A Multi-tool for your Quantum Algorithmic Toolbox

Shelby Kimmel

Middlebury College

Based on work with

- Stacey Jeffery: arXiv: 1704.00765 (Quantum vol 1 p 26)
- Michael Jarret, Stacey Jeffery, Alvaro Piedrafita, arXiv:1804.10591 *(*ESA 2018*)*
- Kai DeLorenzo, Teal Witter, arXiv:1904.05995 (TQC 2019)
- Bohua Zhan, Avinatan Hassidim, arXiv:1101.0796 (ITCS 2012)

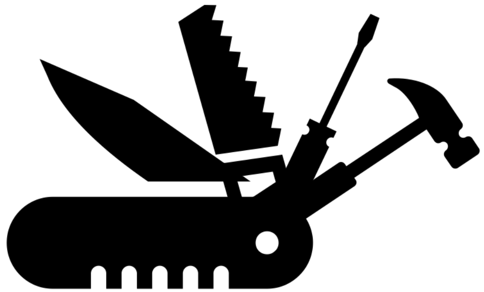
Quantum Tools for Classical Problems

- Quantum Fourier Transform
- Grover Search
- Quantum Approximate Optimization Algorithm (QAOA)

Quantum Tools for Classical Problems

- Quantum Fourier Transform (good if highly structured)
- Grover Search (good if unstructured)
- Quantum Approximate Optimization Algorithm (QAOA) (hard to analyze)

Multi-tool



- ✓ Structured
- ✓ Unstructured
- ✓ Easy creation
- ✓ Easy, provable, non-quantum analysis

- ❖ Query model

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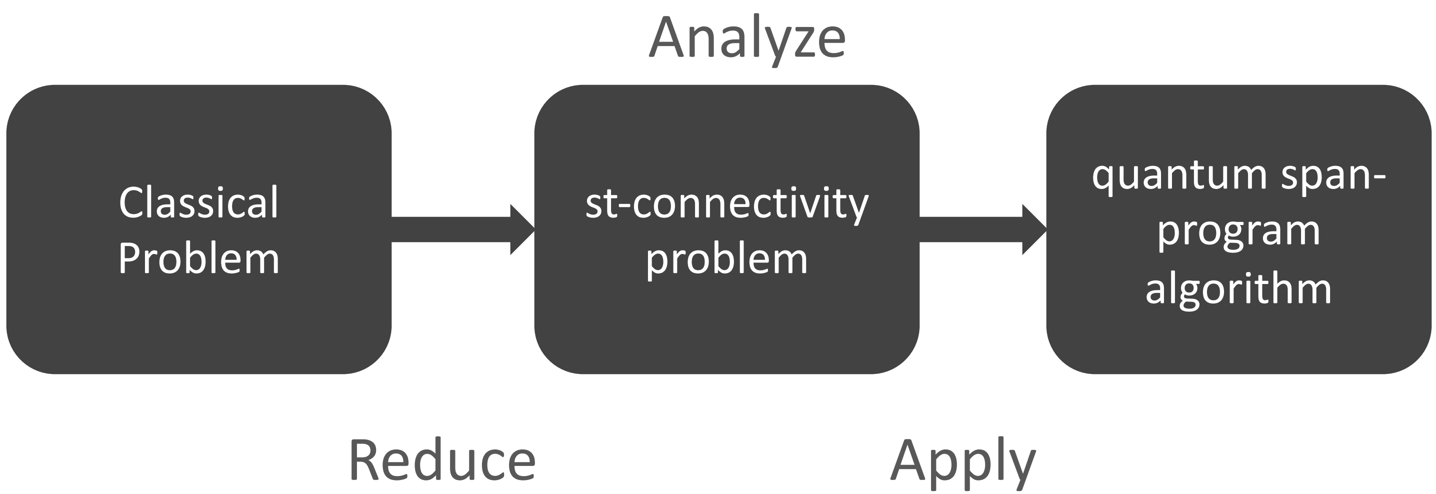

Reduction

Problem: Bit string $x = x_1x_2x_3$ contains a 1?

