Robust Single-Qubit Process Calibration via Robust Phase Estimation

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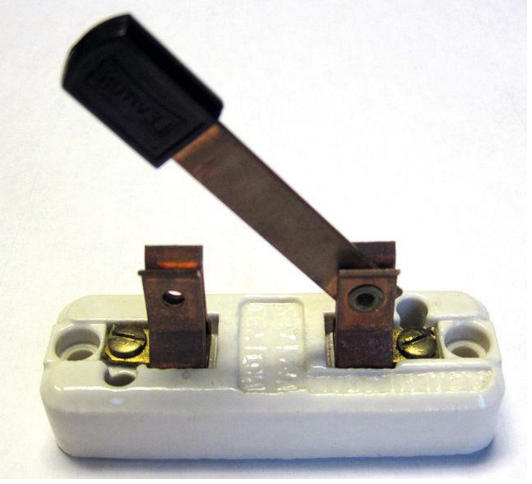
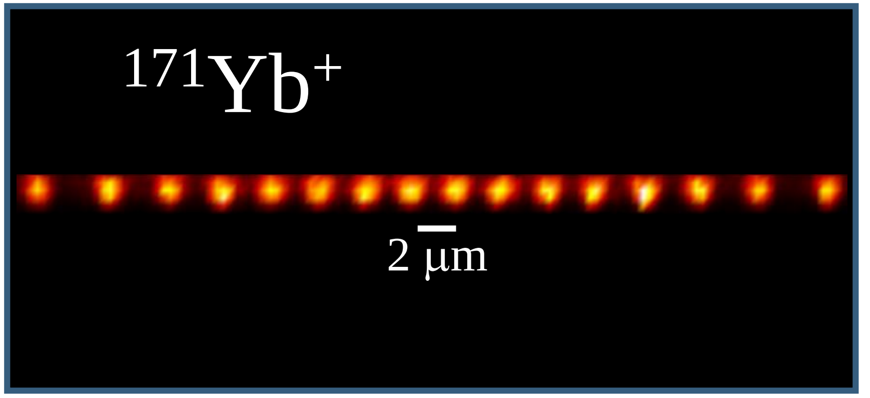
Arxiv: 1502.02677

Joint Center for Quantum Information and Computer Science
UMIACS
University of Maryland Institute for Advanced Computer Studies

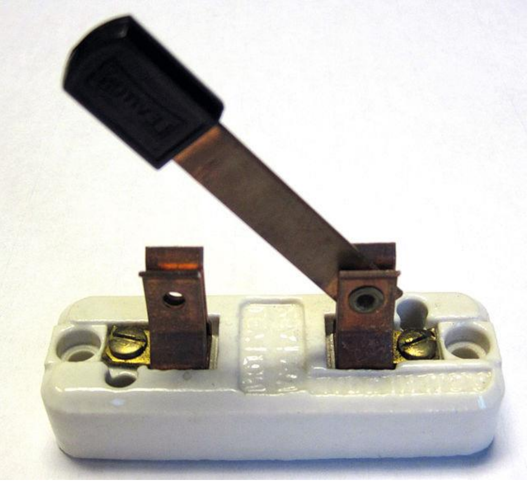
Imagine…

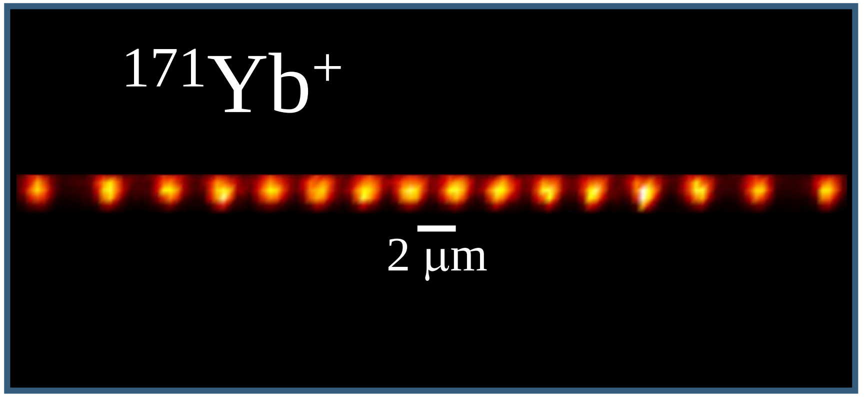




[Monroe Lab]

