Robust Characterization of Quantum Processes

Shelby Kimmel

Center for Theoretical Physics, MIT

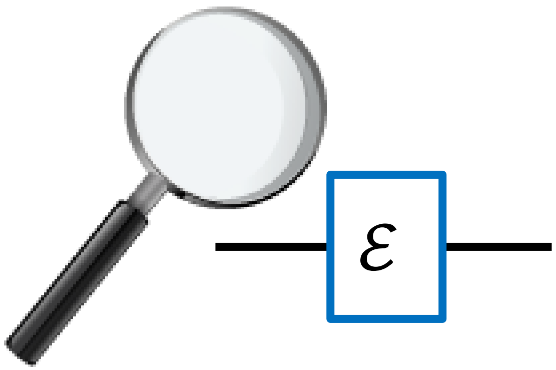
Marcus da Silva, Colm Ryan, Blake Johnson, Tom Ohki
Raytheon BBN Technologies

JQI – Monday Dec. 16

Why don’t we have a working quantum computer?

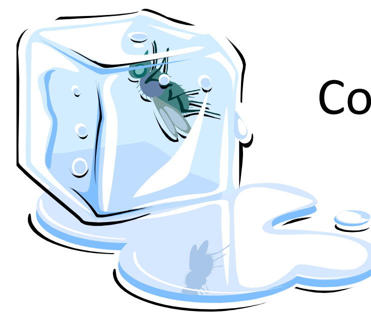
Too Many Errors

Can Improve Operations with Better Characterization of Errors

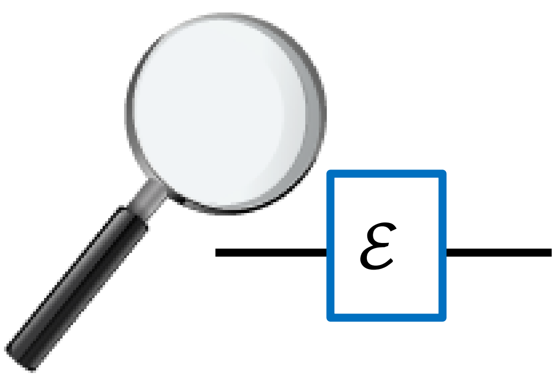


“Depolarizing error”

Improvement to Computer



Cooling

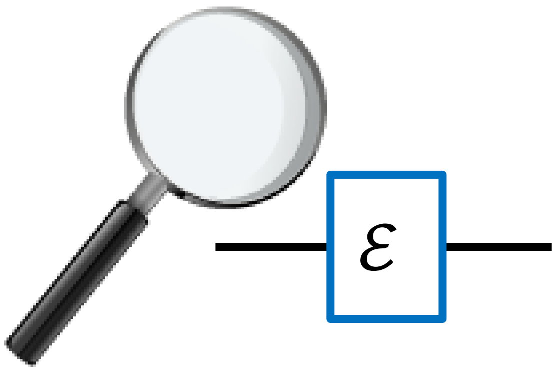


“Extra rotation around z-axis”

Improvement to Computer

Magnetic Shielding

Can Improve Error Correcting Codes with Better Characterization of Errors

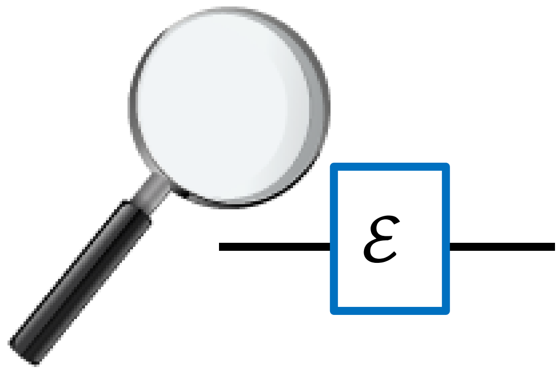


"Non local, correlated error"

Improvement to Error Correcting Code

?

Standard Techniques Have Problems



Need nearly perfect state preparation, measurement and other operations. Otherwise systematic errors give inaccurate or even invalid results.

Not "robust"

Robust Techniques

- Gate Set Tomography Procedures [Stark '13, Blume-Kohout et al. '13, Merkel et al. '12]
  - Characterizes many processes at once
- Randomized Benchmarking (RB) [Emerson et al. '05, Knill et al. '08, Magesan et al. '11, '12]
  - Can only characterize ~~1~~ parameter of ~~1~~ type of process.
    almost all any
  - Can efficiently test performance of a universal gate set.

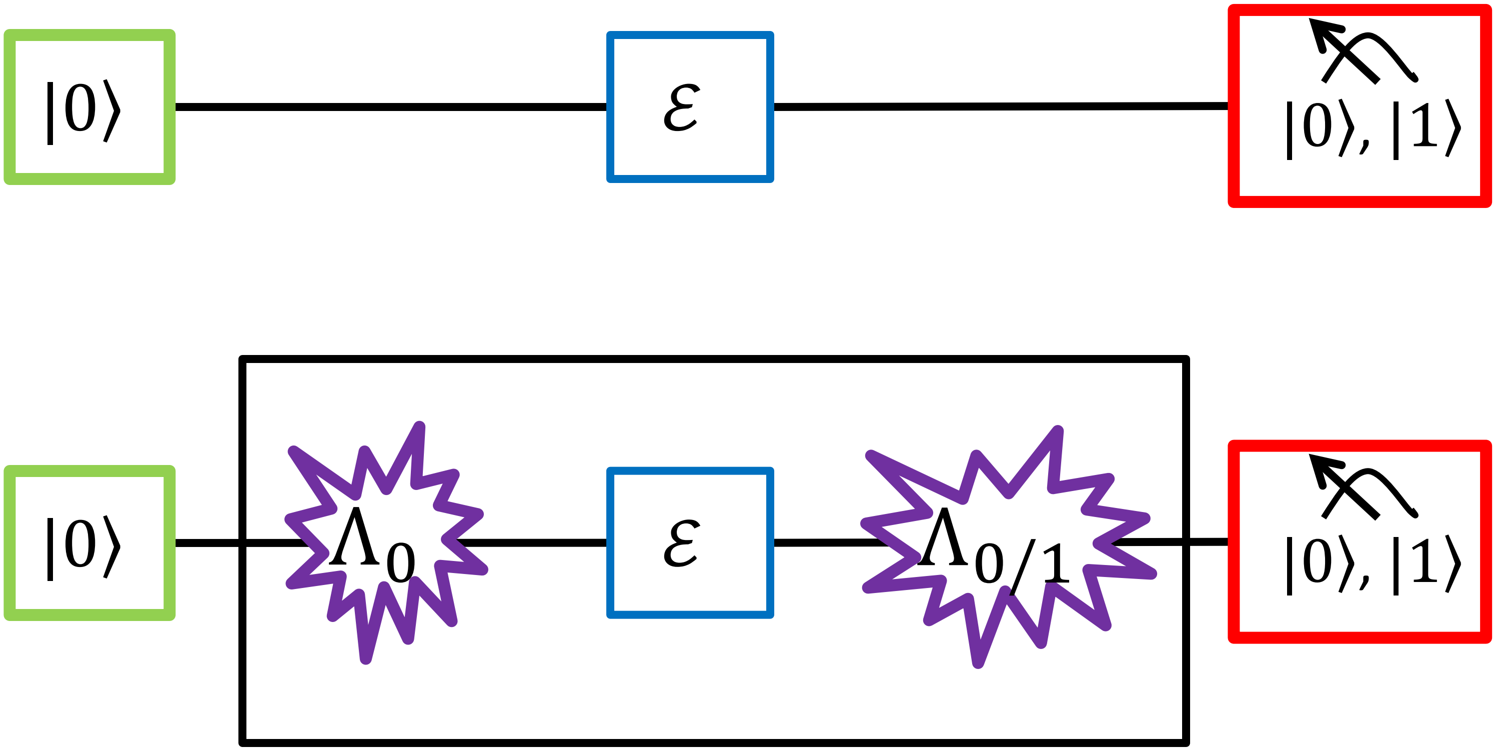
Outline

- **Background**:
  - Issues with standard process characterization
  - Randomized benchmarking framework, challenges of current implementation
- **Our Results**:
  - Robust characterization of unital part of any process
  - Efficient bound on average fidelity of universal gate set.

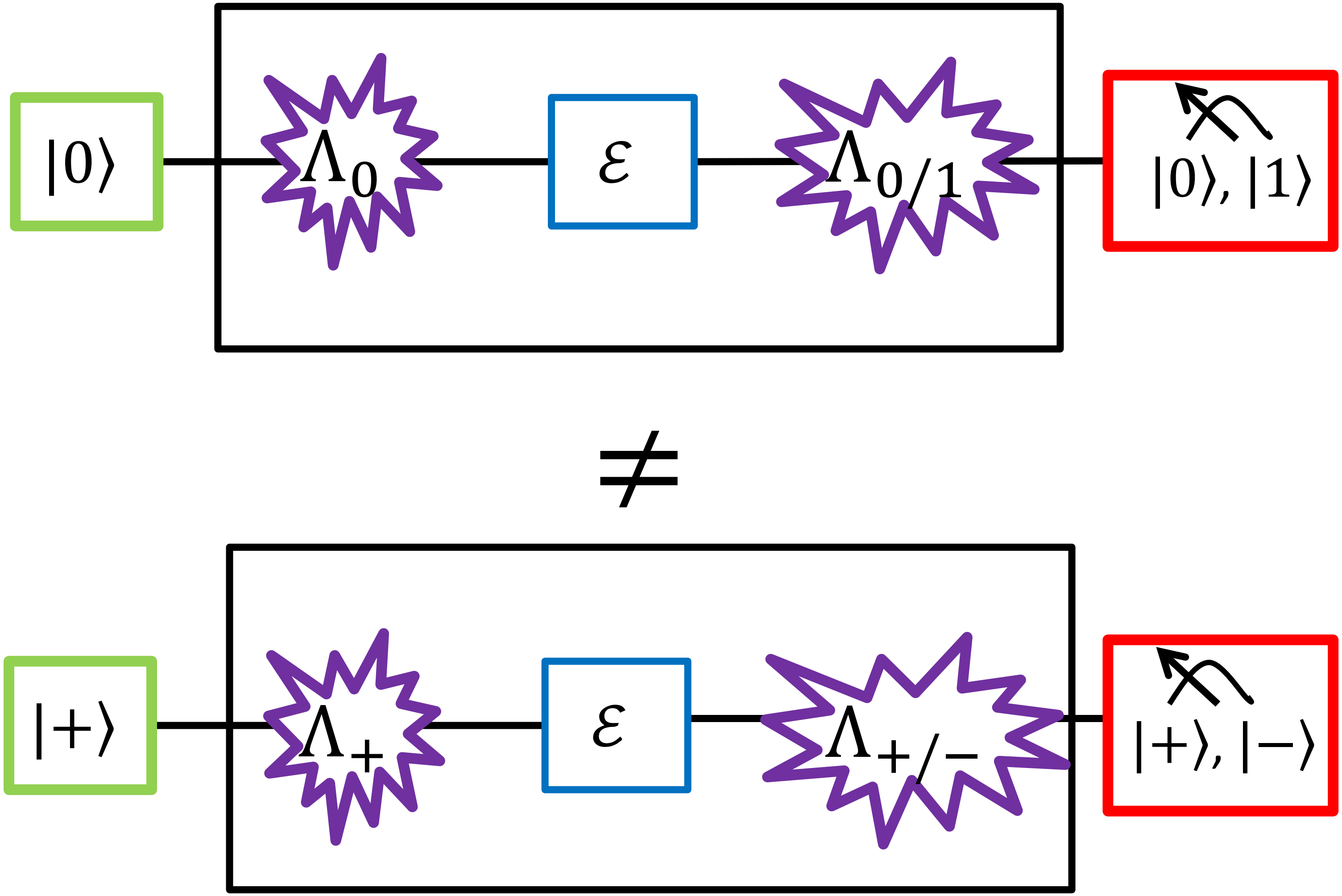
Quantum Process (Map)