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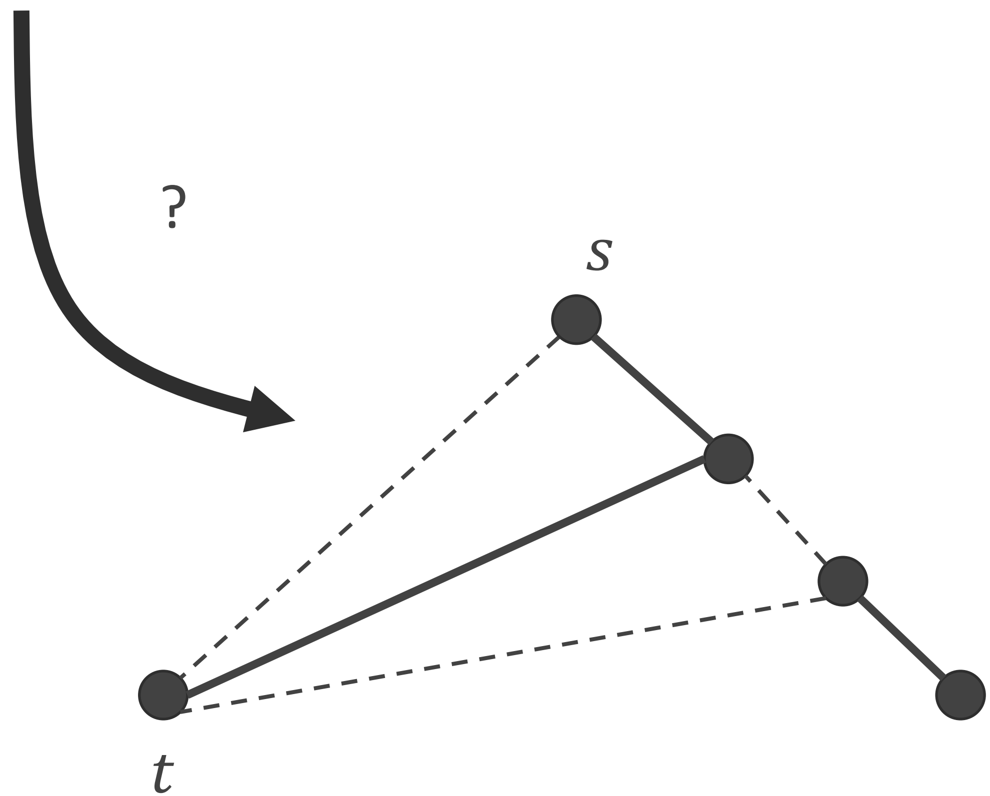

Reduction (Decision Tree Approach)

Problem: Bit string $x = x_1x_2x_3$ contains a 1?

Idea: Turn classical algorithm into decision tree:

Algorithm:

- For each bit:
  - -If 1: Output 1, end
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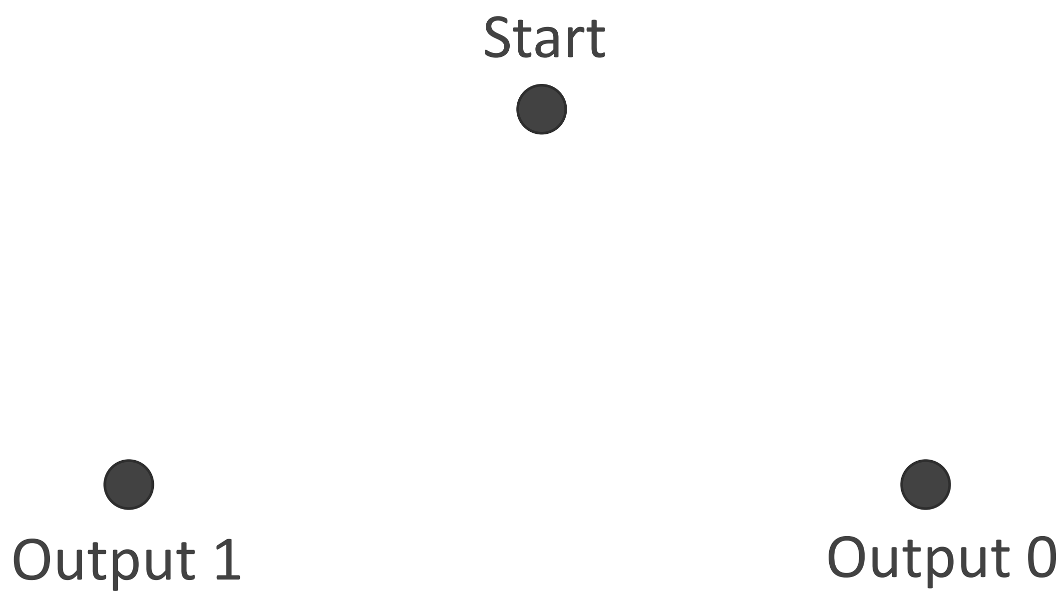