Imagine…

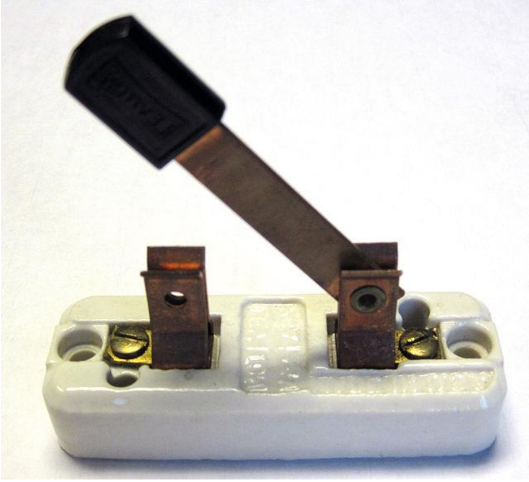


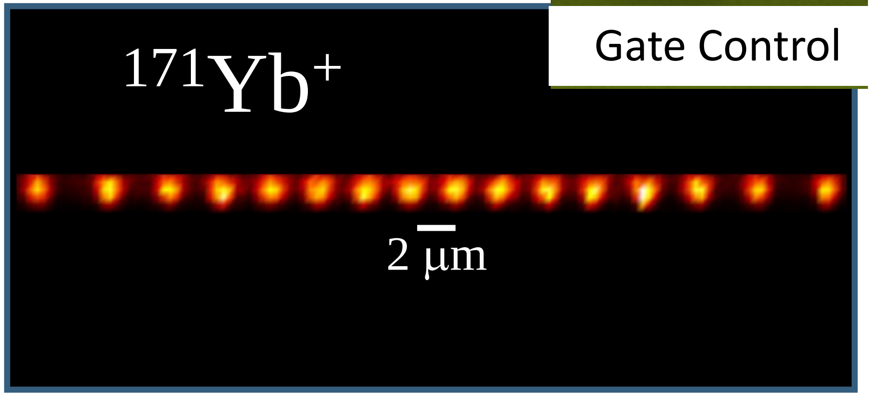


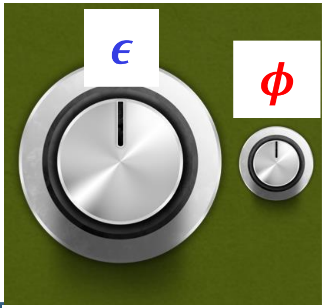
[Monroe Lab]

- All gates have errors
- State preparation has errors
- Measurements has errors

Imagine…







Gate Control

[Monroe Lab]

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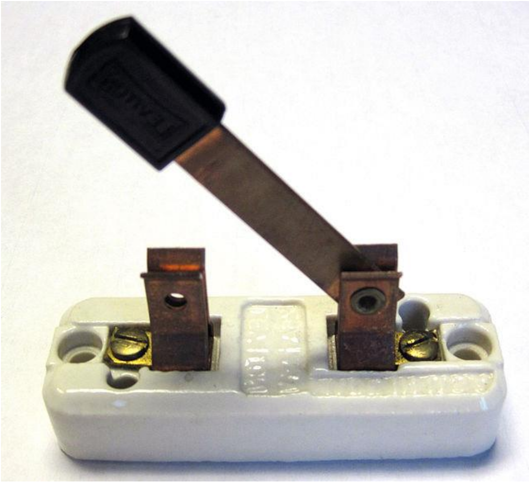
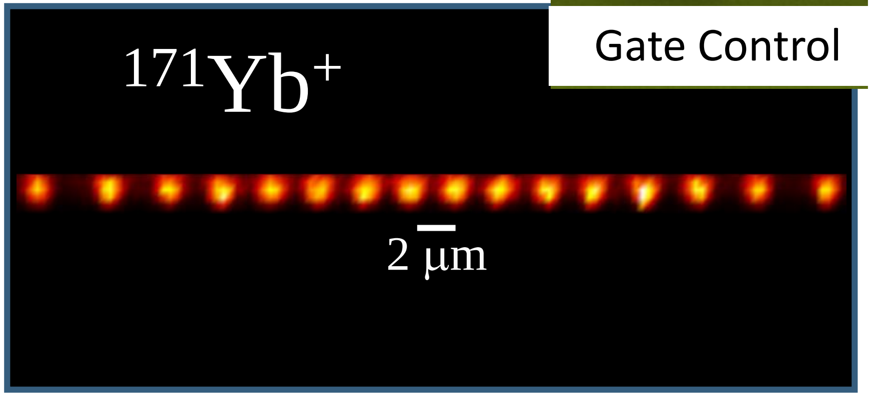
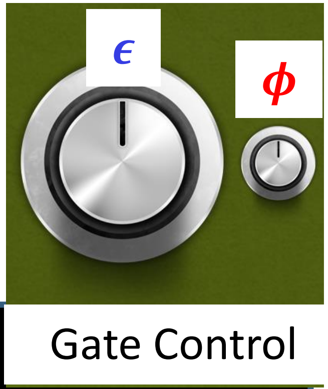
Imagine…





