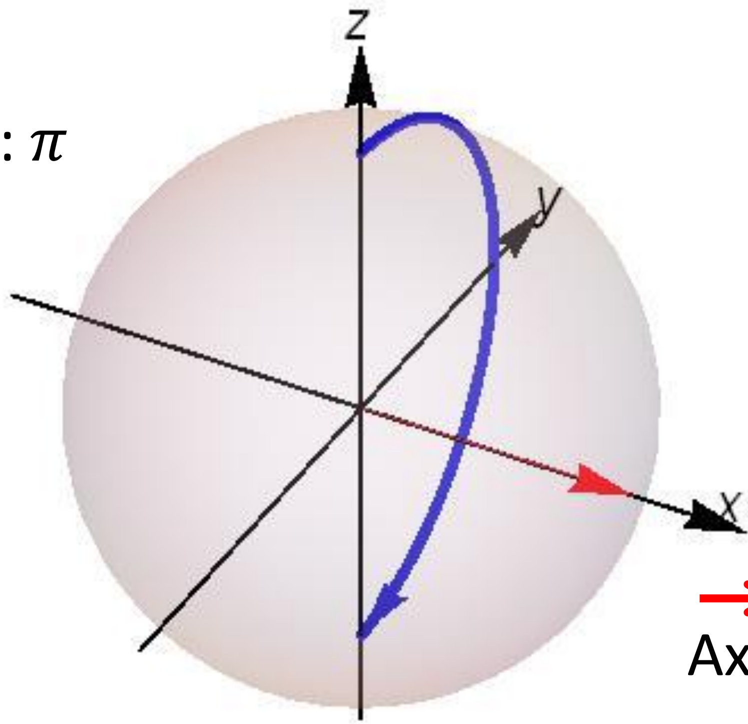
[Monroe Lab]

- All gates need to be tuned
- State preparation is off
- Measurements are off

→ Want to quickly determine imperfections in gate controls and then tune to fix.

Need to Calibrate Operations

Amplitude of Rotation: π

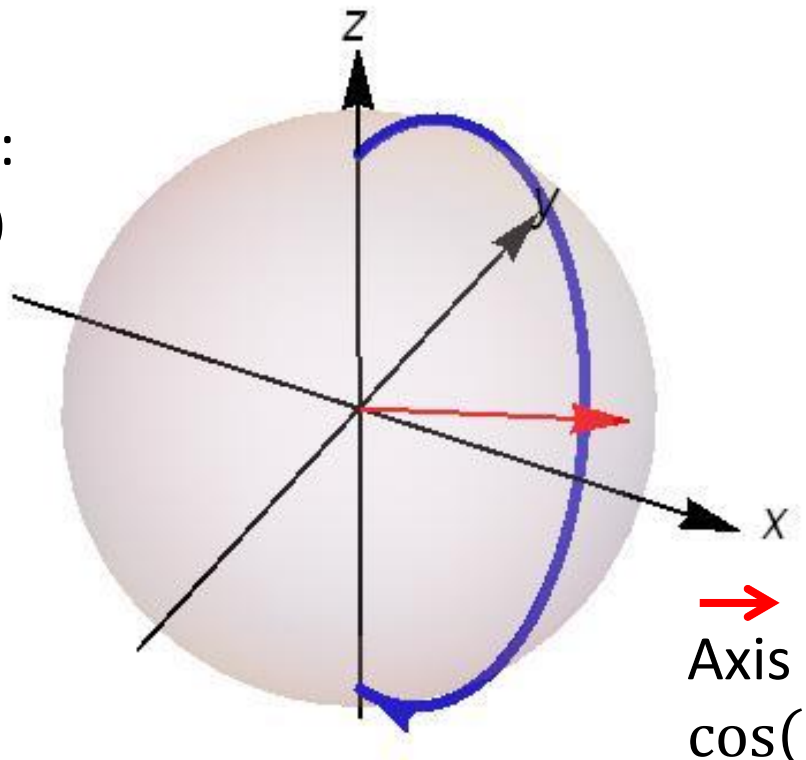


Axis of Rotation: $\hat{x}$

Ideal Unitary U

Need to Calibrate Operations

Amplitude of Rotation:
$A = \pi(1 + \epsilon)$



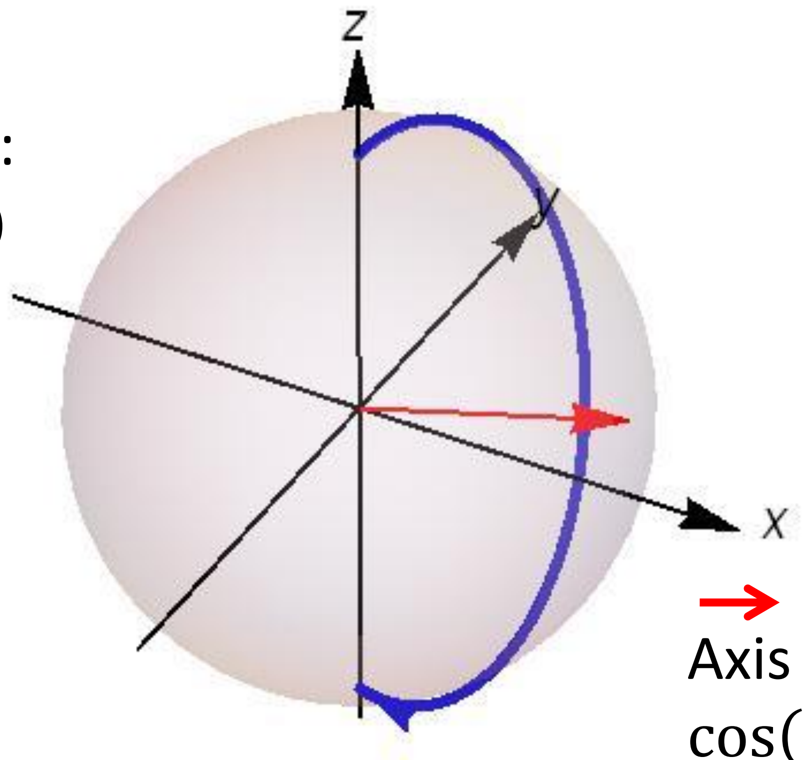
Axis of Rotation:
$\cos(\phi)\hat{x} + \sin(\phi)\hat{z}$