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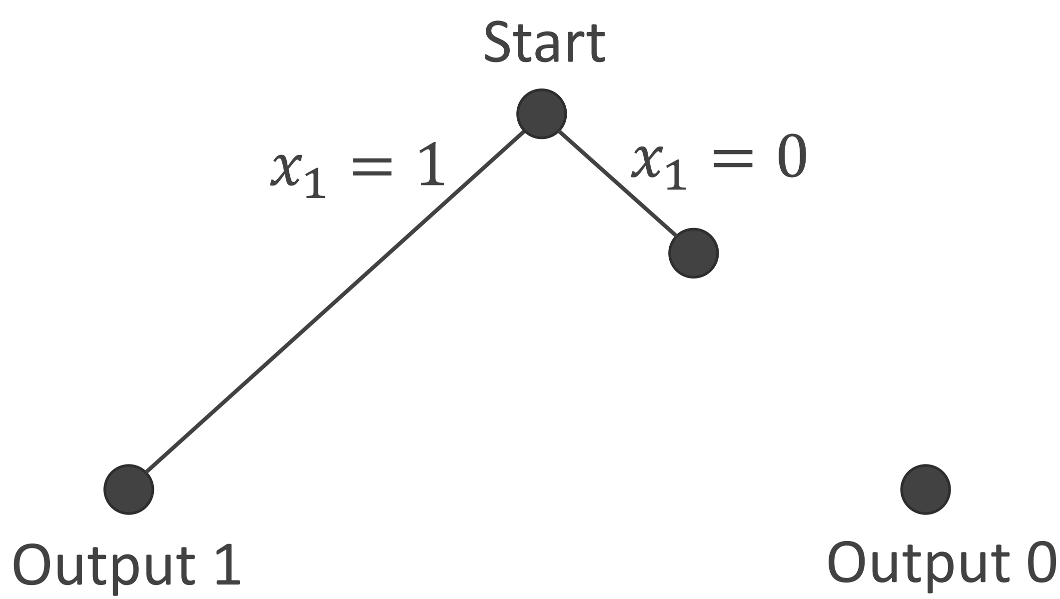

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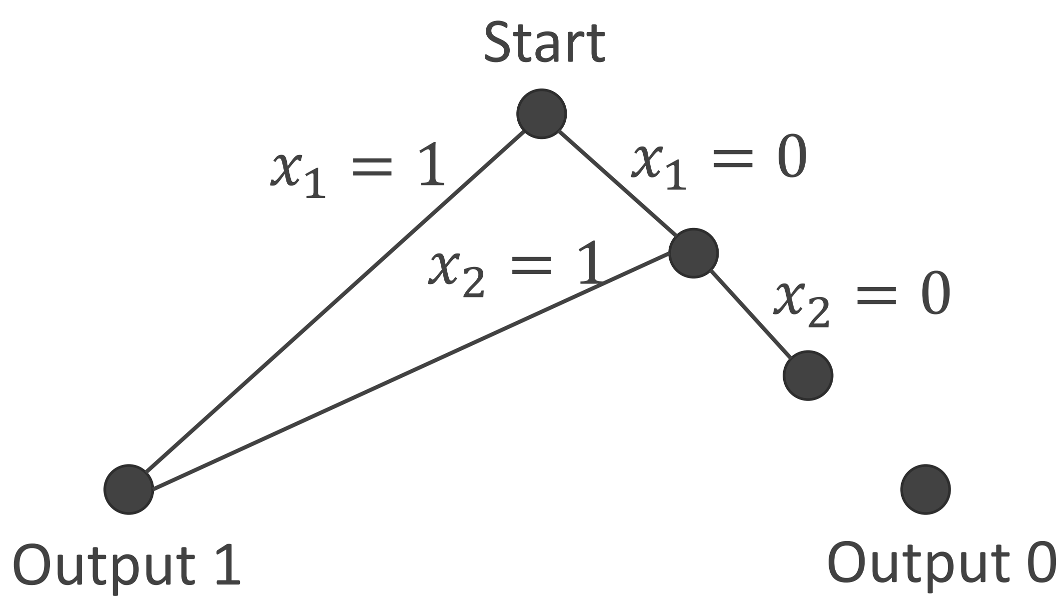

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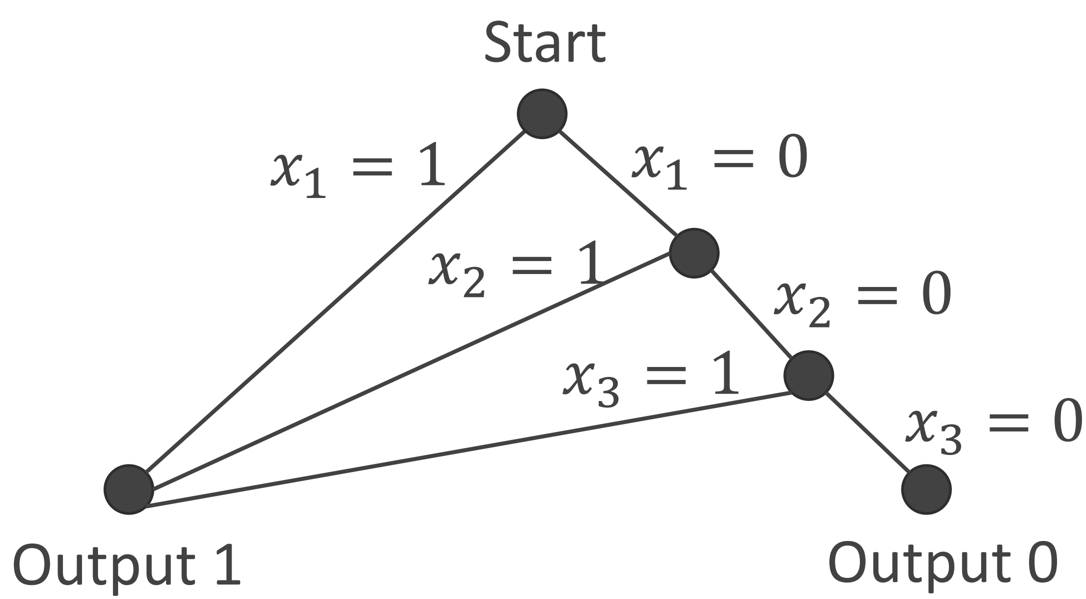