- Completely positive trace preserving (CPTP) map = any process that takes valid quantum states to valid quantum states.
- E.g. unitary, depolarizing process, dephasing process, amplitude damping process
- n qubits, $O(16^n)$ free parameters

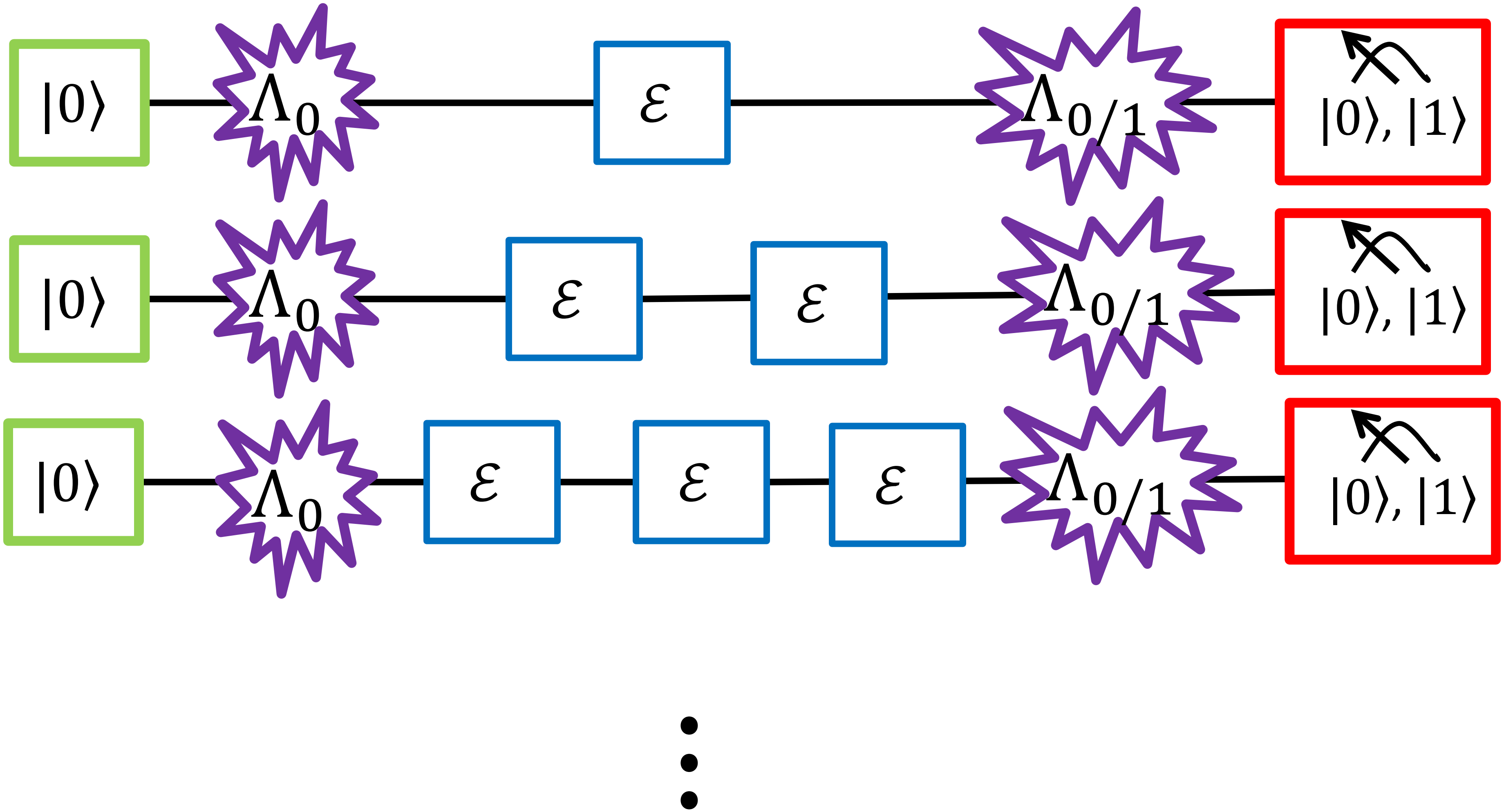
Problem with Standard Process Tomography



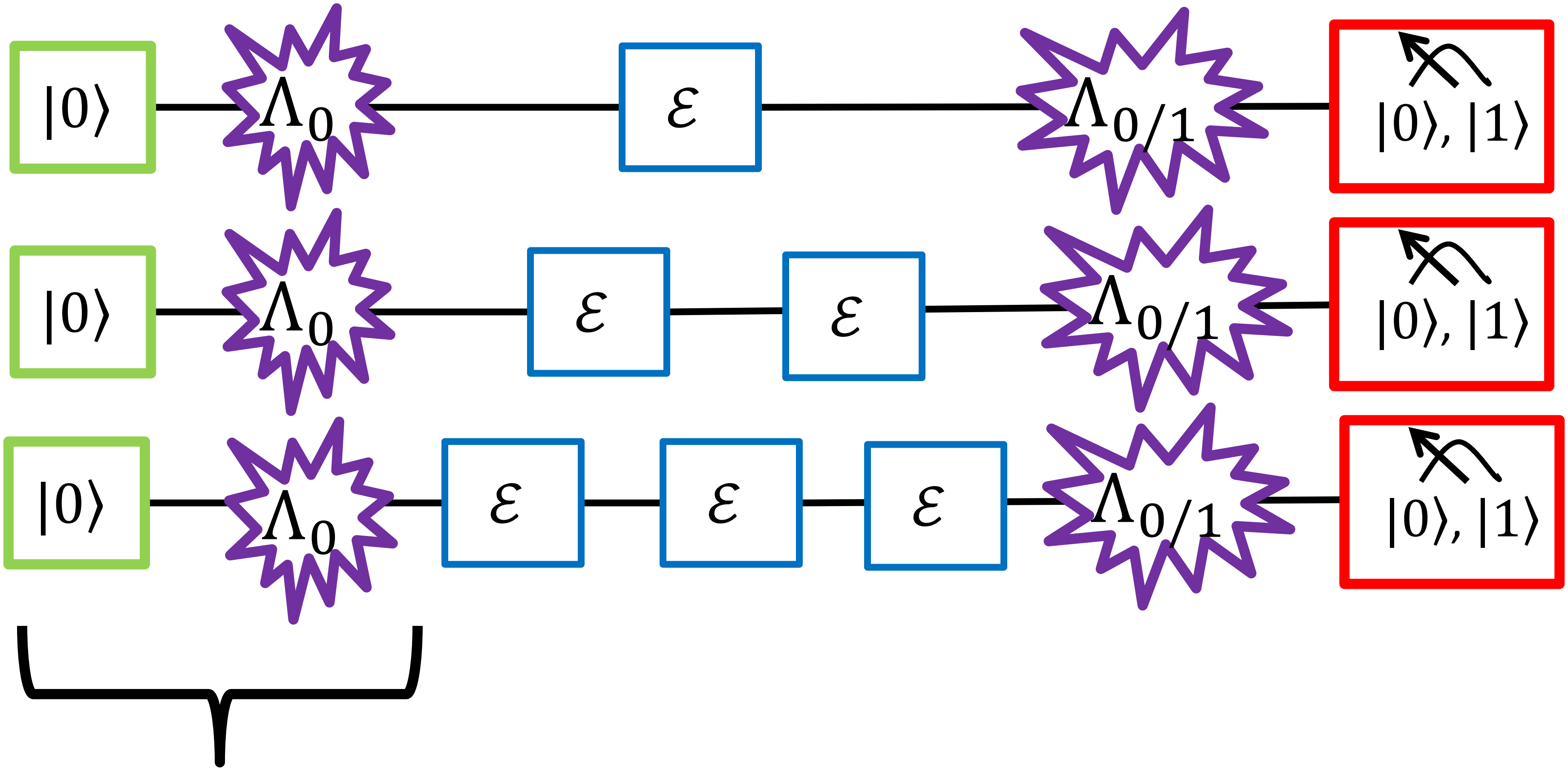
Problem with Standard Process Tomography