Unitary $\tilde{U}$

Need to Calibrate Operations

Amplitude of Rotation:
$A = \pi(1+\epsilon)$

ϵ is an "Amplitude Error"

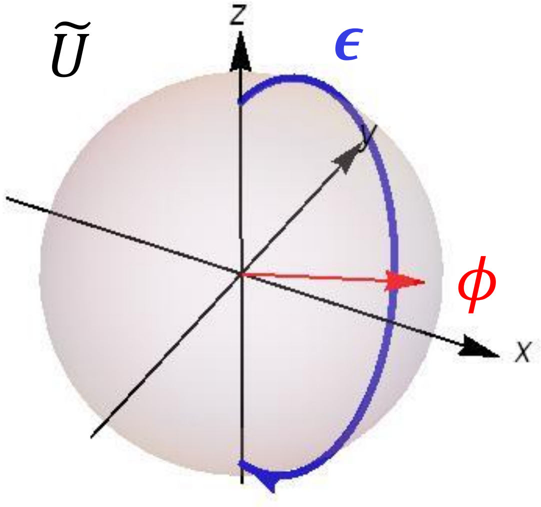


Unitary $\widetilde{U}$

Axis of Rotation:
$\cos(\phi)|x\rangle +$

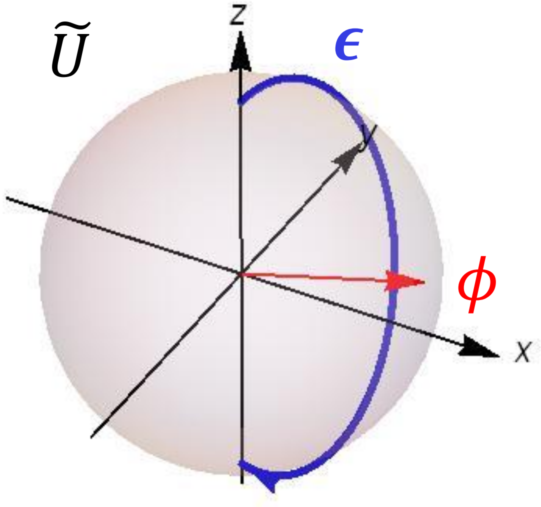
ϕ is an "Off-Resonance Error"

How to Estimate Control Errors



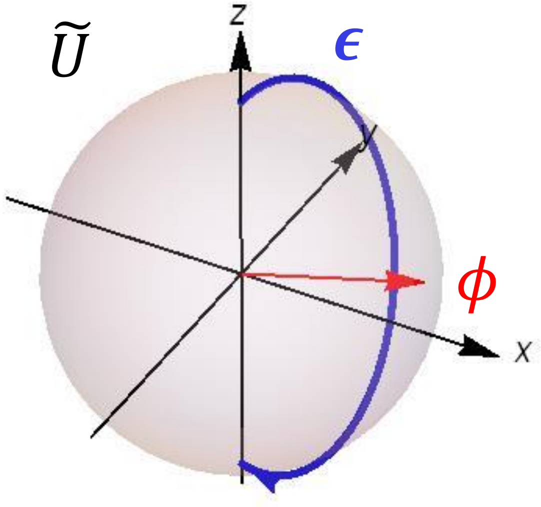
Ad hoc Rabi – Ramsey Sequences.

How to Estimate Control Errors



Process Tomography

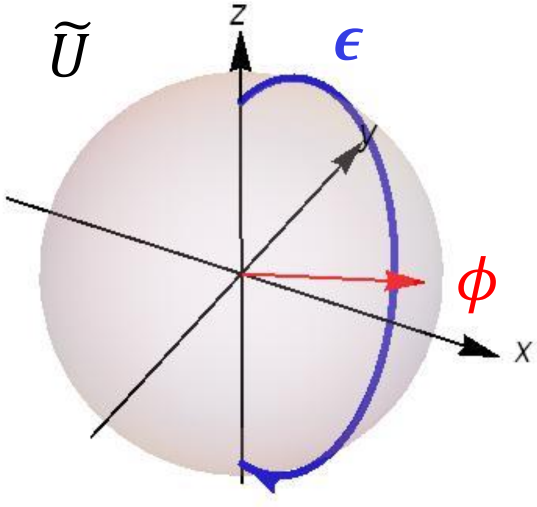
How to Estimate Control Errors



Process Tomography

- Need perfect state preparation and measurement
- Need perfect additional gates
- Time consuming: need to learn 12 parameters to extract ϕ and ϵ