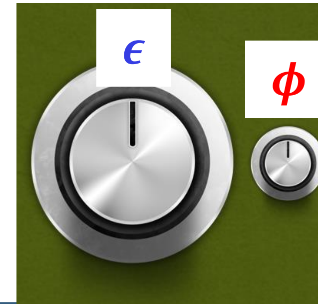


[Monroe Lab]

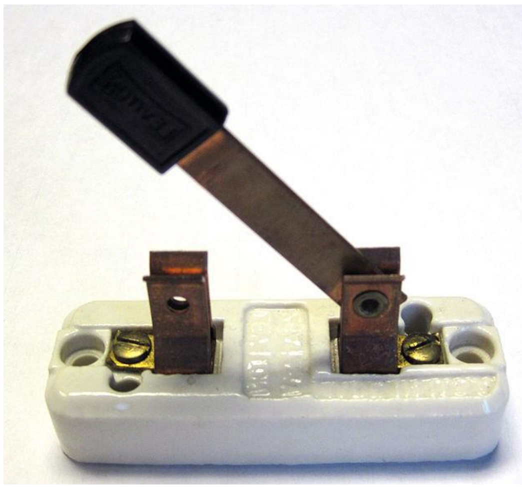
- All gates have errors
- State preparation has errors
- Measurements has errors

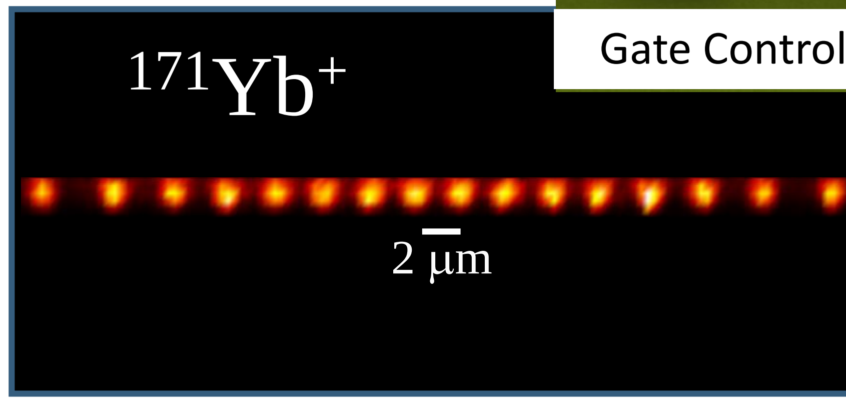
Want to quickly determine gate errors, and then tune to fix.

Imagine…



Gate Control





[Monroe Lab]

- All gates have errors
- State preparation has errors
- Measurements has errors

Robust Phase Estimation

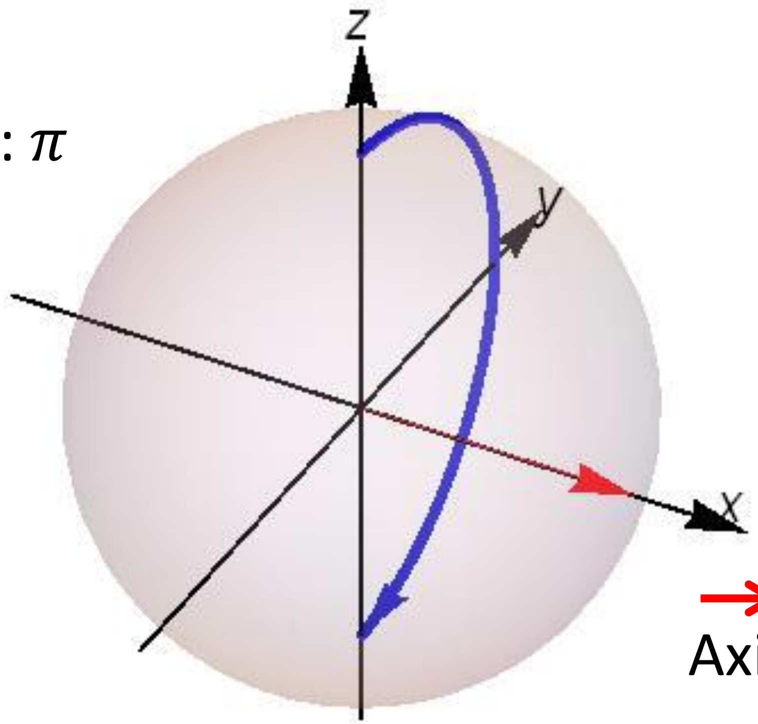
Want to quickly determine gate errors, and then tune to fix.