Repeated Application

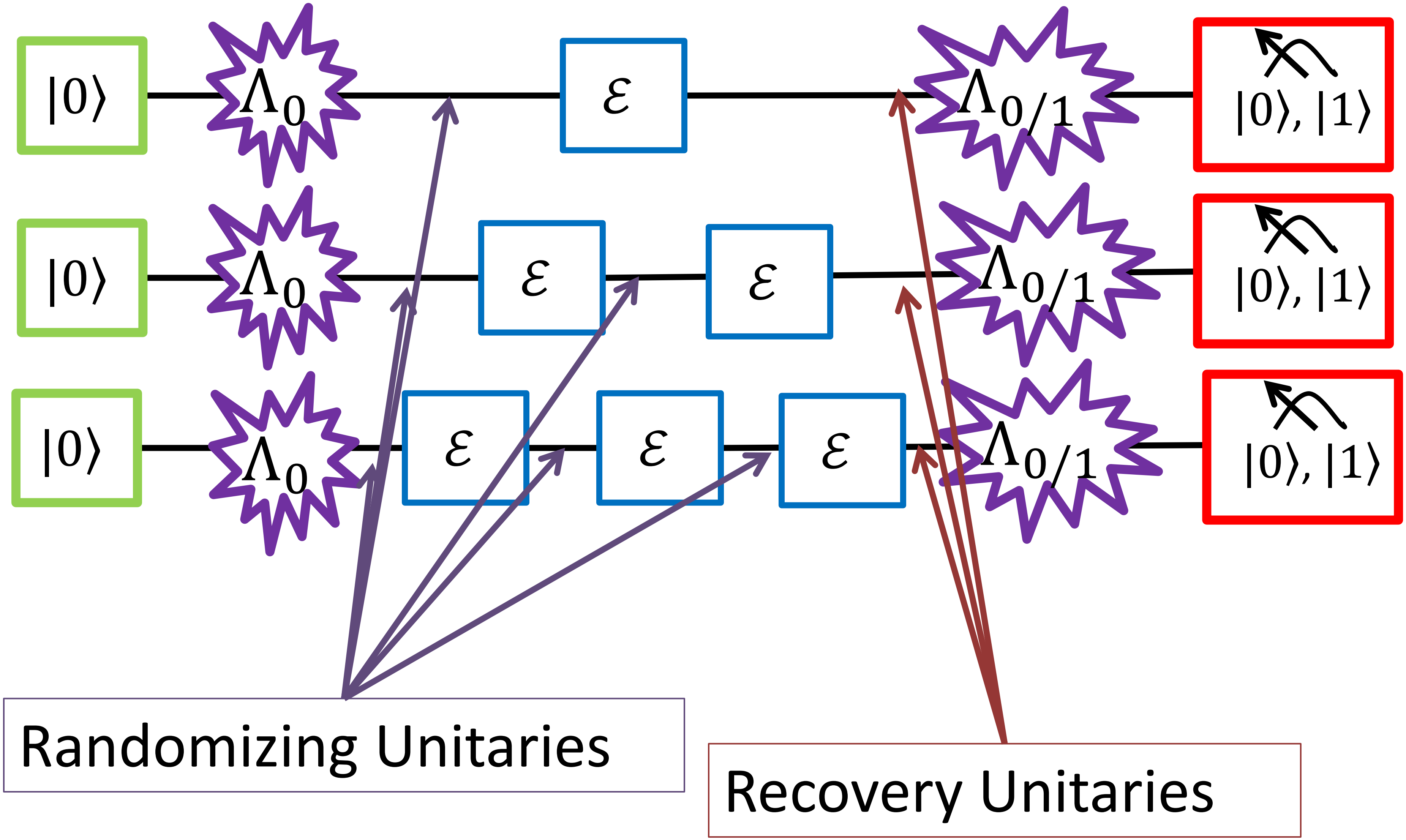


Repeated Application

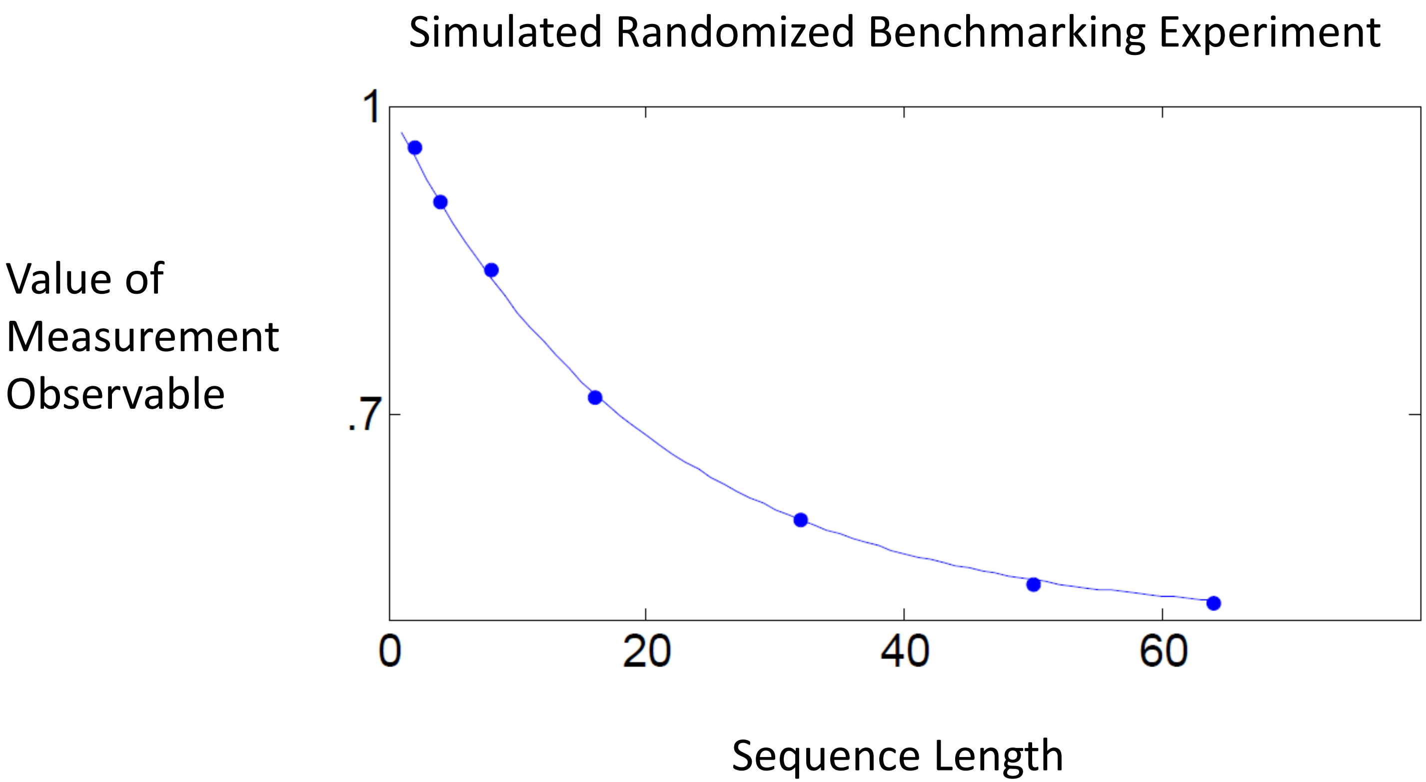


If eigenstate of $\mathcal{E}$, will only see how $\mathcal{E}$ acts on *this* state