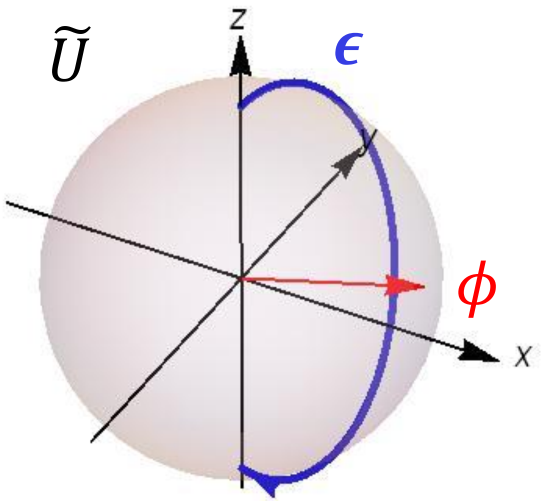
How to Estimate Control Errors



Process Tomography

Randomized Benchmarking

How to Estimate Control Errors

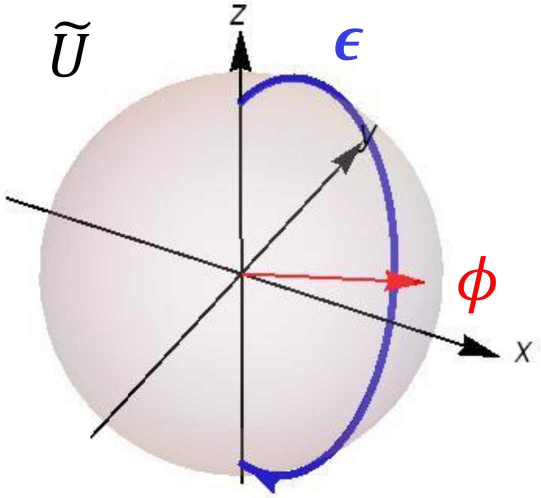


~~Process Tomography~~

Randomized Benchmarking

- Don’t need perfect state preparation and measurement
- Must be able to perform single-qubit Cliffords (although not perfectly)
- Time consuming: need to learn 9 parameters to extract ϕ and ϵ

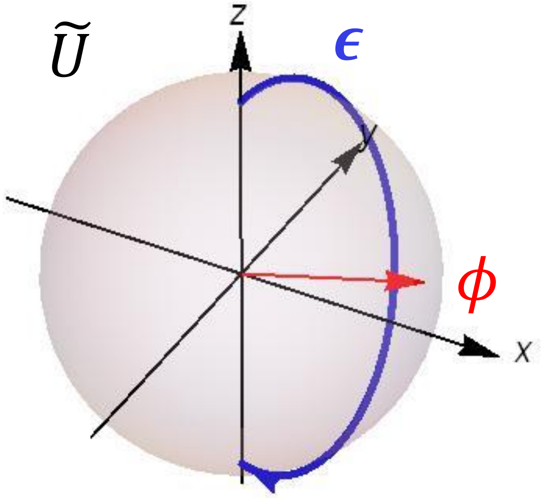
How to Estimate Control Errors



~~Process Tomography~~

~~Randomized Benchmarking~~

How to Estimate Control Errors



~~Process Tomography~~

~~Randomized Benchmarking~~

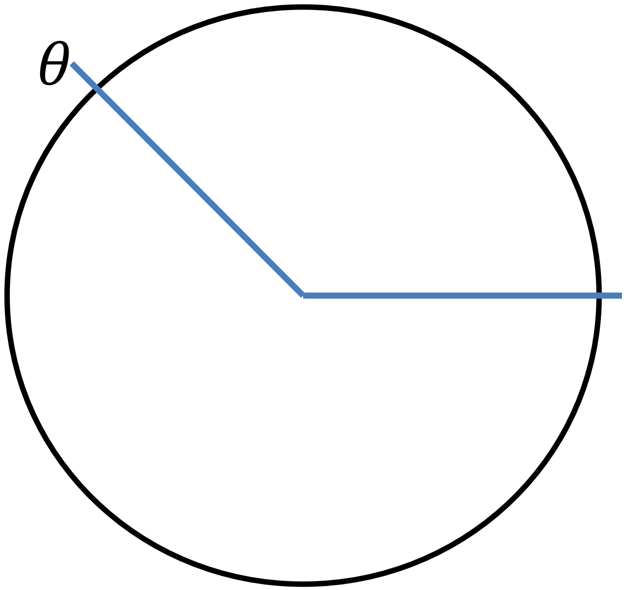
Robust Phase Estimation

- Don’t need perfect state preparation and measurement
- No additional gates, except Z-rotations
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Outline

- Motivation for Robust Phase Estimation
- Robust phase estimation
- Application to Calibration

Phase Estimation [Higgins et al. ‘09]