Outline

1. Motivation for Robust Phase Estimation
2. Control Errors for Single Qubit Gates
3. Comparison to Existing Methods
4. Robust Phase Estimation
5. Application to Parameter Estimation

Control Errors

Amplitude of Rotation: π

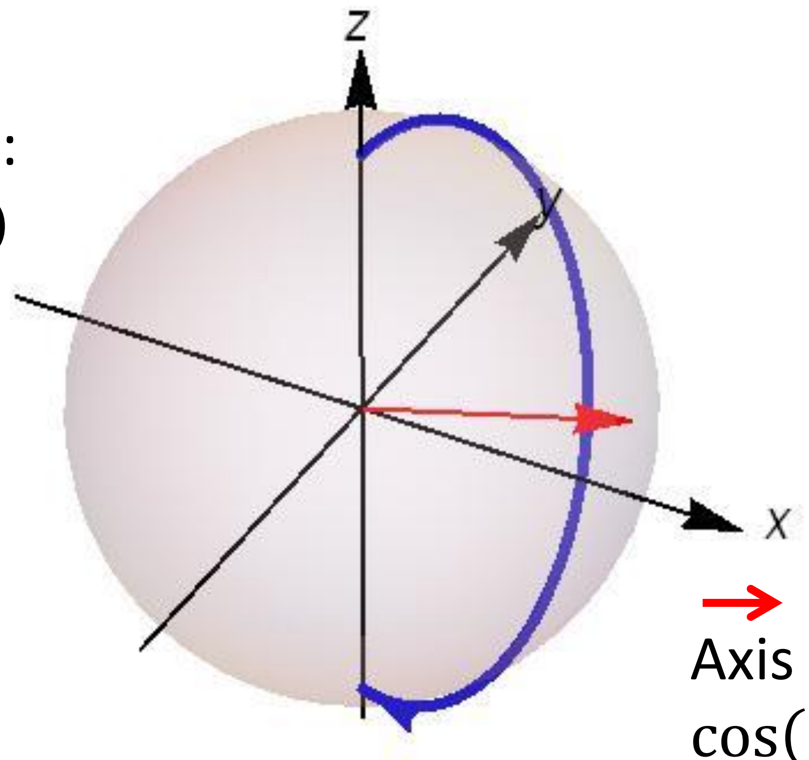


Axis of Rotation: $\hat{x}$

Ideal Unitary U

Control Errors

Amplitude of Rotation:
$A = \pi(1 + \epsilon)$



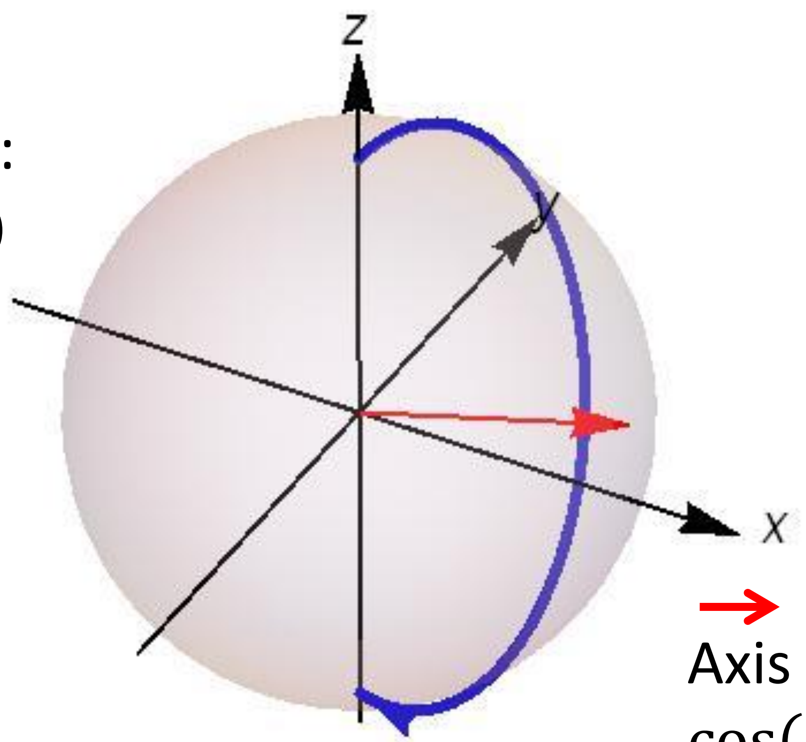
Axis of Rotation:
$\cos(\phi)\hat{x} + \sin(\phi)\hat{z}$

Unitary $\widetilde{U}$

Control Errors

Amplitude of Rotation:
$A = \pi(1 + \epsilon)$

ϵ is an "Amplitude Error"



Unitary $\widetilde{U}$

Axis of Rotation:
$\cos(\phi)|x\rangle +$

ϕ is an "Off-Resonance Error"

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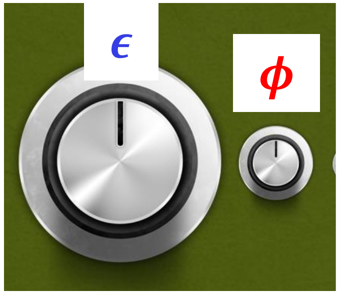
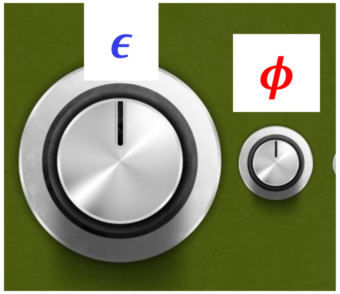
Comparison to Existing Techniques



Gate Control

Ad hoc Rabi – Ramsey Sequences.