Randomized Benchmarking

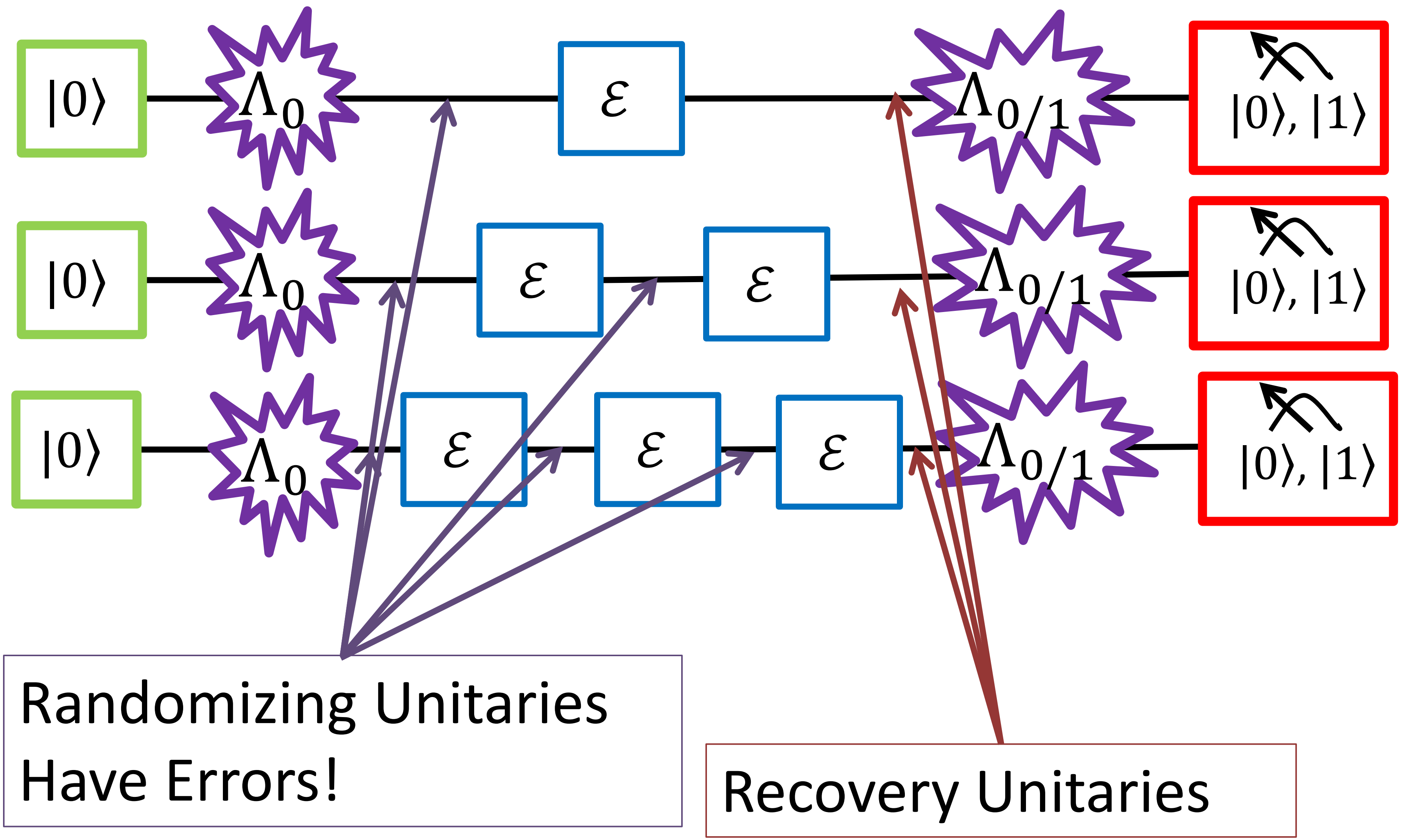


Randomized Benchmarking



Decay constant depends on one parameter of $\mathcal{E}$

Randomized Benchmarking



Two Issues with RB

1. How can we extract more than just 1 parameter?
2. How can we deal with errors on the randomizing operations?

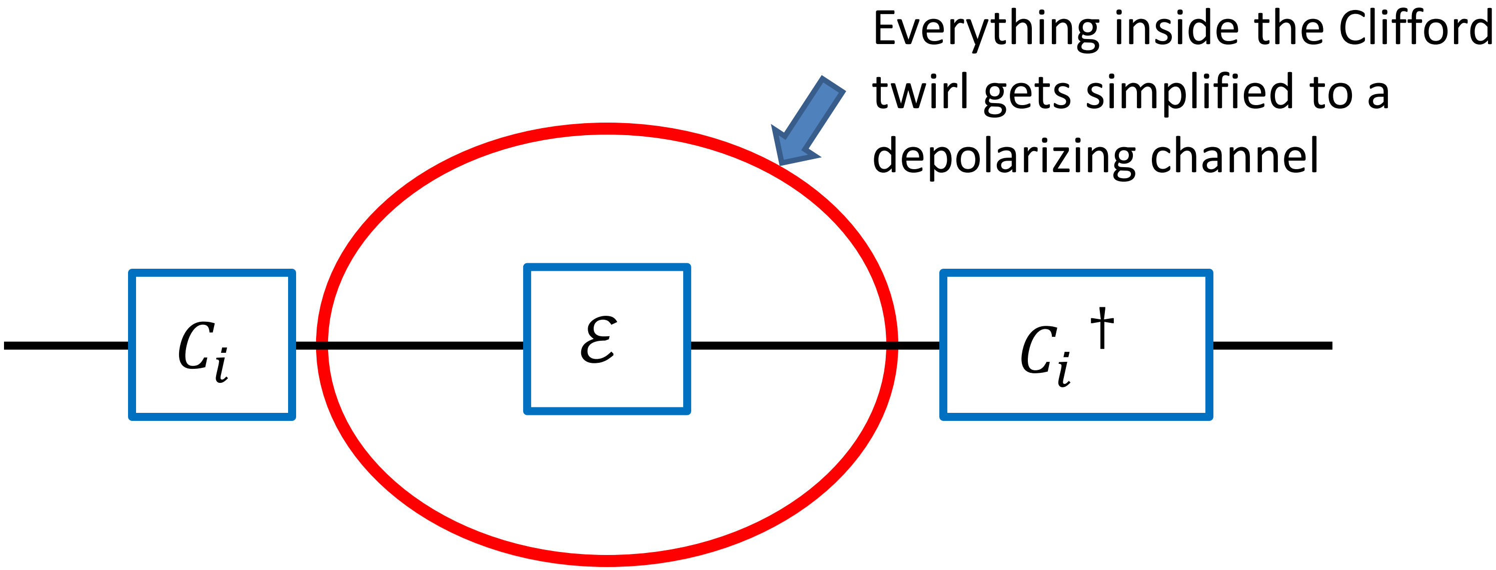
Randomizing Operation: Clifford Twirl

$$\frac{1}{|C_i|} \sum_{c_i \text{ in Cliffords}} C_i^{\dagger} \circ \mathcal{E} \circ C_i(\rho) = (1-q)\rho + q\frac{\mathbb{I}}{d}$$

Result is depolarizing channel (very simple process) that depends on only one parameter of $\mathcal{E}$: Average fidelity of $\mathcal{E}$ to the identity

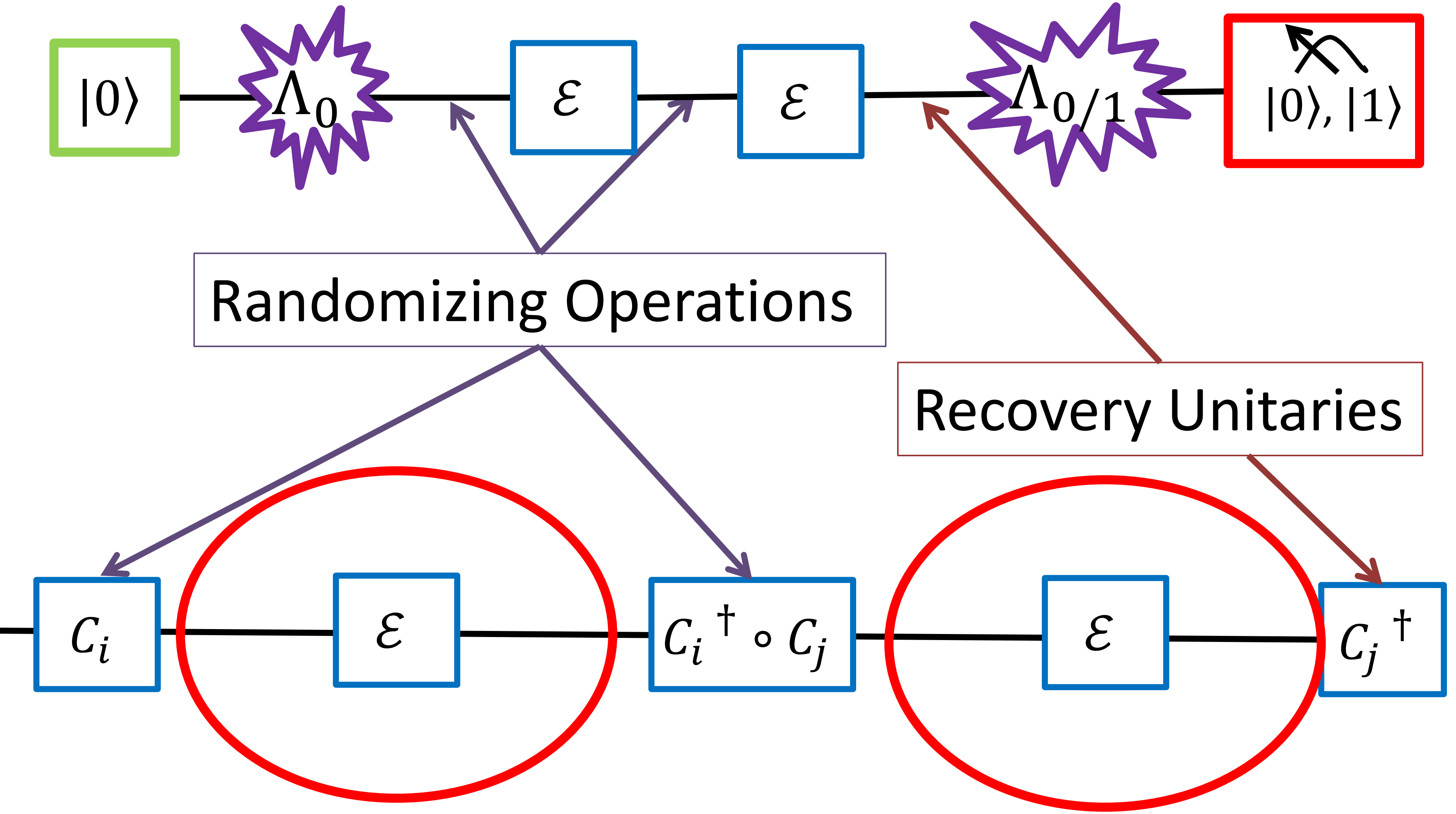
Average fidelity of $\mathcal{E}$ = $\int d\,|\psi\rangle\ \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle$

Randomizing Operation: Clifford Twirl

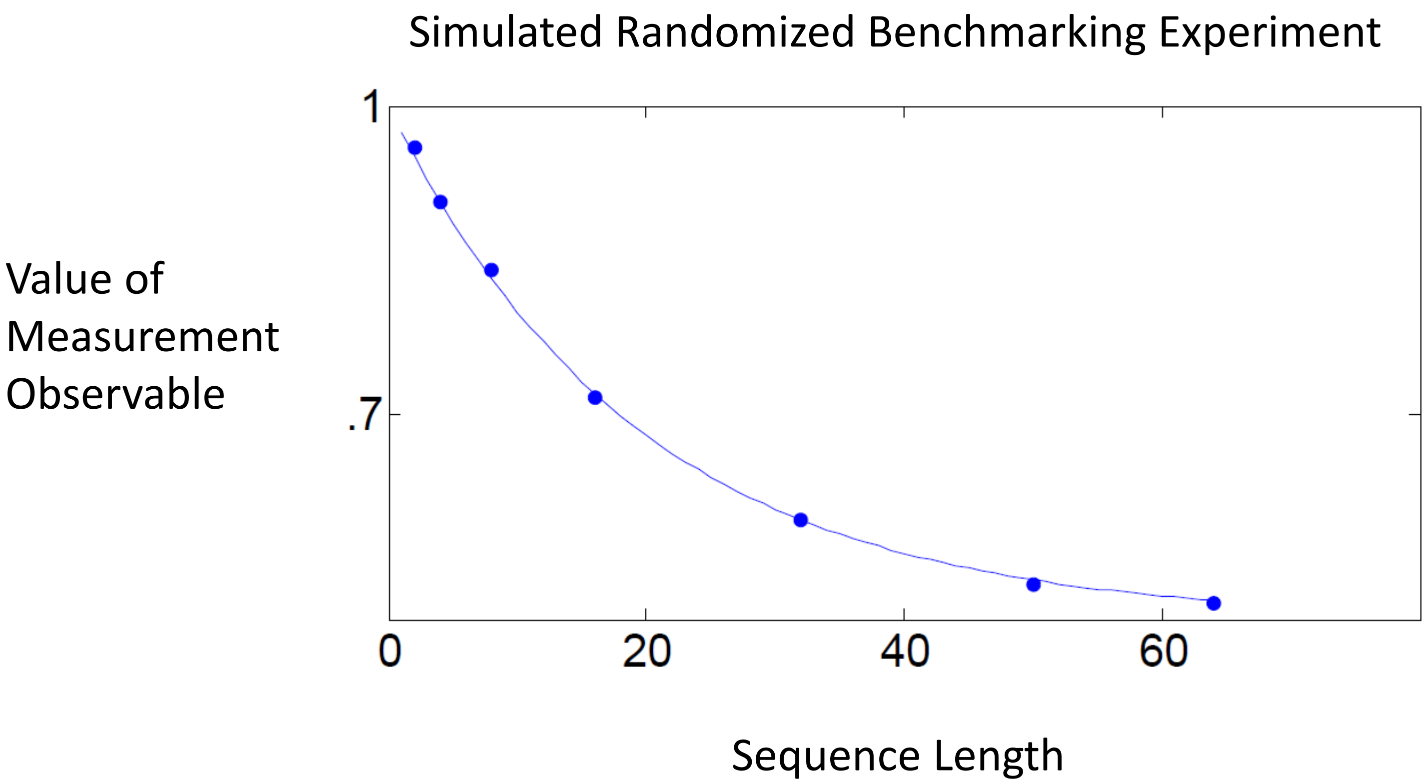


To implement (approximately), repeat many times, each time randomly choosing C_i, and average results