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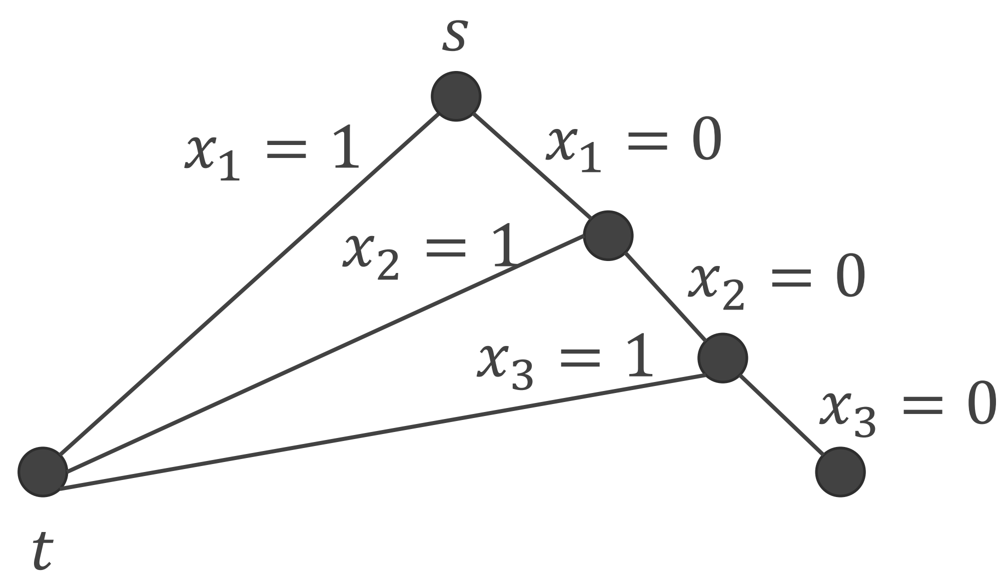

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Problem: Bit string $x = x_1x_2x_3$ contains a 1?

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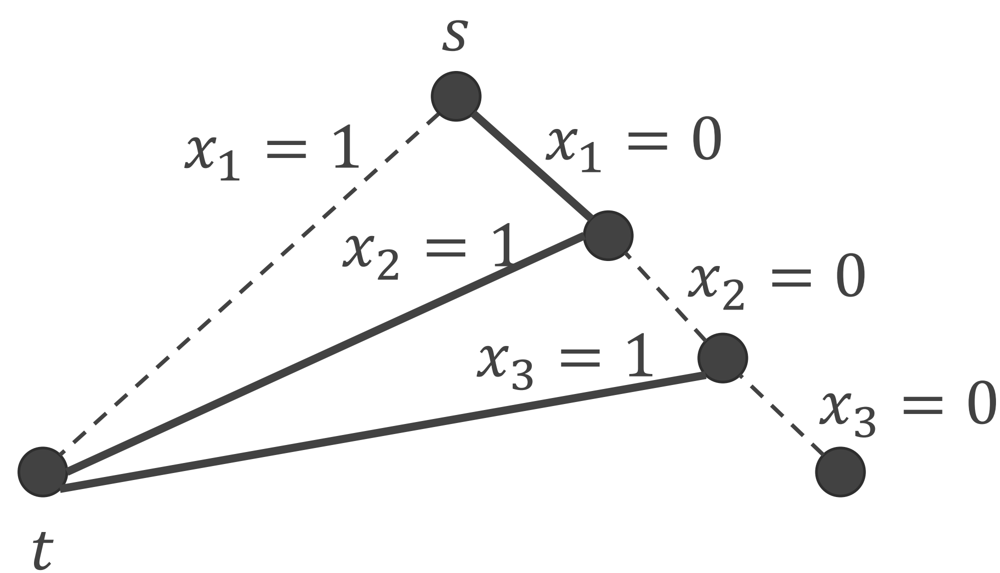