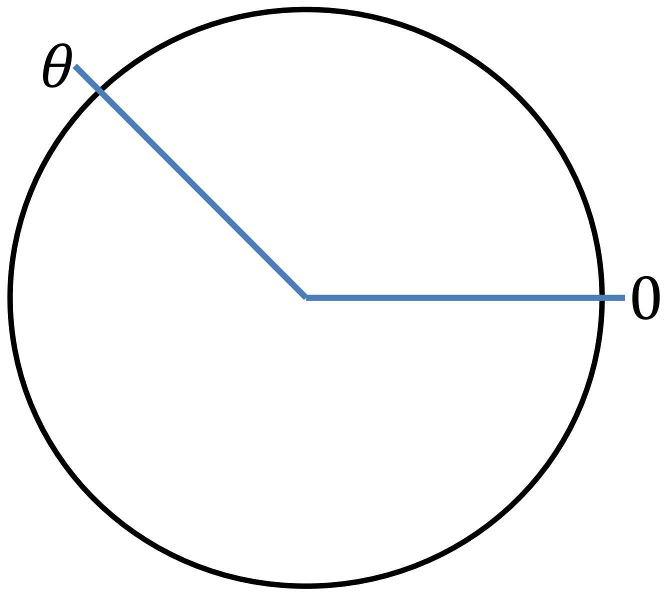


Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1+\sin k\theta}{2}, \qquad \frac{1+\cos k\theta}{2}$$

For k in $\mathbb{Z}$, each in time k

Phase Estimation [Higgins et al. '09]



Can sample from 2 binomial random variables with probability of "heads"

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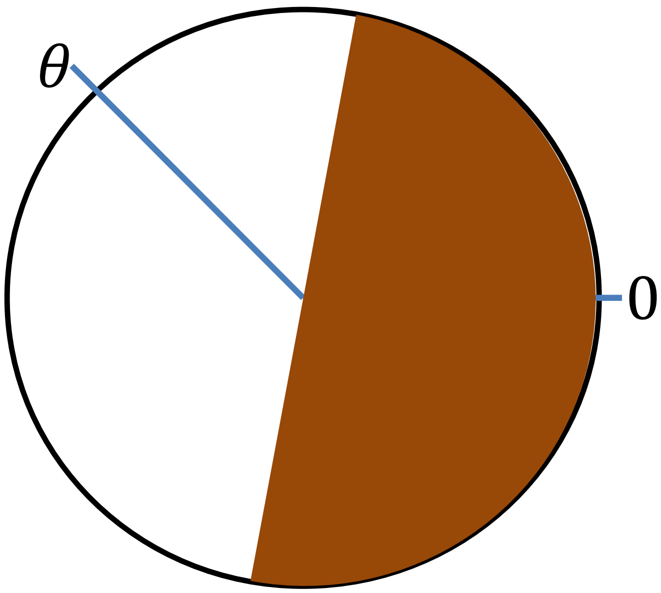
For k in $\mathbb{Z}$, each in time k

$k = 1$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{\sqrt{T}}$

Phase Estimation [Higgins et al. '09]



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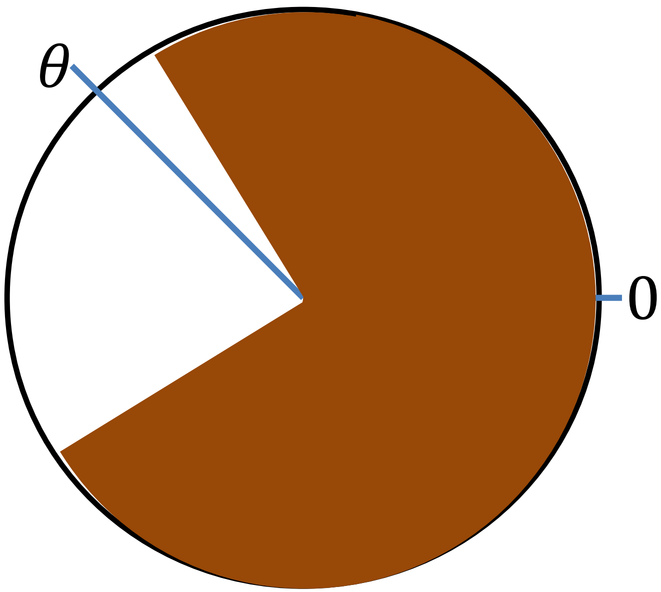


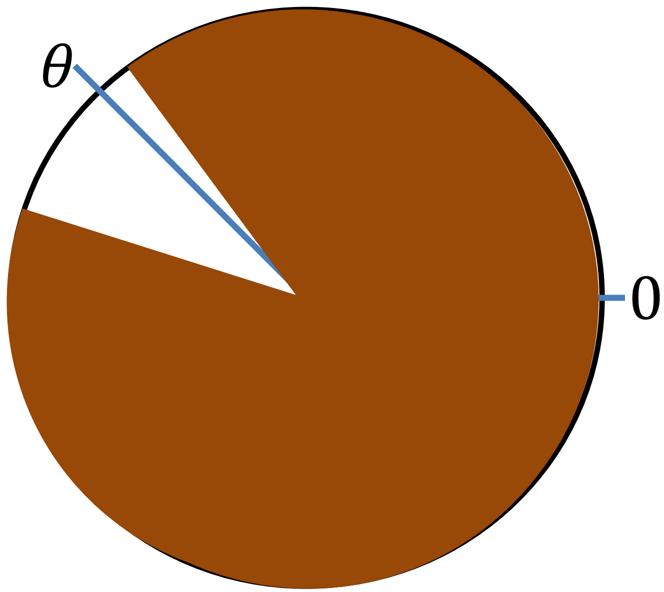
Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1+\sin k\theta}{2}, \qquad \frac{1+\cos k\theta}{2}$$