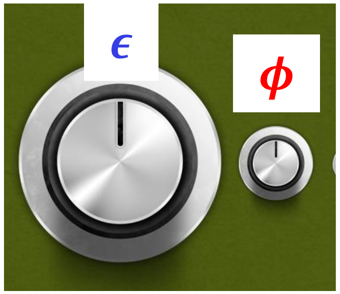
Comparison to Existing Techniques



Gate Control

Process Tomography [Chuang & Nielsen '97]

Comparison to Existing Techniques



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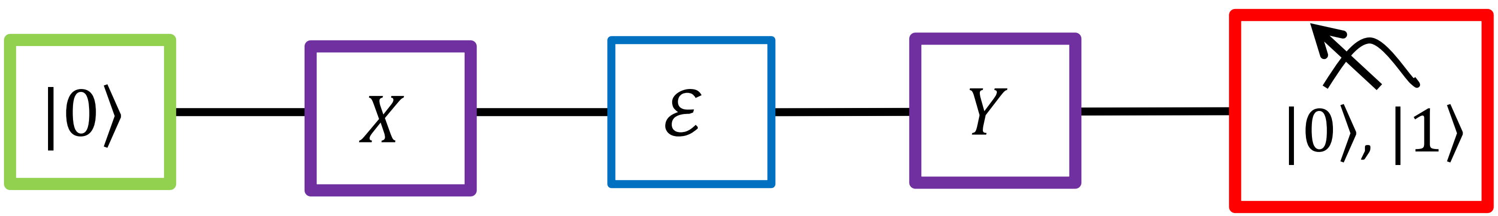

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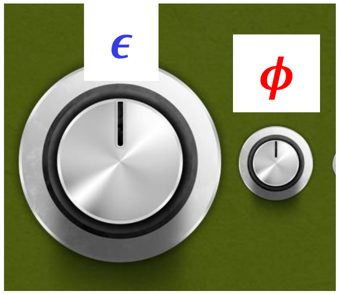

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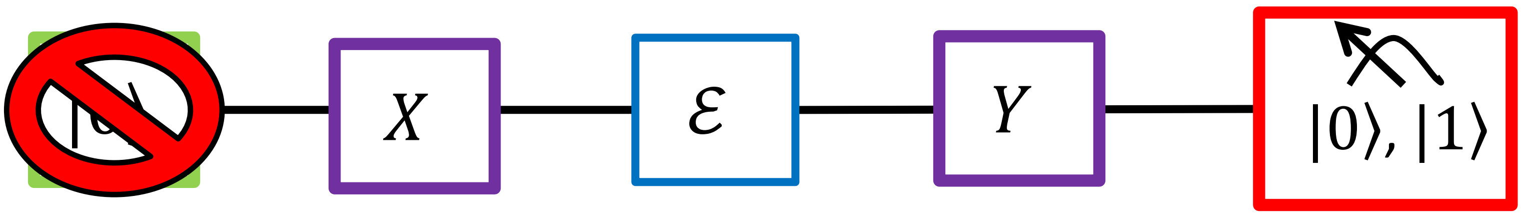

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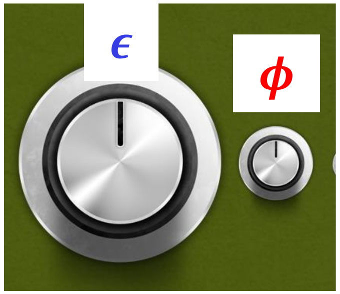

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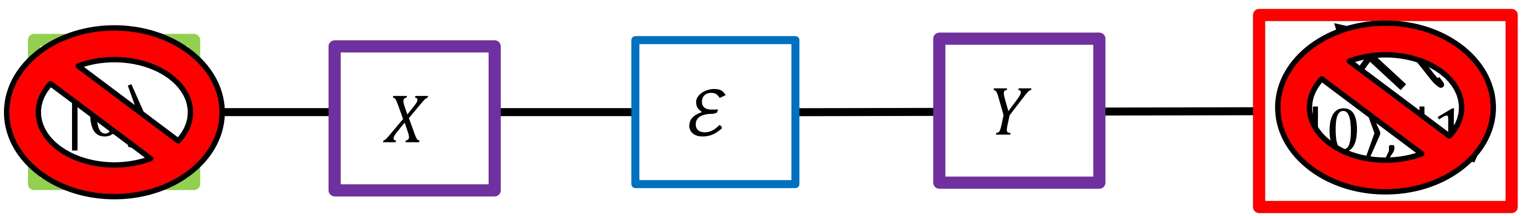