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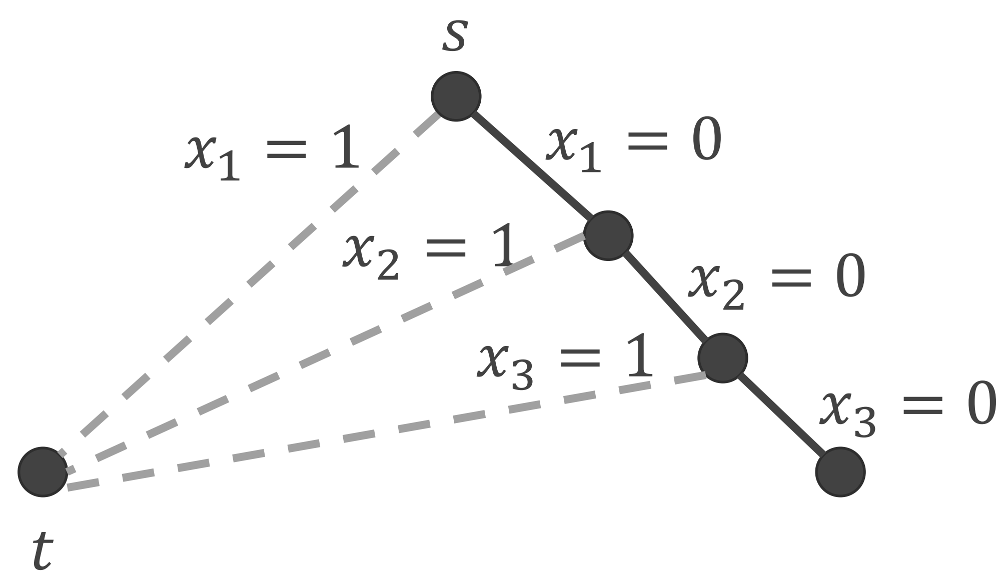
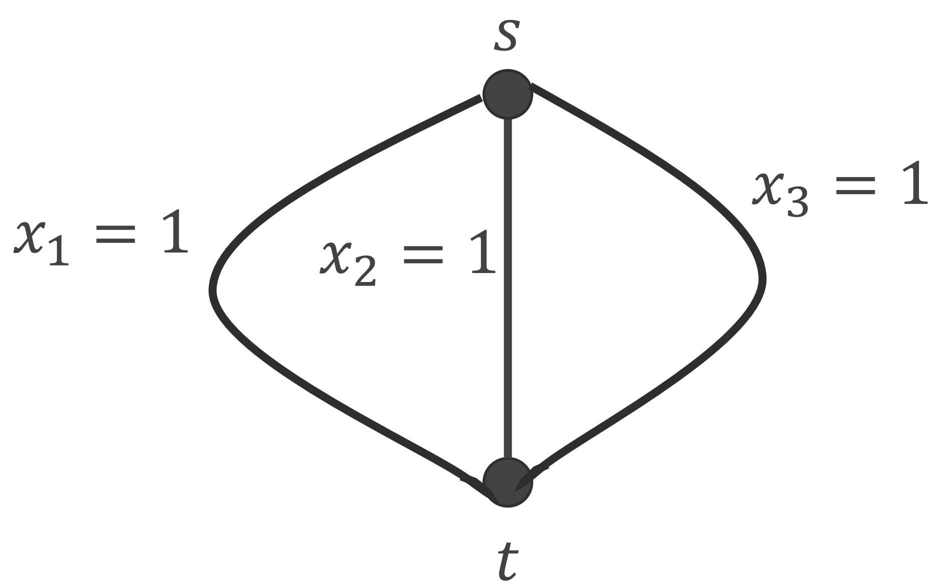


Reduction (Parallel Approach)

Problem: Bit string $x = x_1 x_2 x_3$ contains a 1?

Idea: Take advantage of parallel paths



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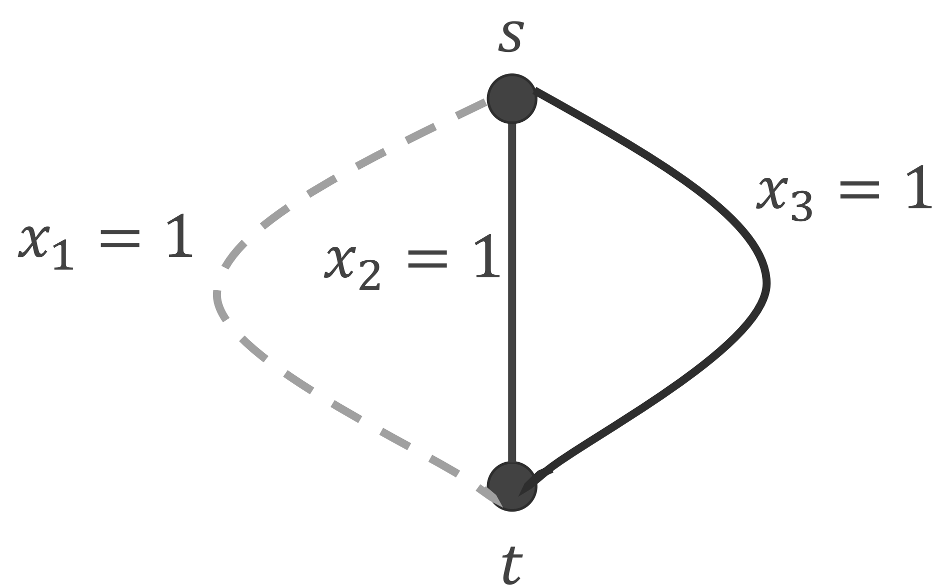