Randomizing Operation: Clifford Twirl

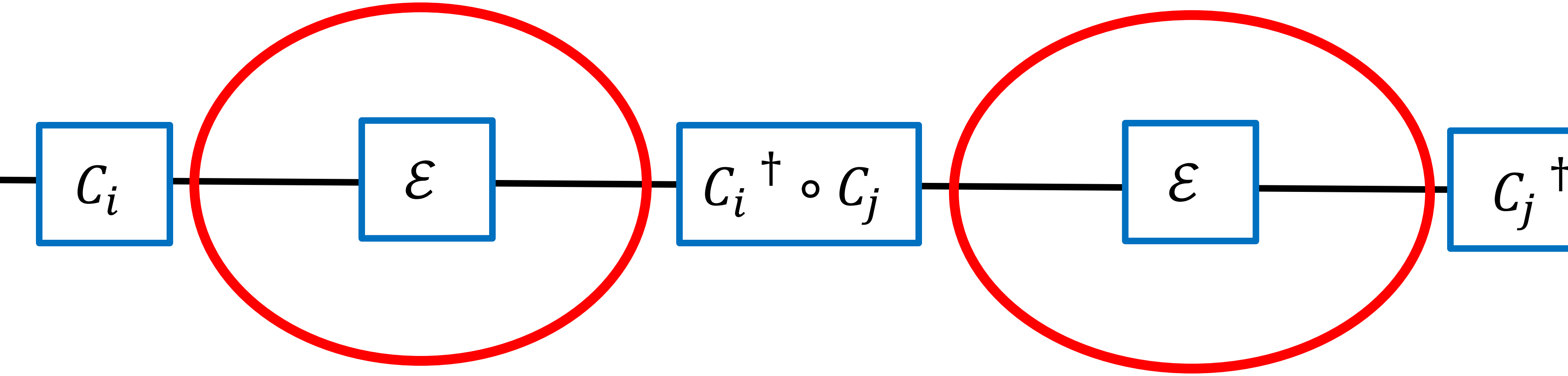


Randomizing Operations



Decay constant depends on 1 parameter of $\mathcal{E}$: Average fidelity of $\mathcal{E}$ to the identity.

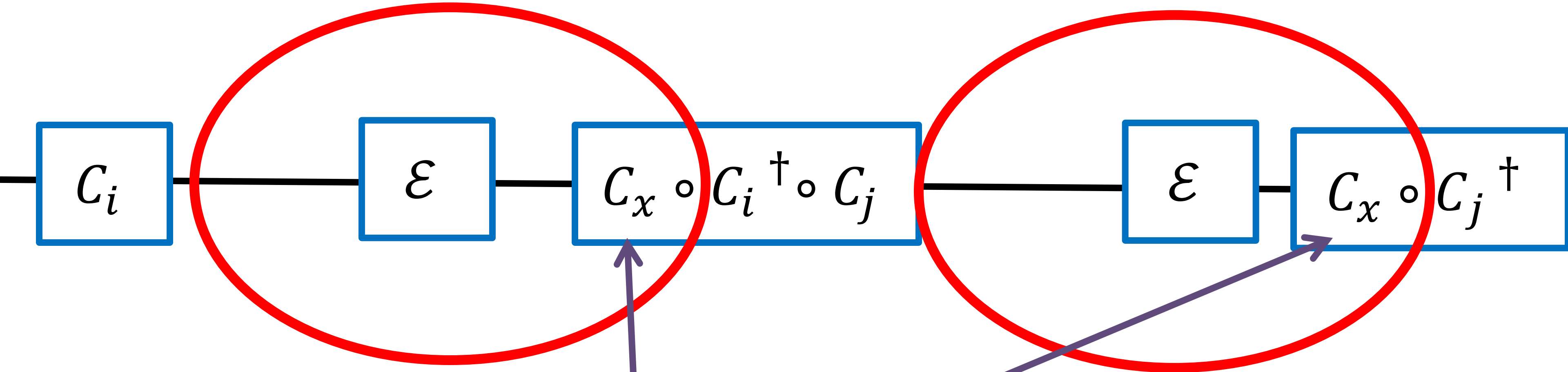
1. Extracting More Information



Twirl simplifies too much!

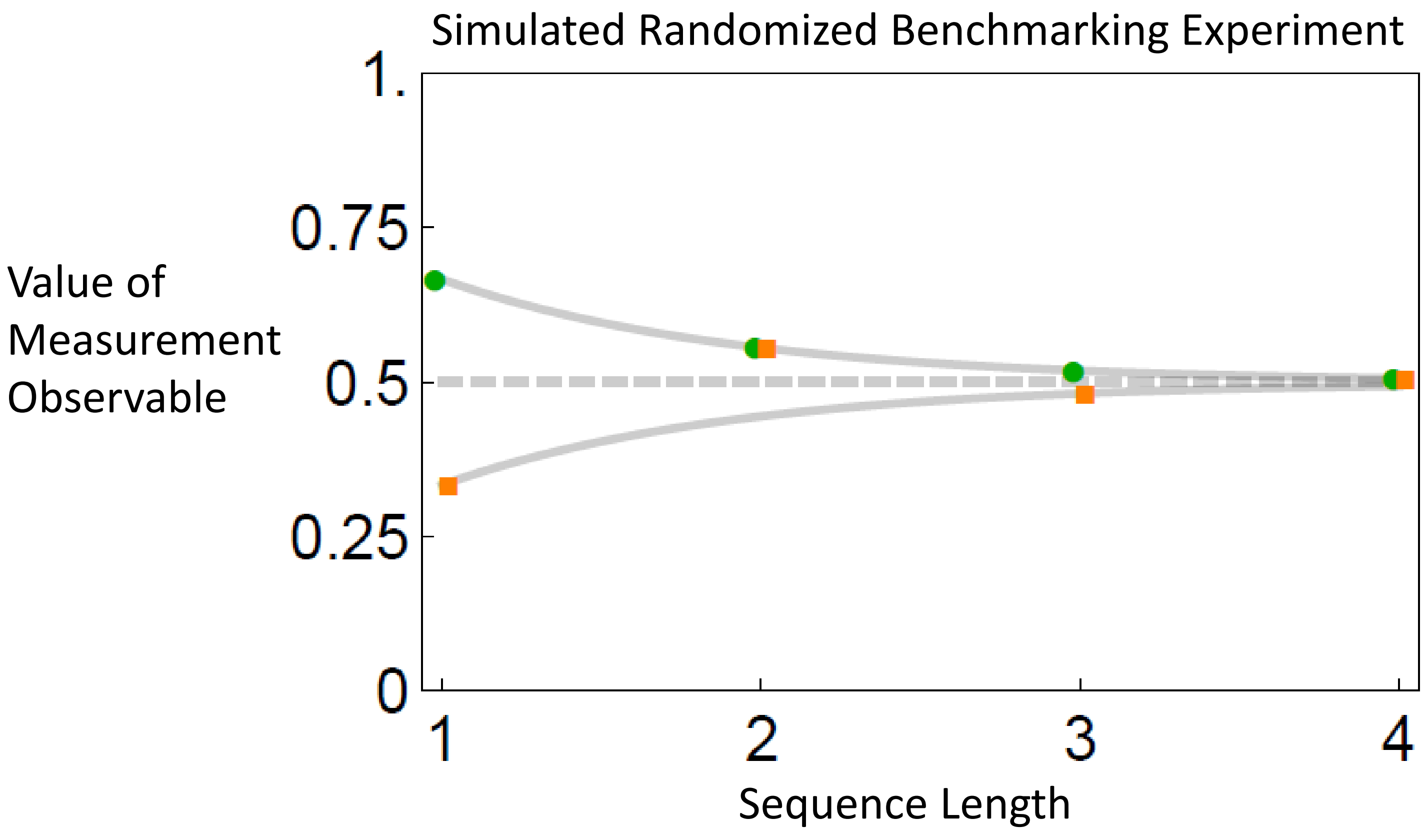
- no twirl
- stick additional information inside twirl

1. Extracting More Information



C_x is fixed – not random. The same C_x is applied in each twirl.

1. Extracting More Information



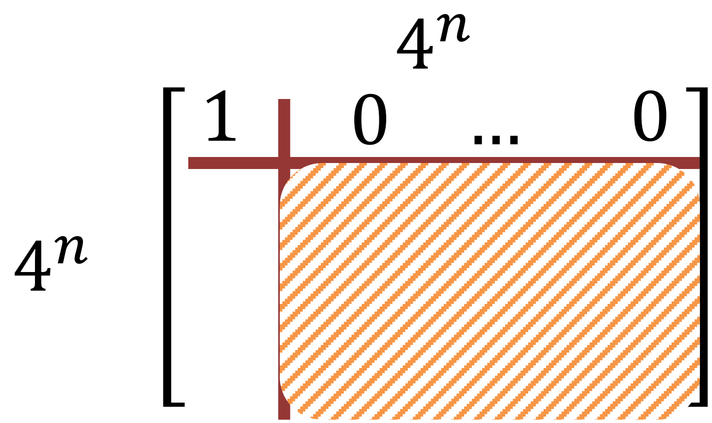
Decay constant depends on 1 parameter of $\mathcal{E}$:

Average Fidelity of $\mathcal{E}$ to $C_x{}^{\dagger}$ (can have fast decays)

1. Extracting More Information

CPTP map: $16^n - 4^n$ parameters for n-qubit map

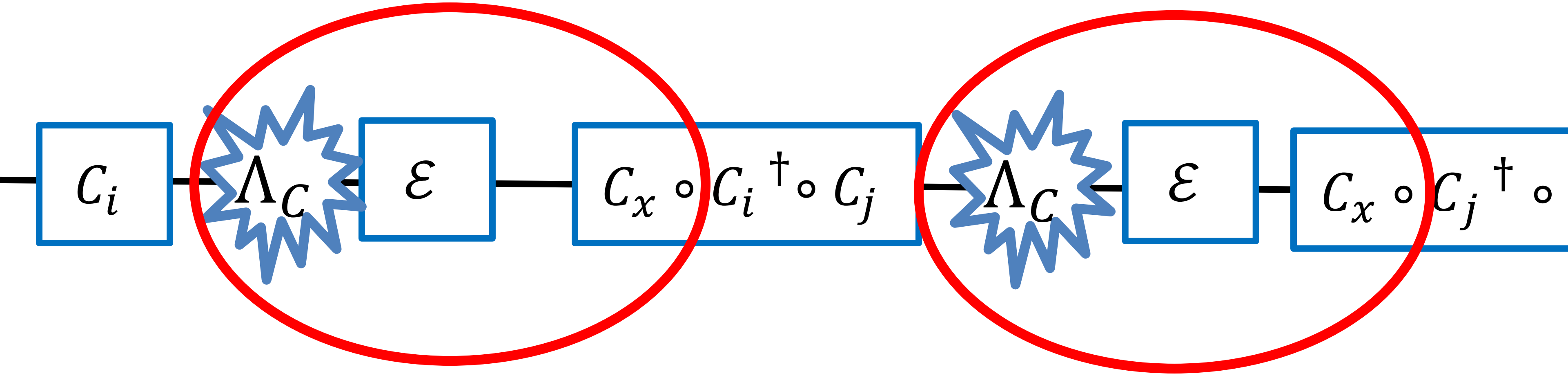
To compose two maps, just multiply matrices!