For k in $\mathbb{Z}$, each in time k

$k = 1 \quad k = 2$

Phase Estimation [Higgins et al. '09]



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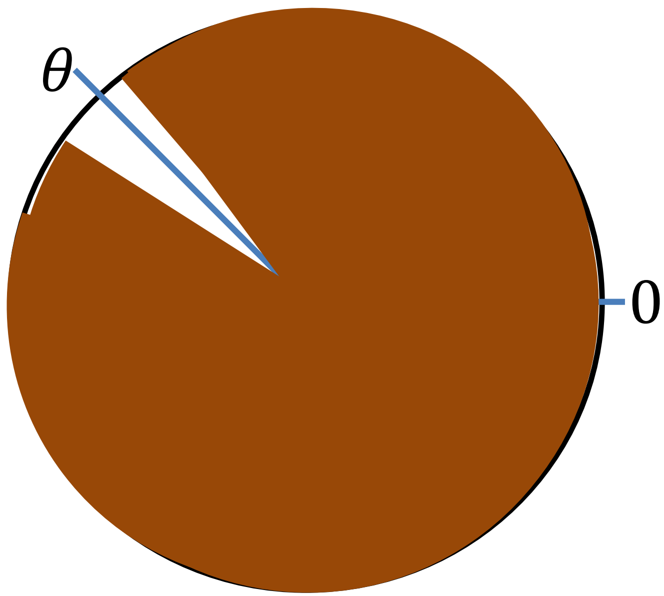
For k in $\mathbb{Z}$, each in time k

$k=1 \quad k=2 \quad k=4$

Phase Estimation [Higgins et al. '09]



Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1+\sin k\theta}{2}, \qquad \frac{1+\cos k\theta}{2}$$

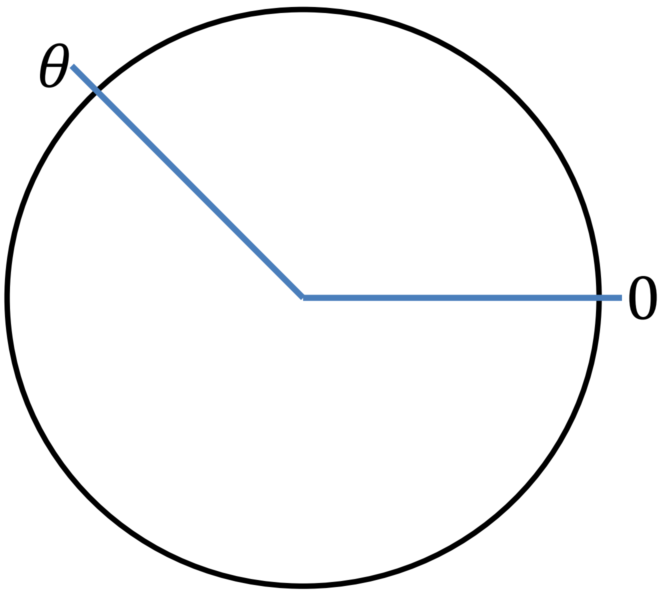
For k in $\mathbb{Z}$, each in time k

$k=1$ $\quad k=2$ $\quad k=4$ $\quad k=8$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$

Optimal – by information theory.

Robust Phase Estimation

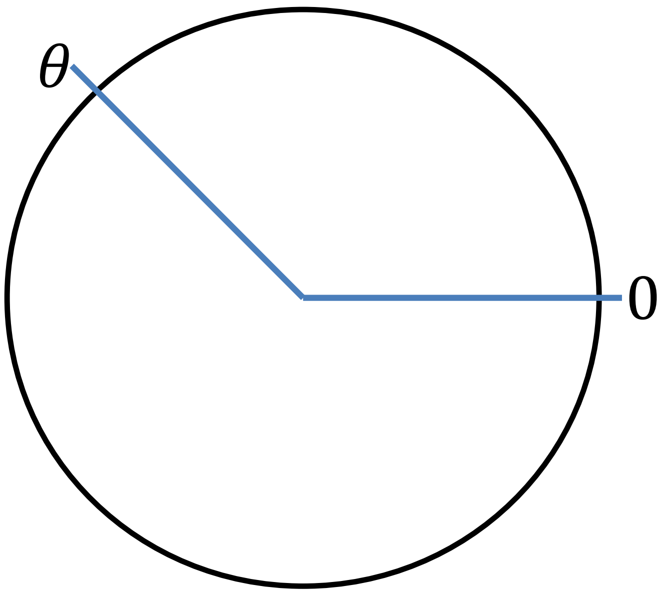


Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

For k in $\mathbb{Z}$, each in time k

Robust Phase Estimation

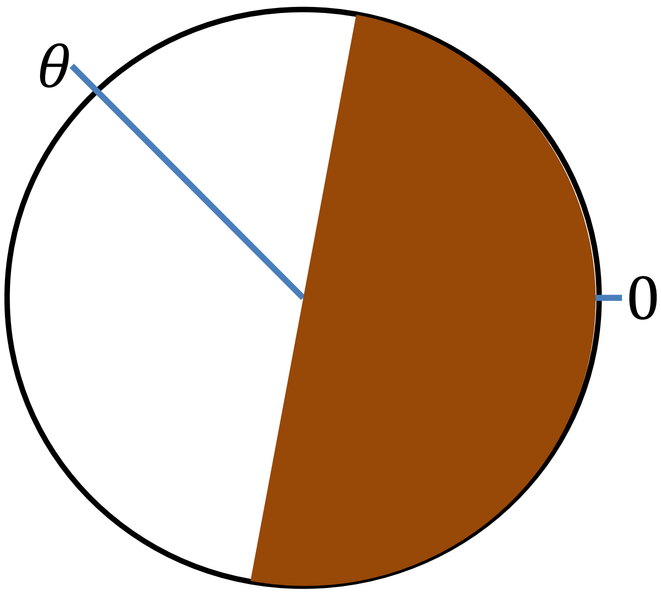


Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1+\sin\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos\theta}{2}+\delta_{k2}$$

Using only $k = 1$ can’t get an accurate estimate!