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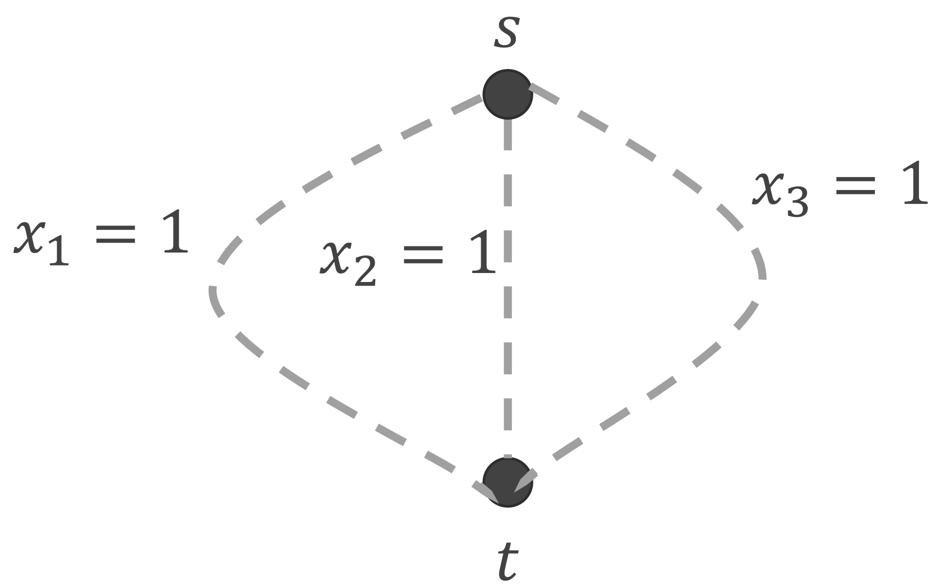

[Beigi, Taghavi `12], [JJ**K**P, `18]

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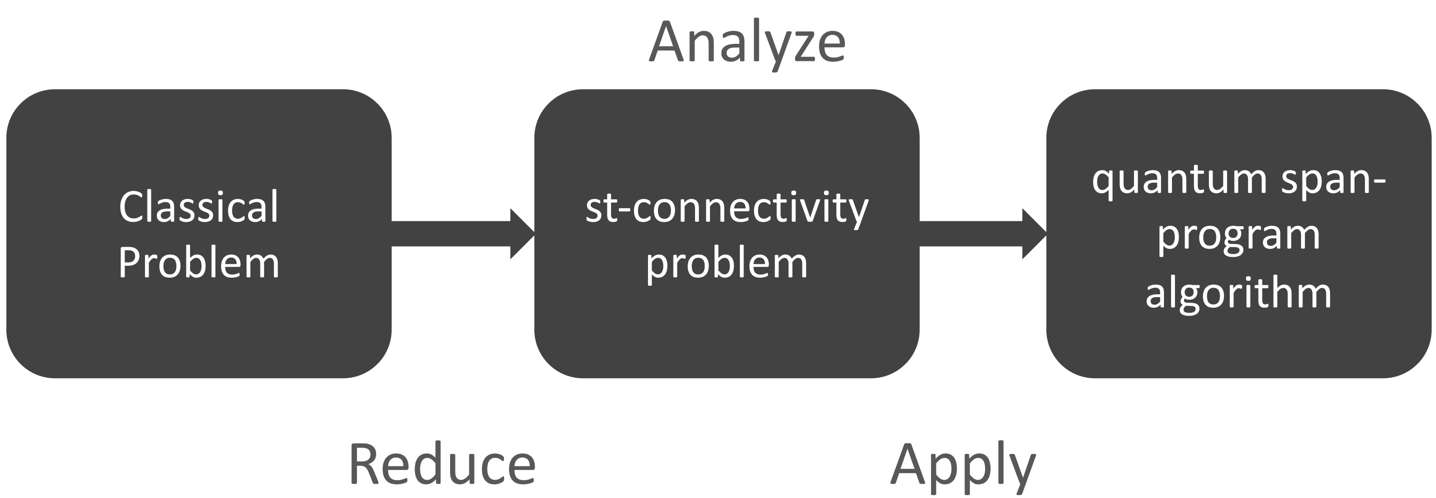
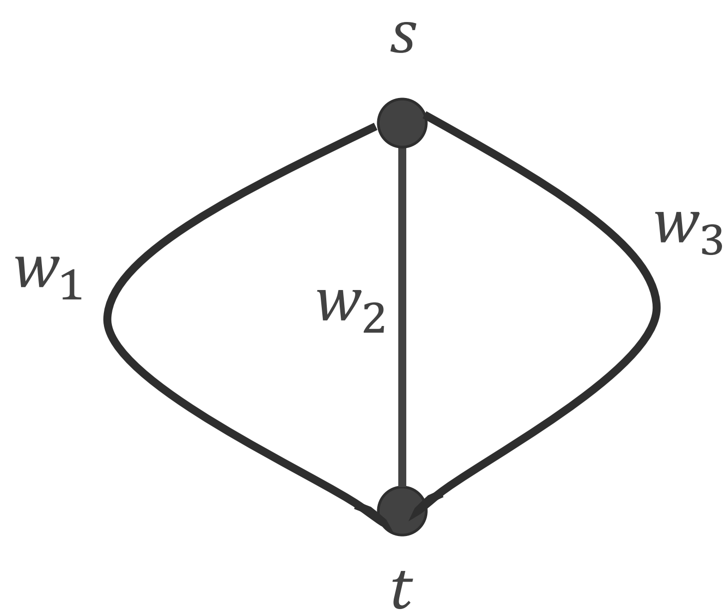