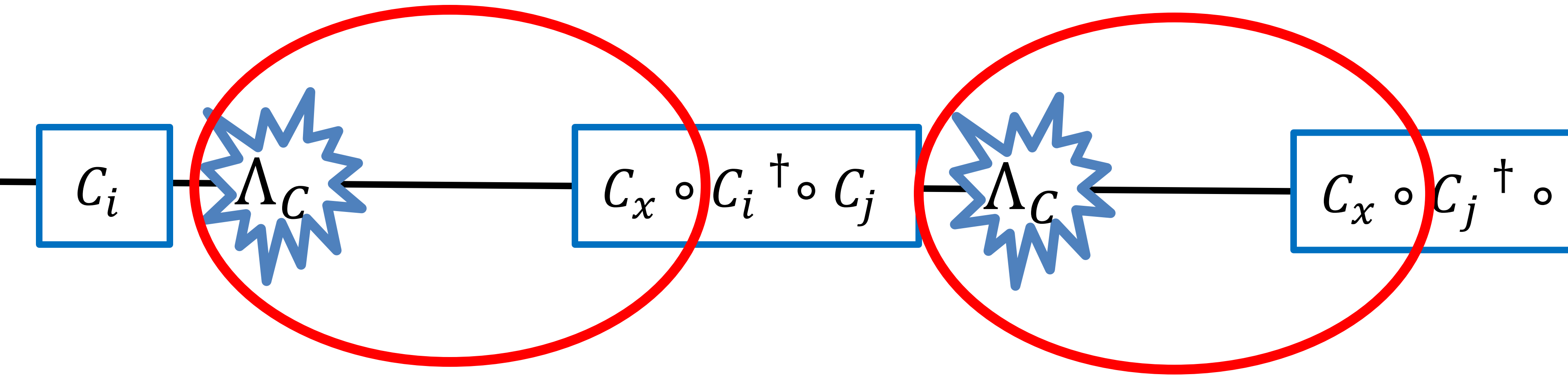
- Vectors V span a subspace S
- Learn inner product between V and unknown vector u
- Can learn projection of u onto S

- Cliffords span unital part
- Learn inner product between Cliffords and $\mathcal{E}$
- Learn projection of $\mathcal{E}$ onto unital subspace

2. Dealing with Errors

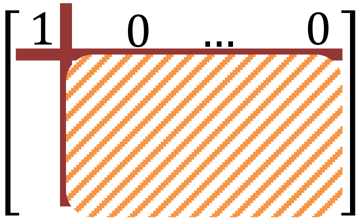


2. Dealing with Errors



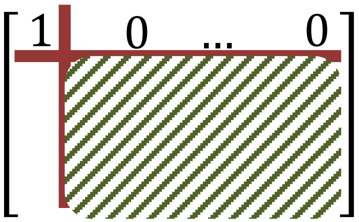
2. Dealing with Errors

almost complete characterization of Λ_C



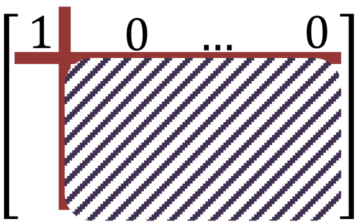
$+$

almost complete characterization of $\Lambda_C \circ \mathcal{E}$



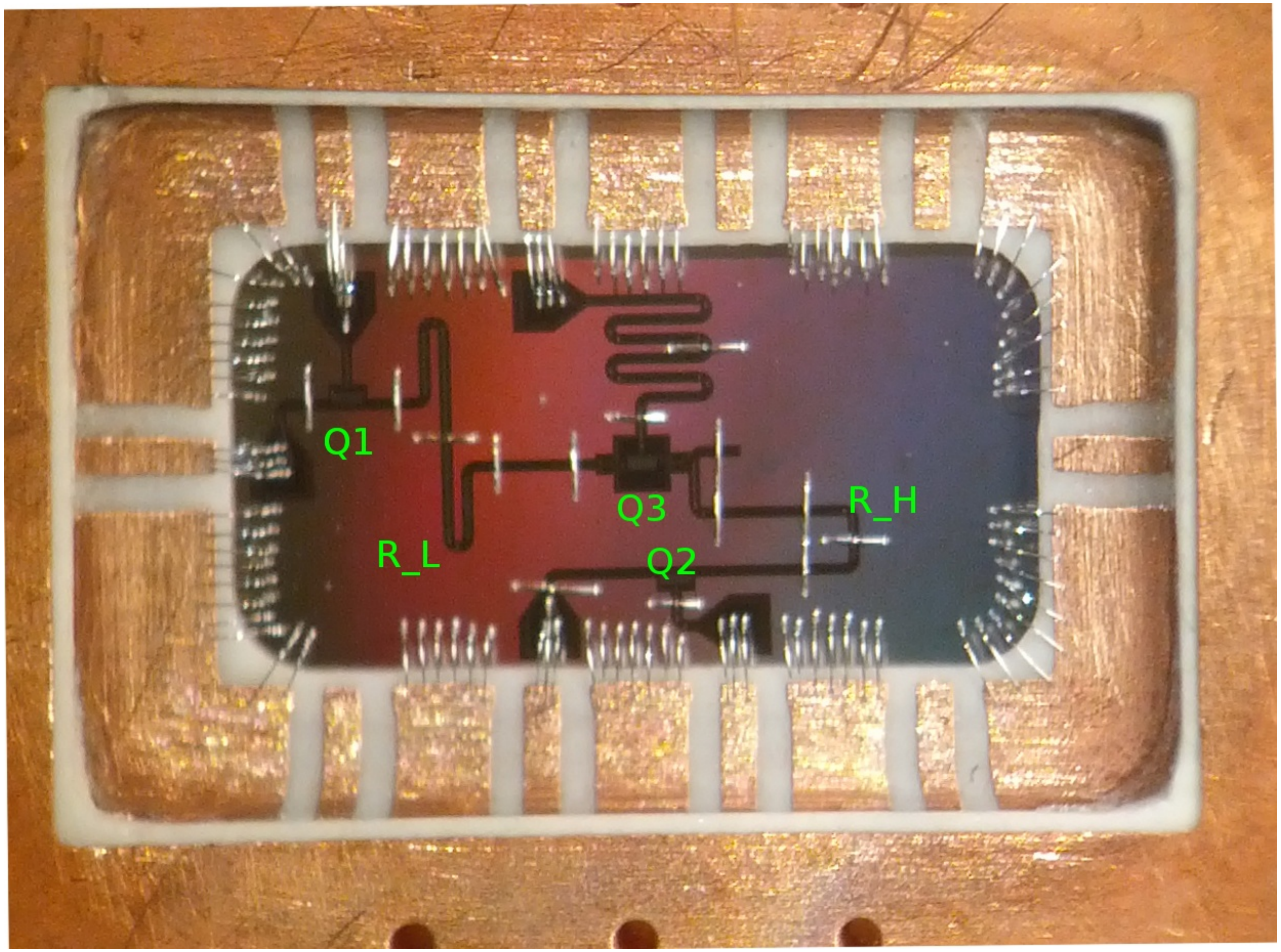
$=$

almost complete characterization of $\mathcal{E}$



All without the systematic errors of previous procedures!

Experimental Implementation

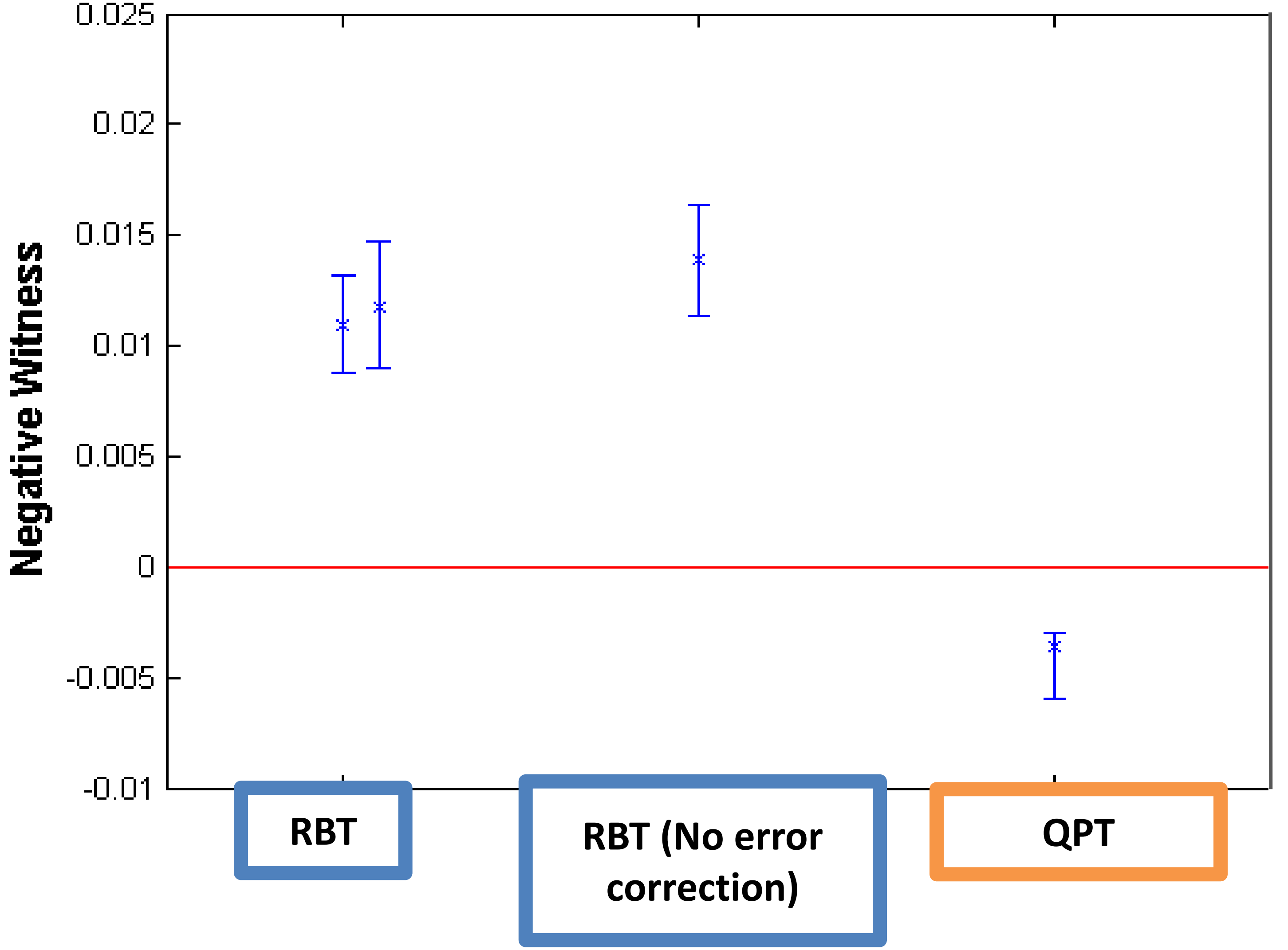


Negative Witness Test [Moroder et al. ‘13]

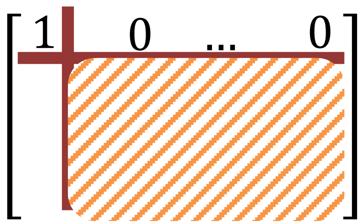
- To be a valid quantum process, must be trace preserving and completely positive
- Complete positivity = in Choi representation, all eigenvalues must be positive

- Negative witness test:
  - Look at value of smallest eigenvalues of reconstructed map in Choi representation.
  - If negative, BAD!