Robust Phase Estimation



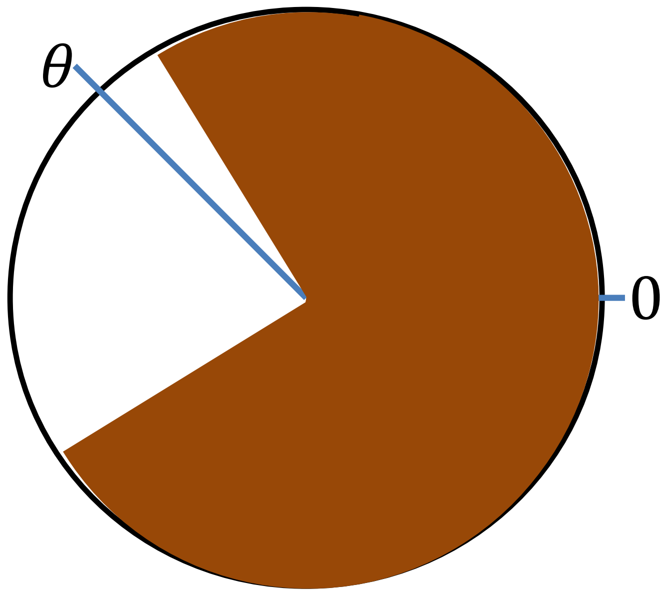
Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1 + \sin k\theta}{2} + \delta_{k1}, \qquad \frac{1 + \cos k\theta}{2} + \delta_{k2}$$

For k in $\mathbb{Z}$, each in time k

$k = 1$

Robust Phase Estimation



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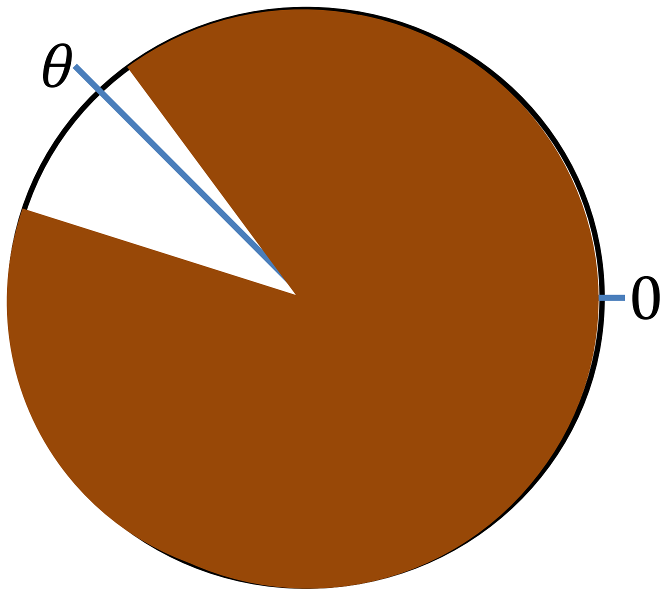
Robust Phase Estimation



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For k in $\mathbb{Z}$, each in time k

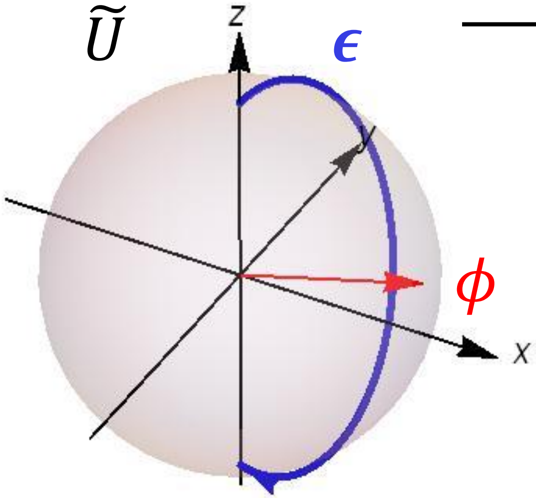
$k=1 \quad k=2 \quad k=4$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$,
as long as $|\delta_k| < .35$ for all k.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities like:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