Analysis

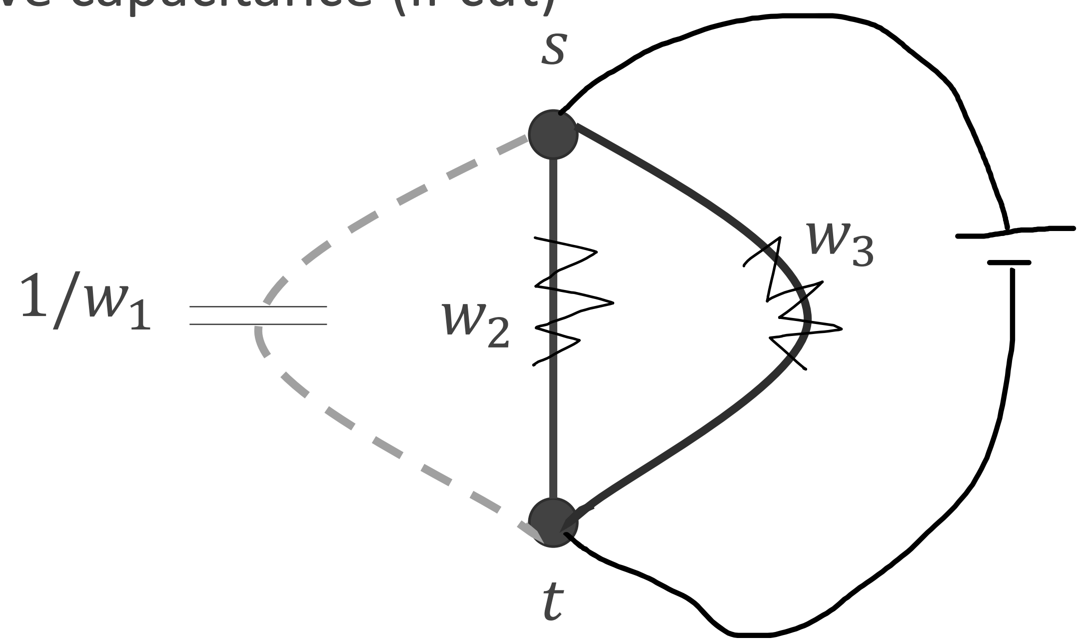
Assign a weight to each edge:



Analysis

For each input, can calculate

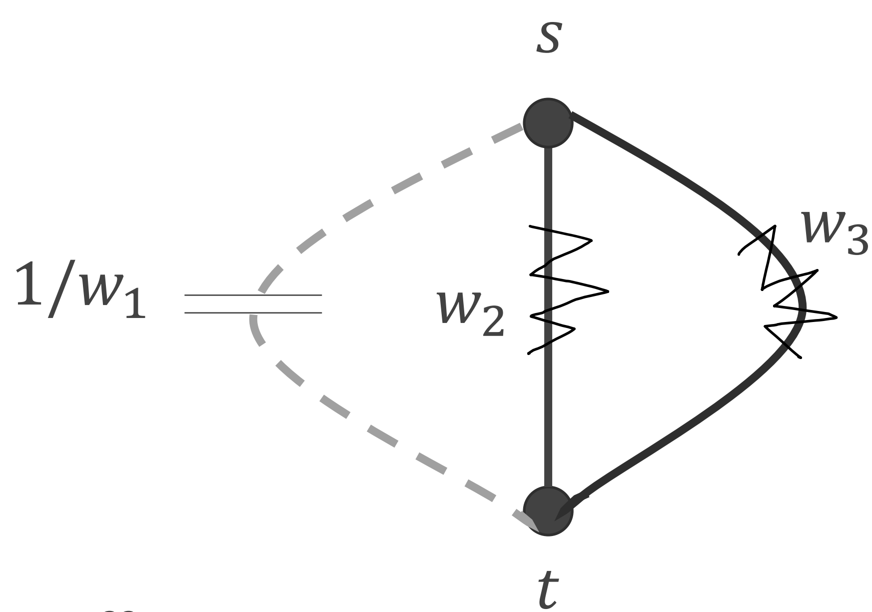
- effective resistance (if path)
- effective capacitance (if cut)



