Robust Characterization of Gates

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2015 Korea-US Joint Workshop on Quantum Information
November 16, 2015
The Dream

| 𝜓 |
The Reality

$X_{\pi/4}$
The Reality

$X_{\pi/4}$
The Reality

\( X_{\pi/4} \)

What Happened?
The Reality

Standard Tomography:
• Inaccurate
• Results not valid quantum operations
The Solution

Robust Tomography:

\[ \text{Dephase} \quad X_{\pi/4.001} \quad \text{Amp. D} \]
Outline

1. Why Standard Tomography Fails
2. Methods for Robust Tomography
Problem with Standard Process Tomography  
[Chuang & Nielsen ’97]

Some imperfect gate $\mathcal{G}$. Want to learn errors on $\mathcal{G}$
Problem with Standard Process Tomography

State preparation and measurement errors (SPAM)
Problem with Standard Process
Tomography

\[ |0\rangle, |1\rangle \]
Problem with Standard Process Tomography

\[ |0\rangle \xrightarrow{\Lambda_0} g \xrightarrow{\Lambda_0/1} |0\rangle, |1\rangle \neq |+\rangle, |-\rangle \]
Methods that are Robust to SPAM

Two Approaches:
1. Repeated Application
2. Learn Everything at Once
Repeated Application

Any change outcome is due only to $\mathcal{G}$, not SPAM
Methods that are Robust to SPAM

Two Approaches:

1. Repeated Application
2. Learn Everything at Once
Learn Everything at Once

Actions that can be performed

State Prep:

Measurement:

Noisy Gates:

\[ |0\rangle \overset{\Lambda_0}{\rightarrow} \]

\[ \Lambda_0/1 \]

|0\rangle, |1\rangle

\[ G_1 \quad G_2 \quad G_3 \]
Learn Everything at Once

Get a ton of data, then solve for everything at once, including SPAM rather than just one gate at a time
Methods that are Robust to SPAM

Two Approaches:

1. Repeated Application
   a) Randomized Benchmarking
      [Emerson et al ‘05, Knill et al ‘08, Magesan et al ‘12, Kimmel, da Silva et al ‘14, Wallman et al ‘15, etc]
   a) Robust Phase Estimation [Kimmel, Low, Yoder ‘15]

2. Learn Everything All At Once
   a) Gate Set Tomography
      [Stark ‘12, Merkel et al ‘13, Blume-Kohout et al ‘13 (Sandia National Labs)]
## Comparing Methods

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<tr>
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<th>Randomized Benchmarking</th>
<th>Robust Phase Estimation</th>
<th>Gate Set Tomography</th>
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<tbody>
<tr>
<td><strong>Efficiency</strong></td>
<td>Polynomial in # qubits</td>
<td>Heisenberg scaling</td>
<td>Exponential in # of qubits</td>
</tr>
<tr>
<td><strong>Size of system to characterize</strong></td>
<td>Multiple qubits</td>
<td>Single qubit</td>
<td>3 qubits max in practice</td>
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<tr>
<td><strong>Type of Parameters Extracted</strong></td>
<td>Course-grained</td>
<td>Specific, Experimentally relevant</td>
<td>Everything</td>
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<tr>
<td><strong>Additional Resources</strong></td>
<td>Clifford operations</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td><strong>Ease of use</strong></td>
<td>Very easy to use and analyze</td>
<td>Easy to use and analyze</td>
<td>Easy to use b/c software, harder to analyze</td>
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</tbody>
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Goals for the Future

• Targeted, experimentally relevant information efficiently, robustly.
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

\[
\frac{1 + \sin k\theta}{2} + \delta_{k1}, \quad \frac{1 + \cos k\theta}{2} + \delta_{k2}
\]

For \( k \) in \( \mathbb{Z} \), each in time \( k \)
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1 + \sin \theta}{2} + \delta_{k1}, \quad \frac{1 + \cos \theta}{2} + \delta_{k2}$$

Using only $k = 1$ can’t get an accurate estimate!
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

\[
\frac{1 + \sin k\theta}{2} + \delta_{k1}, \quad \frac{1 + \cos k\theta}{2} + \delta_{k2}
\]

For \( k \) in \( \mathbb{Z} \), each in time \( k \)

\[k = 1\]
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

\[
\frac{1 + \sin k\theta}{2} + \delta_{k1}, \quad \frac{1 + \cos k\theta}{2} + \delta_{k2}
\]

For \( k \) in \( \mathbb{Z} \), each in time \( k \)

\( k = 1 \quad k = 2 \)
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

\[
\frac{1 + \sin k\theta}{2} + \delta_{k1}, \quad \frac{1 + \cos k\theta}{2} + \delta_{k2}
\]

For \( k \) in \( \mathbb{Z} \), each in time \( k \)

\( k = 1 \quad k = 2 \quad k = 4 \)

Can estimate \( \theta \) with standard deviation \( \sigma(\theta) \sim \frac{1}{T} \),
as long as \( |\delta_k| < \frac{1}{\sqrt{8}} \approx .35 \) for all \( k \).
Robust Phase Estimation

Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1 + \sin k\theta}{2} + \delta_{k1}, \quad \frac{1 + \cos k\theta}{2} + \delta_{k2}$$

For $k$ in $\mathbb{Z}$, each in time $k$

$$k = 1 \quad k = 2 \quad k = 4$$

Can estimate $\theta$ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$,

as long as $|\delta_k| < \frac{1}{\sqrt{8}} \approx .35$ for all $k$.

...but need upper bound on size of $\delta$ to know how many extra samples to take.