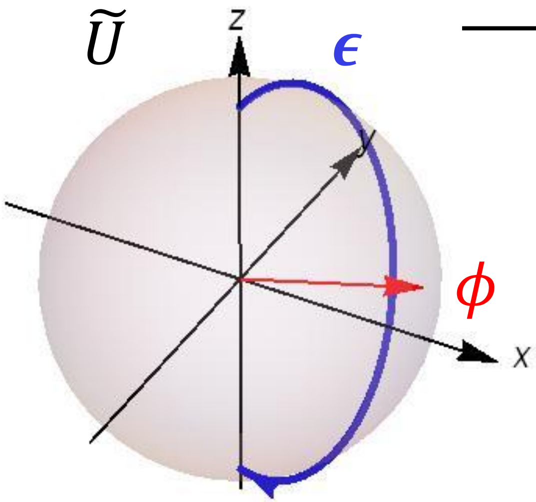


Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

$\widetilde{U}$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

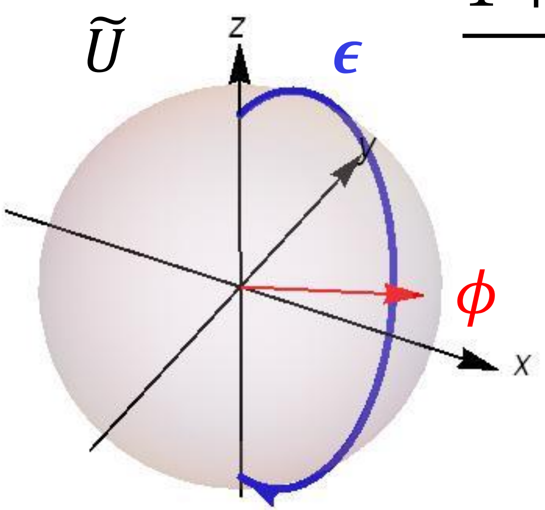
$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

$A=\pi(1+\epsilon)$ is total amplitude of rotation

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2 = \frac{1+\cos kA}{2} + \sin^2\frac{kA}{2}\sin^2\phi$$

$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2 = \frac{1+\sin kA}{2} - \sin kA \sin^2\frac{\phi}{2}$$

Size less $< \phi^2$

$A = \pi(1+\epsilon)$ is total amplitude of rotation

Robust Phase Estimation for Gate Estimation

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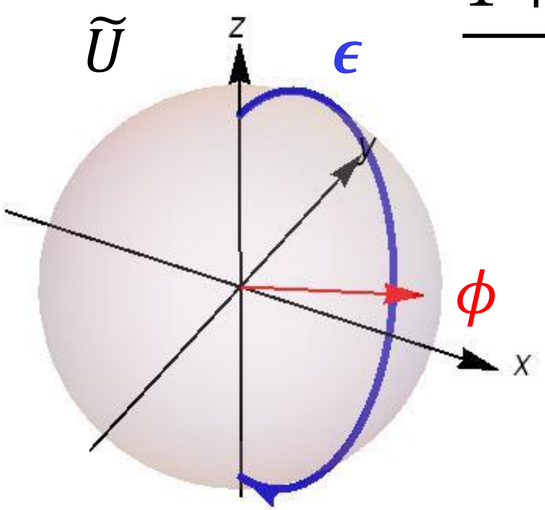
$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2 = \frac{1+\cos kA}{2} + \sin^2\frac{kA}{2}\sin^2\phi$$

$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2 = \frac{1+\sin kA}{2} - \sin kA\sin^2\frac{\phi}{2}$$

Don't have perfect state prep and measurement? OK! Just add to δ error.

Robust Phase Estimation for Gate Estimation

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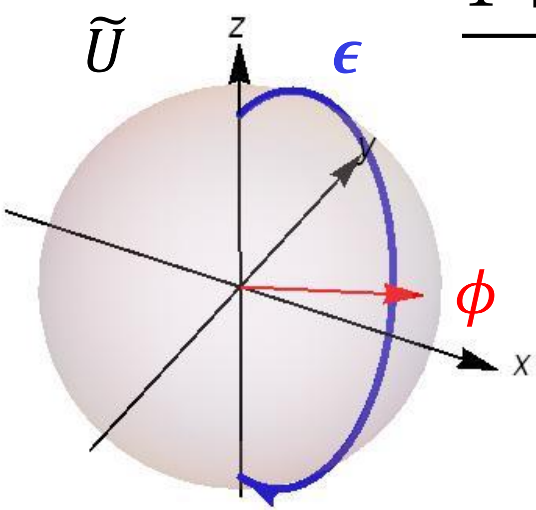
$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

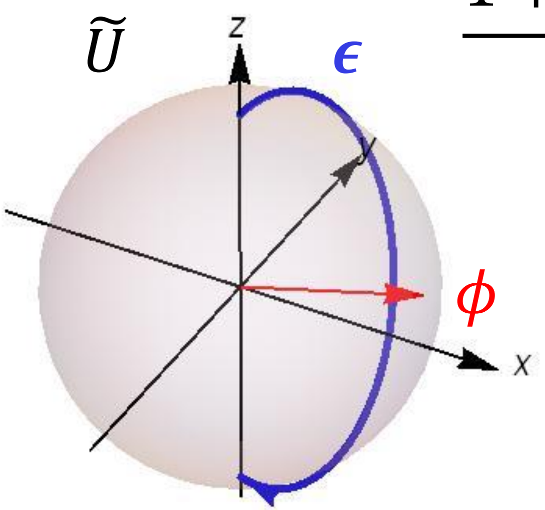
$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

Have depolarizing errors? OK! Just add to δ errors.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2 = \frac{1+\cos kA}{2} + \sin^2\frac{kA}{2}\sin^2\phi$$