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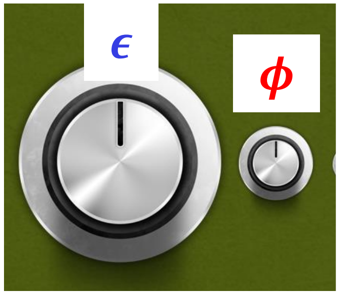

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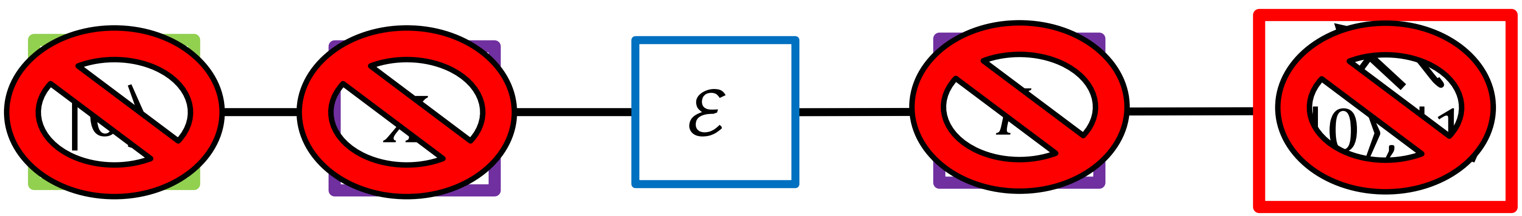

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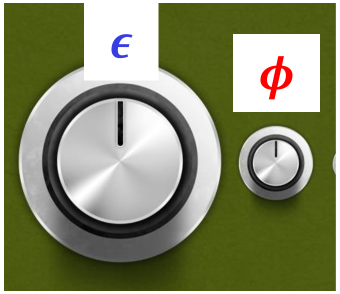
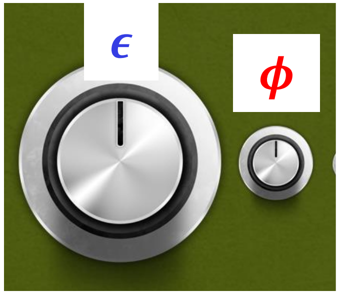


Gate Control

Process Tomography

- Need perfect state preparation and measurement
- Need perfect additional gates
- Time consuming: need to learn 12 parameters to extract ϕ and ϵ

Comparison to Existing Techniques



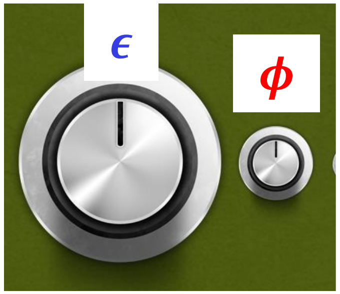
Gate Control

Process Tomography

Randomized Benchmarking

Tomography

Comparison to Existing Techniques



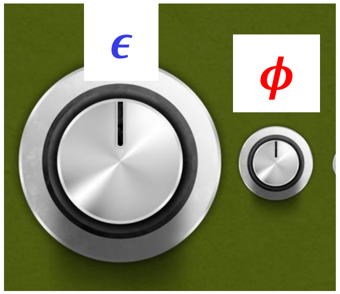
Gate Control

~~Process Tomography~~

Randomized Benchmarking

- Don't need perfect state preparation and measurement
- Must be able to perform single-qubit Cliffords (although not perfectly)
- Time consuming: need to learn 9 parameters to extract ϕ and ϵ

Comparison to Existing Techniques

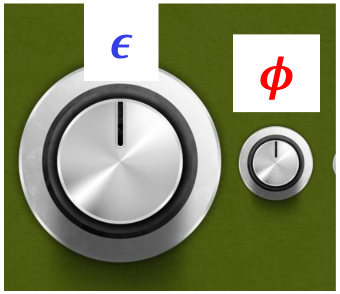


Gate Control

Process Tomography

Randomized Benchmarking

Comparison to Existing Techniques



Gate Control

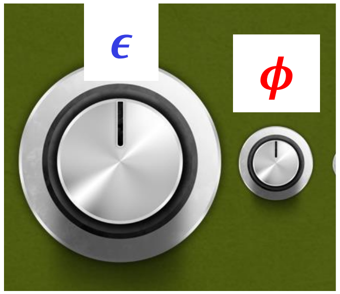
Process Tomography

Randomized Benchmarking

Gate Set Tomography

[Blume-Kohout et al ‘13]

Comparison to Existing Techniques



Gate Control

Process Tomography

Randomized Benchmarking

Gate Set Tomography

[Blume-Kohout et al ‘13]