Analysis

Then Quantum Query Complexity is:

$$O(\sqrt{(\max C)(\max R)})$$



$\max R$: max effective resistance over connected instances

$\max C$: max effective capacitance over not connected instances

[Belovs, Reichardt, `12], [JJ**K**P, `18]

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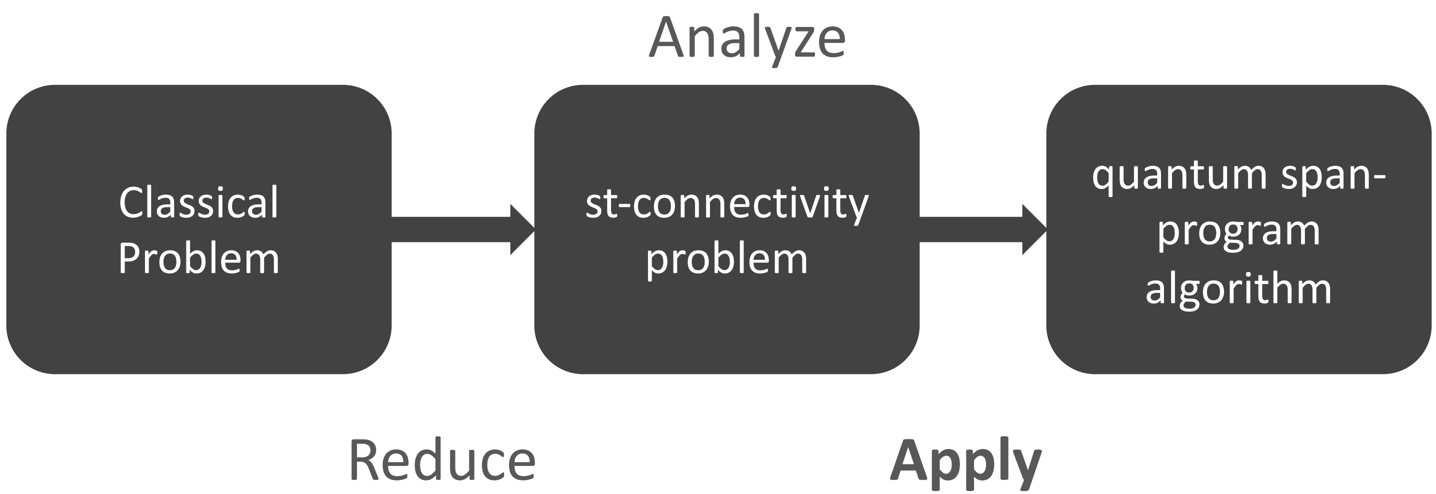

Algorithm

Apply phase estimation to a unitary that is a product of two unitaries,

- One depends on input x
- One depends on underlying graph

Space complexity scales log(no. edges, vertices) in graph.

[Belovs, Reichardt `12]

Performance

*There are alternative optimal algorithms for some problems

- Read-once Boolean formulas (query optimal) [J**K**]
- Total connectivity (query optimal) [JJ**K**P]
- Cycle detection (query optimal) [D**K**W]
- Even length cycle detection [D**K**W]
- Bipartiteness (query optimal) [D**K**W]
- Directed st-connectivity (query optimal) (Beigi, Taghavi `19)
- Directed smallest cycle (query optimal) (Beigi, Taghavi `19)
- Topological sort (Beigi, Taghavi `19)
- Connected components (Beigi, Taghavi `19)
- Strongly connected components (Beigi, Taghavi `19)
- k-cycle at vertex v (Beigi, Taghavi `19)
- st-connectivity (query optimal) (Reichardt, Belovs `12)
- Maximum bipartite matching (Lin, Lin `16; Beigi and Taghavi `19
- Maximum matching (**K**, Witter, in preparation)
- Super-polynomial speed-up for game evaluation [ZH**K**]

Summary

- A multi-tool for algorithm design that is accessible

Open Problems

- How to set weights?
- When is this approach optimal?

Funding:

Middlebury

ARO

People:

Stacey Jeffery

Michael Jarret

Alvaro Piedrafita

Teal Witter

Kai De Lorenzo