Negative Witness Test for Hadamard



Efficient Fidelity Estimate

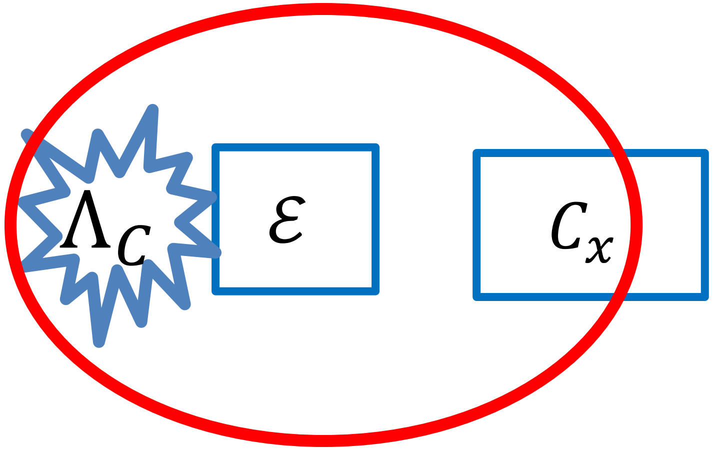


Requires an exponential number of measurement settings with different C_x

Instead, only want to check that your operations are good enough.

Want to check implementation of Clifford Gates and T gates = universal gate set

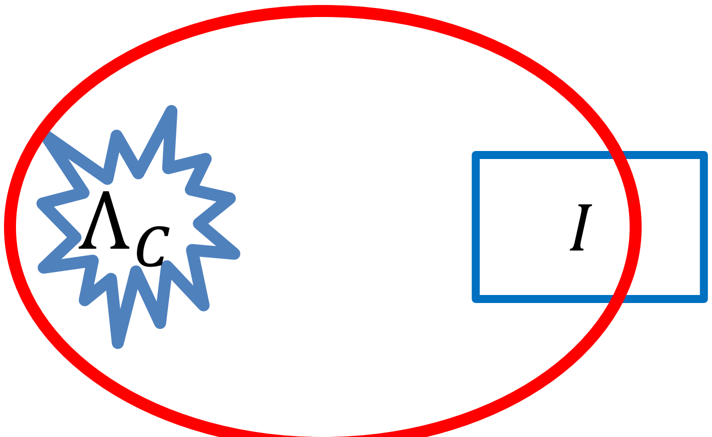
Efficient Fidelity Estimate



Average fidelity to any unitary $\mathcal{U}$ of

- $\mathrm{O}(\log n)$ T gates
- $\mathrm{O}(\mathrm{poly}\, n)$ Cliffords

only need to repeat for $\mathrm{O}(\mathrm{poly}\, n)$ different C_x.



If Λ_C is close to Identity, can closely bound the average fidelity of $\mathcal{E}$ to $\mathcal{U}$.

Can test a universal gate set!

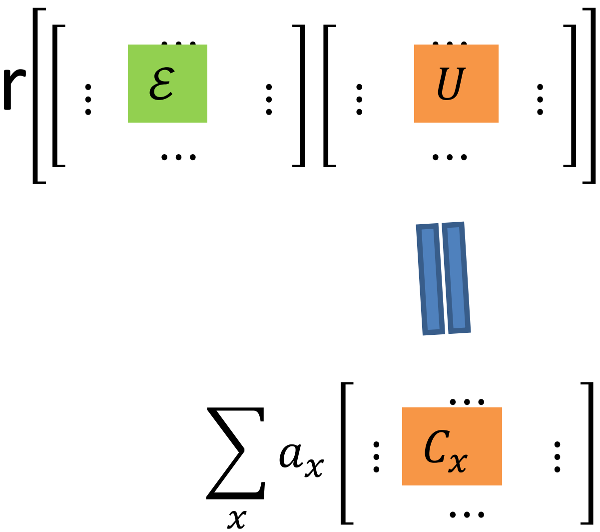
Conclusions and Open Questions

- Can robustly measure unital part of any quantum process
- Can efficiently and robustly test fidelity of universal quantum gate set operations.
- Experimentally implemented with superconducting qubit system at BBN

- What about the non-unital part?
- Can we extract other information efficiently and robustly (compressed sensing?)
- How does RB compare to Gate Set Tomography methods?

Efficient Fidelity Estimate

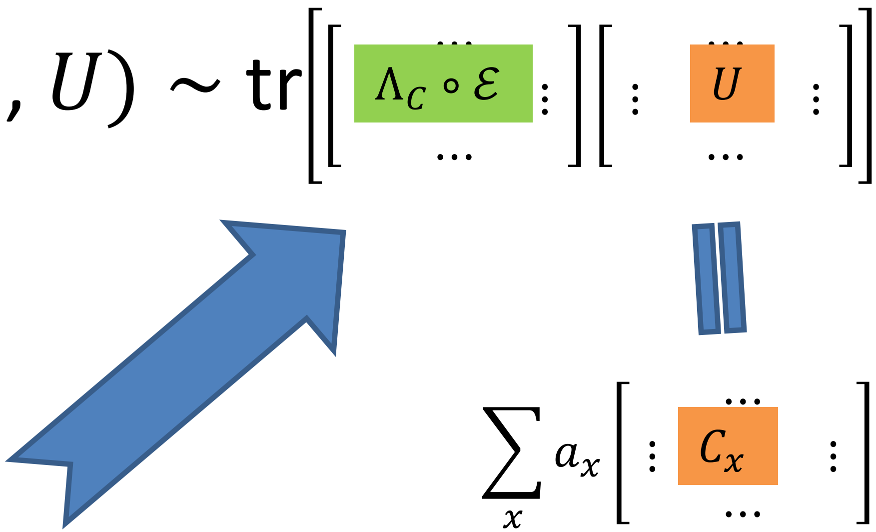
Average Fidelity $(\mathcal{E}, U) \sim \mathrm{tr}$



Unitaries composed of Cliffords and $\mathrm{O}(\log n)$ T gates can be written as a linear combination of $\mathrm{O}(\mathrm{poly}\, n)$ Cliffords.
Only need to measure $\mathrm{O}(\mathrm{poly}\, n)$ traces, each of which can be done efficiently.

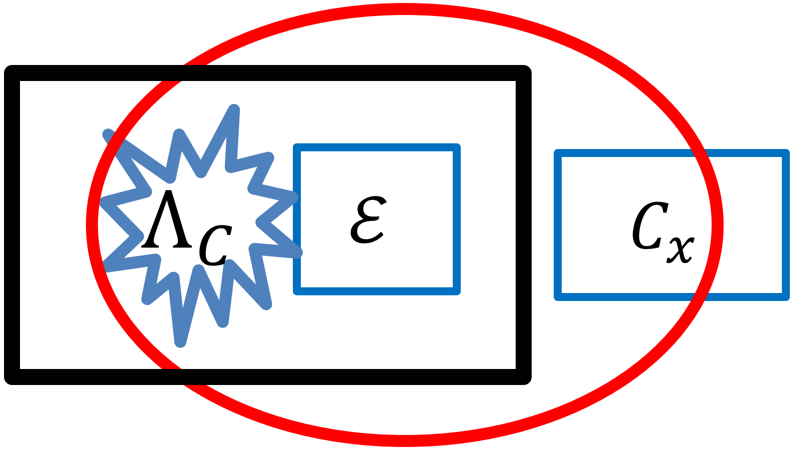
Efficient Fidelity Estimate

Average Fidelity $(\mathcal{E}\,,\,U) \sim \mathrm{tr}$

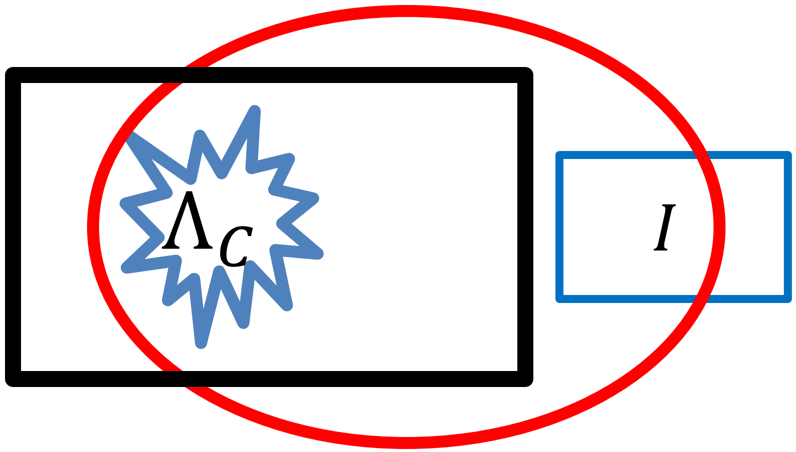


Since we haven't characterized Λ_C, we can't get rid of its effect. However, we can measure its average fidelity to the identity, and if it is close to the identity, we can bound its effect