- Don’t need to assume anything about state preparation, measurements or other gates
- Extremely Inefficient: need to learn ~25 parameters to extract ϕ and ϵ

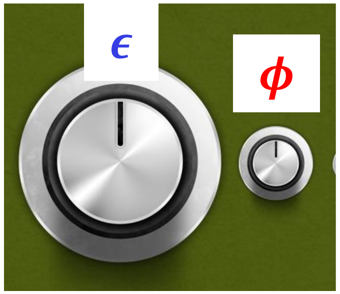
Comparison to Existing Techniques

Process Tomography

GST

Randomized Benchmarking

Comparison to Existing Techniques



Gate Control

Process Tomography GST

Randomized Benchmarking

Robust Phase Estimation

- Don't need perfect state preparation and measurement
- No additional gates*
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Outline

1. Motivation for Robust Phase Estimation
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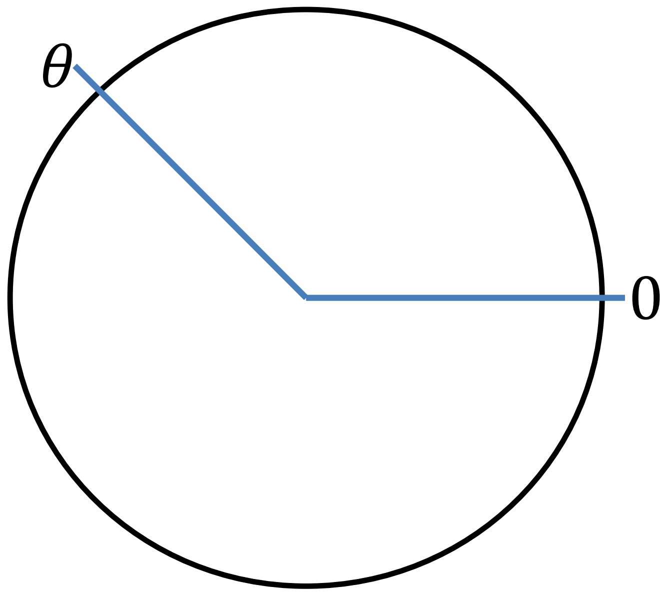
Phase Estimation [Higgins et al. '09]

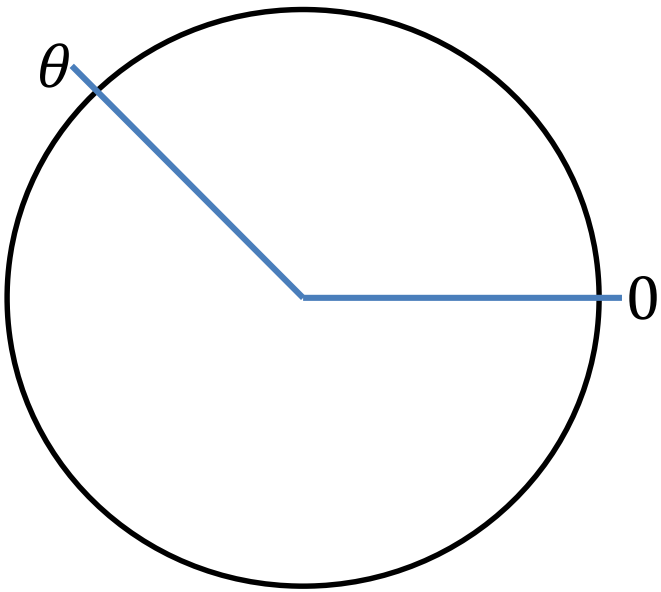


Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1+\sin k\theta}{2}, \qquad \frac{1+\cos k\theta}{2}$$

For k in $\mathbb{Z}$, each in time k

Phase Estimation [Higgins et al. '09]



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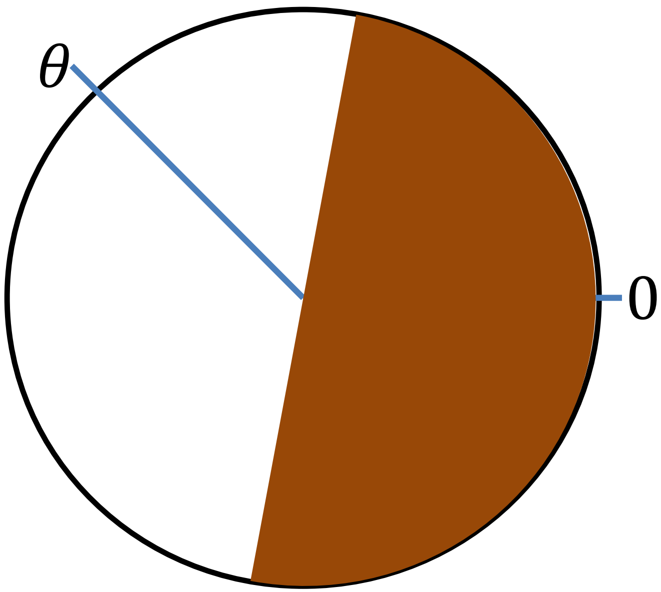
$k = 1$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{\sqrt{T}}$