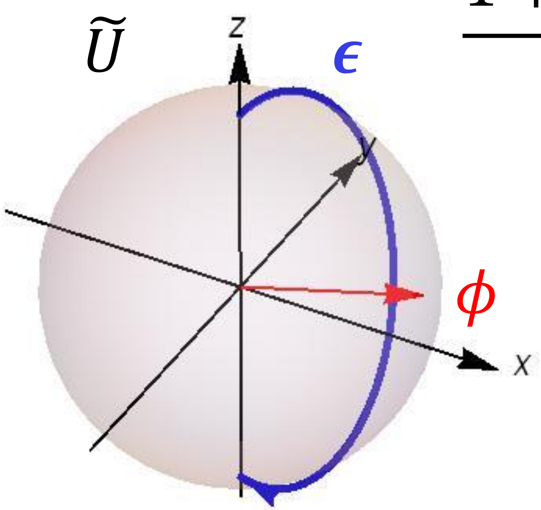
$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2 = \frac{1+\sin kA}{2} - \sin kA\sin^2\frac{\phi}{2}$$

Heisenberg limited! Estimate of ϵ with standard deviation $\sigma(\epsilon)\sim\frac{1}{N}$, where N is the number of times $\widetilde{U}$ is applied.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

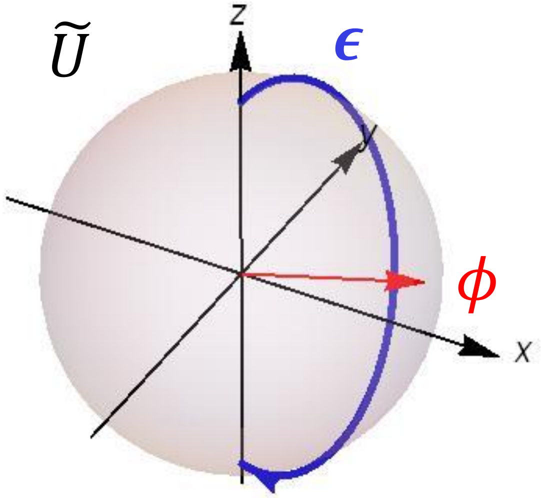


$$\left|\langle 0|(Z\tilde{U}Z\tilde{U})^k|0\rangle\right|^2 \approx \frac{1+\cos mk\phi}{2}+O(\epsilon^2)$$

$$\left|\langle 0|(Z\tilde{U}Z\tilde{U})^k|\rightarrow\rangle\right|^2 \approx \frac{1+\sin mk\phi}{2}+O(\epsilon^2)$$

Heisenberg limited! Estimate of ϕ with standard deviation $\sigma(\phi) \sim \frac{1}{N}$, where N is the number of times $\tilde{U}$ is applied.

Recap:



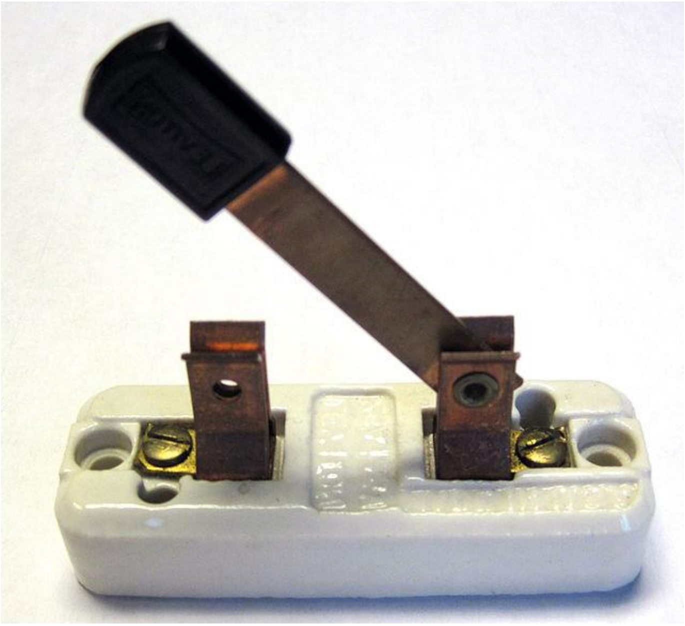
Robust Phase Estimation

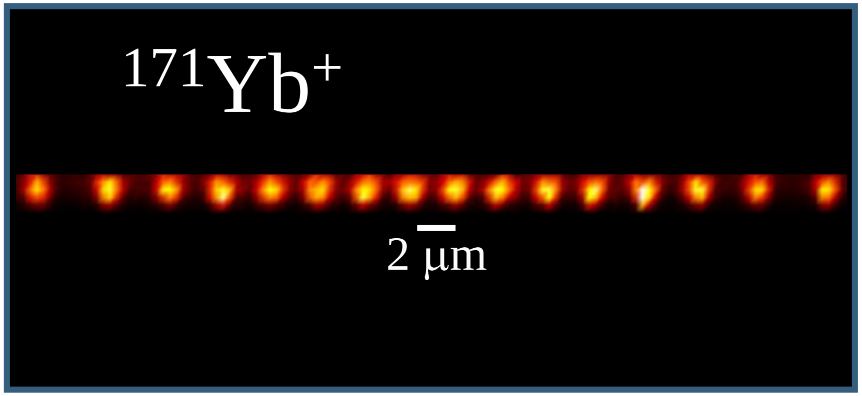
- Don't need perfect state preparation and measurement
- No additional gates, except Z-rotations
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Open Questions

- Multi-Qubit Operations?
- No perfect Z-rotation?
- Connection to Randomized Benchmarking

Think this might be useful?





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