Efficient Fidelity Estimate



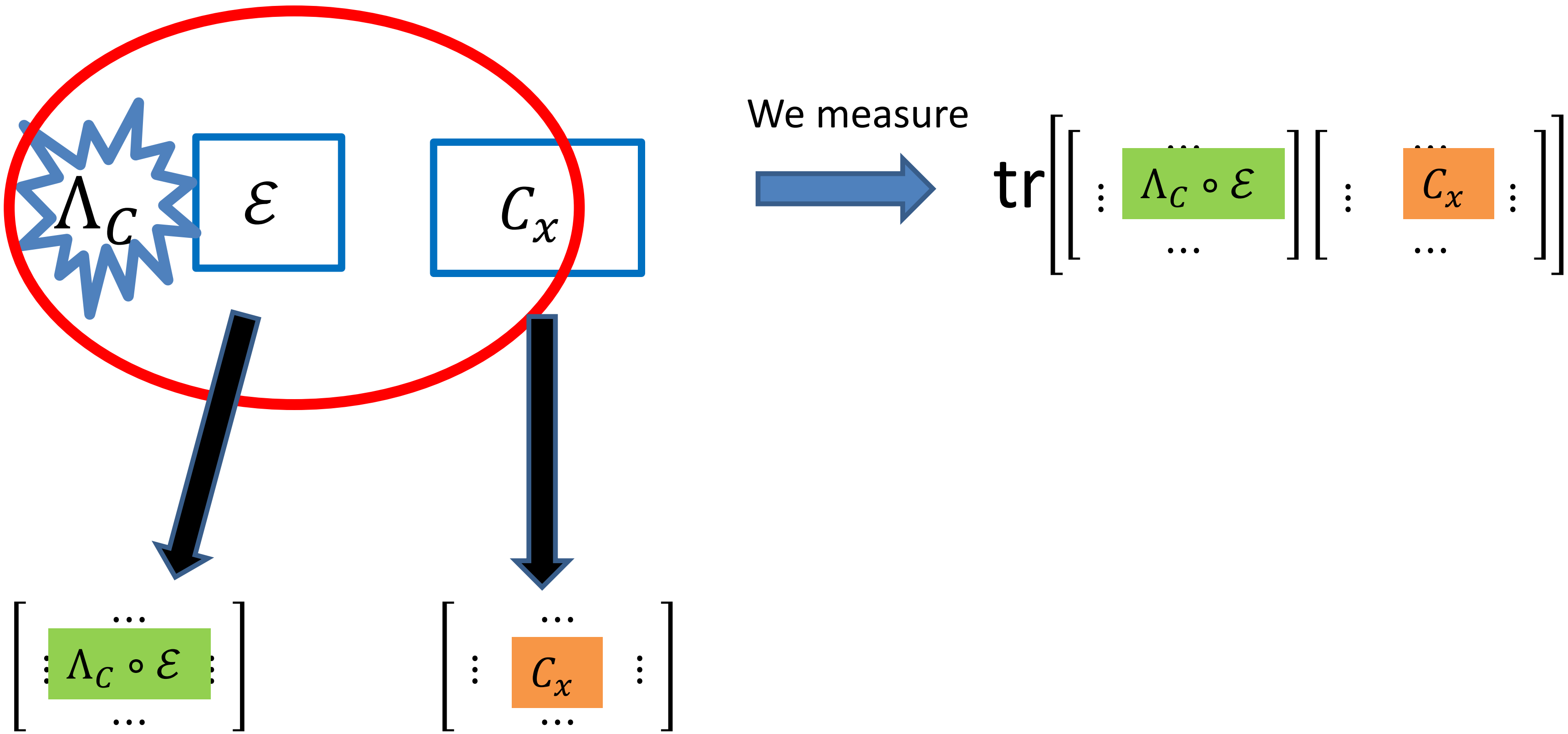
To get average fidelity to any unitary $\mathcal{U}$ of $\mathrm{O}(\log n)$ T gates and $\mathrm{O}(\mathrm{poly}\, n)$ Cliffords, only need to repeat for $\mathrm{O}(\mathrm{poly}\, n)$ overlaps C_x.



If Λ_C is close to Identity, can closely bound the average fidelity of $\mathcal{E}$ to $\mathcal{U}$.

Can test a universal gate set!

What do we measure?



What can we measure?

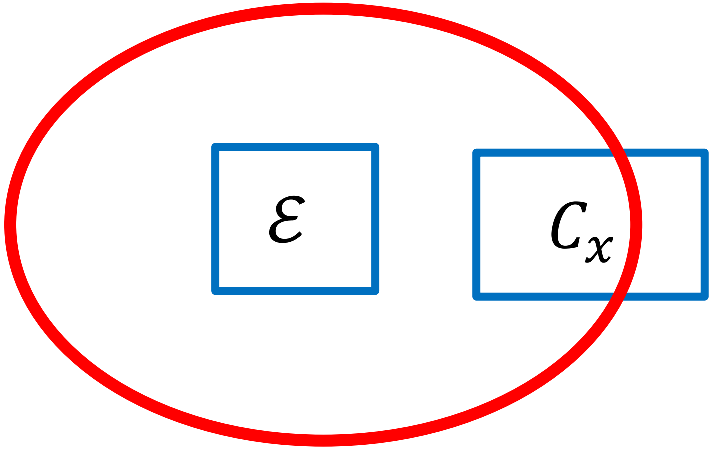
CPTP map: $16^n - 4^n$ parameters for n qubit map

$$\begin{array}{cc} & 4^n \\ 4^n & \left[\begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline & & & \\ & & & \\ & & & \end{array}\right] \end{array}$$

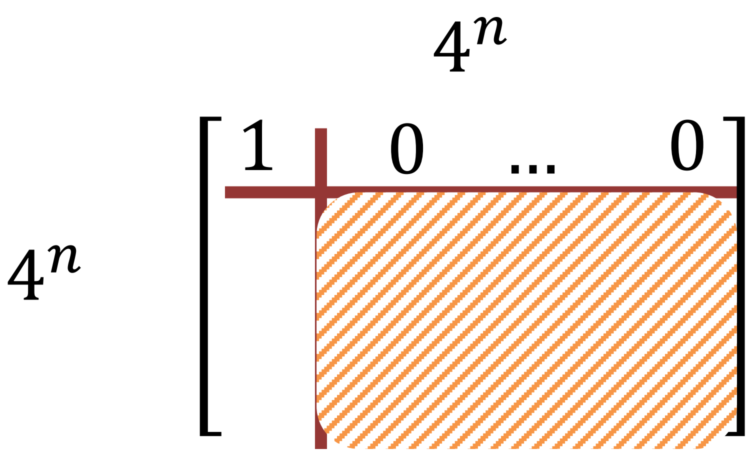
Choose many different $C_x{}'s$ and measure trace. By measuring enough traces can learn unital part

We learn: $16^n - 2 * 4^n + 1$ parameters

What do we characterize?



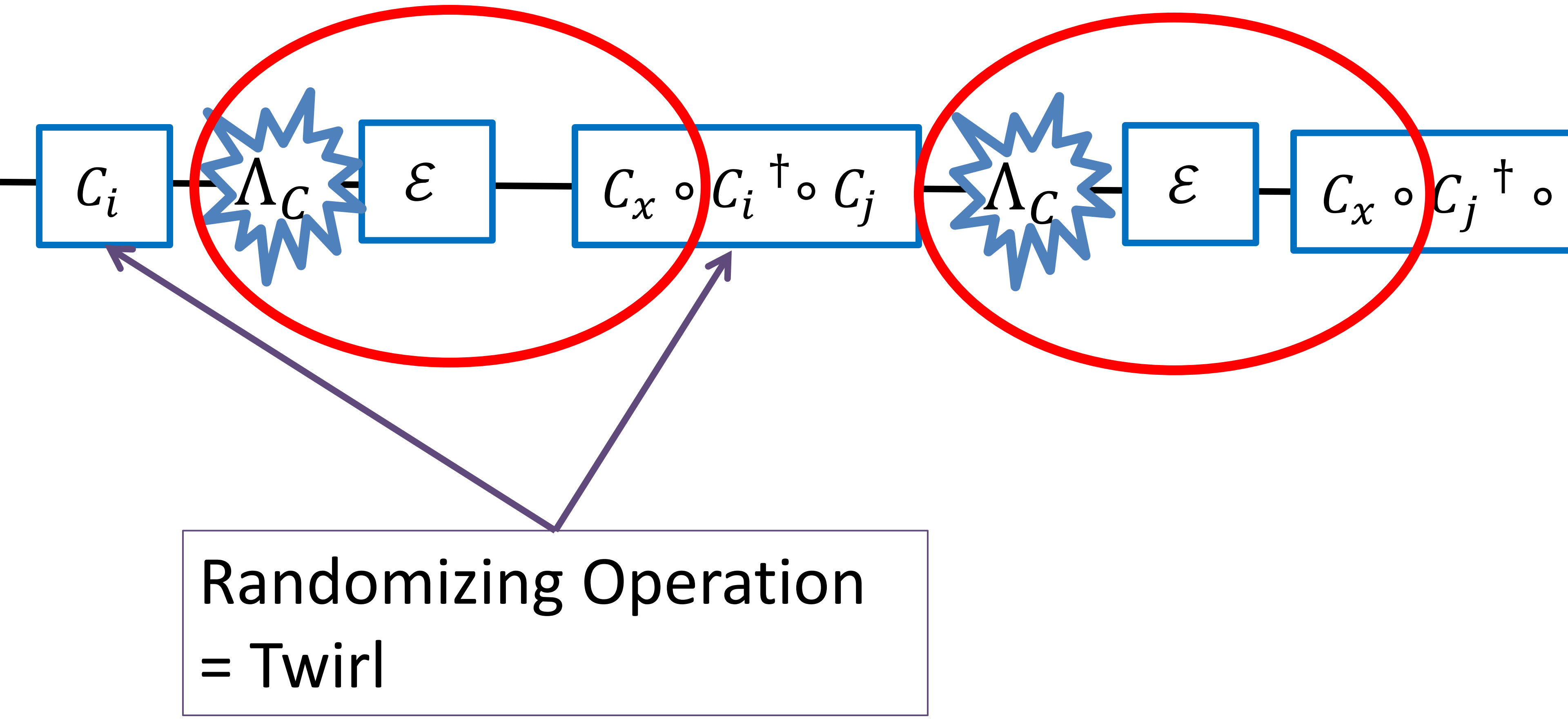
Need to repeat for 16^n different C_x



Unital part.

(Pauli-Liouville Representation)

2. Dealing with Errors



Can we do better with Randomized Benchmarking?

- Can we robustly characterize many parameters of any operation?

Can characterize almost all parameters of any quantum map

- We show Cliffords span unital part of quantum maps. By learning average fidelity of $\mathcal{E}$ to many $C_x{}'s$, can learn projection of $\mathcal{E}$ into unital subspace.

- What information can we obtain robustly *and* efficiently?

Can test performance of a universal gate set.