Phase Estimation [Higgins et al. '09]



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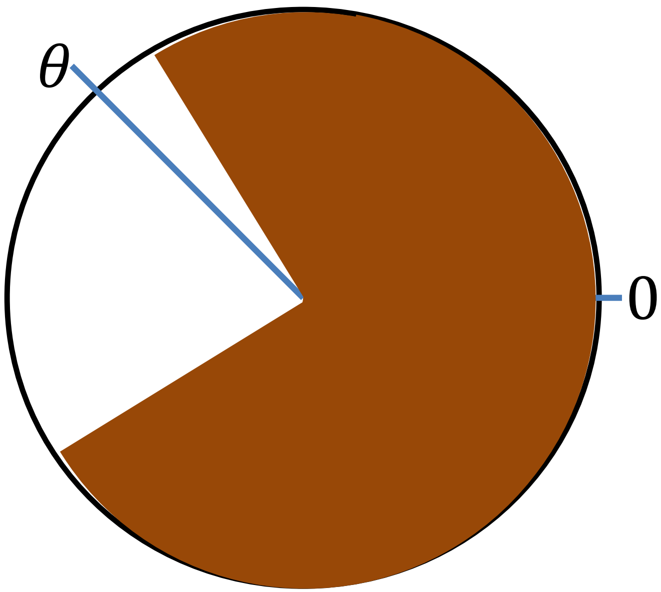

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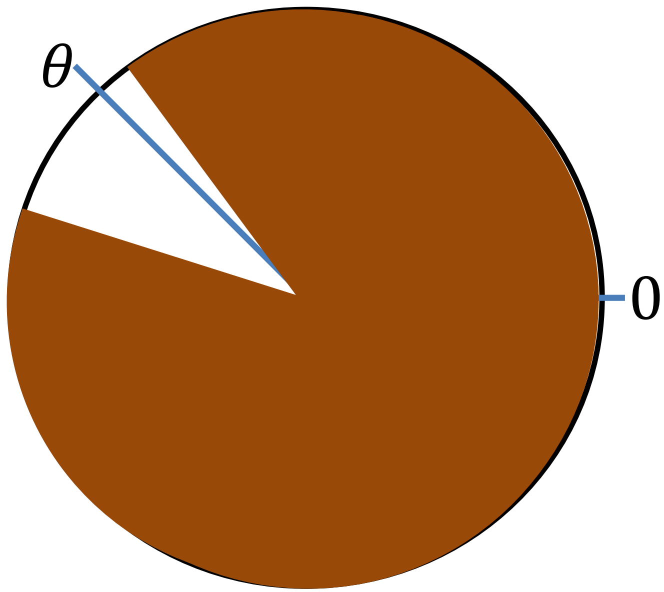
For k in $\mathbb{Z}$, each in time k

$k=1 \qquad k=2$

Phase Estimation [Higgins et al. '09]



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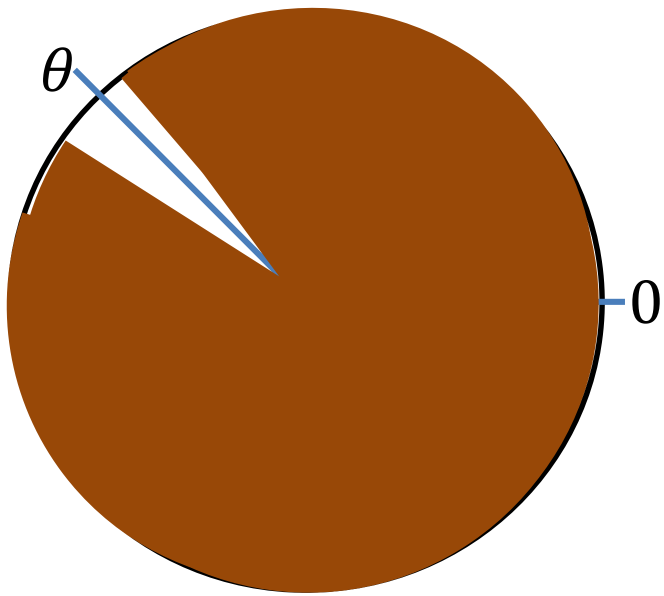
For k in $\mathbb{Z}$, each in time k

$k = 1 \quad k = 2 \quad k = 4$

Phase Estimation [Higgins et al. ‘09]



Can sample from 2 binomial random variables with probability of “heads”

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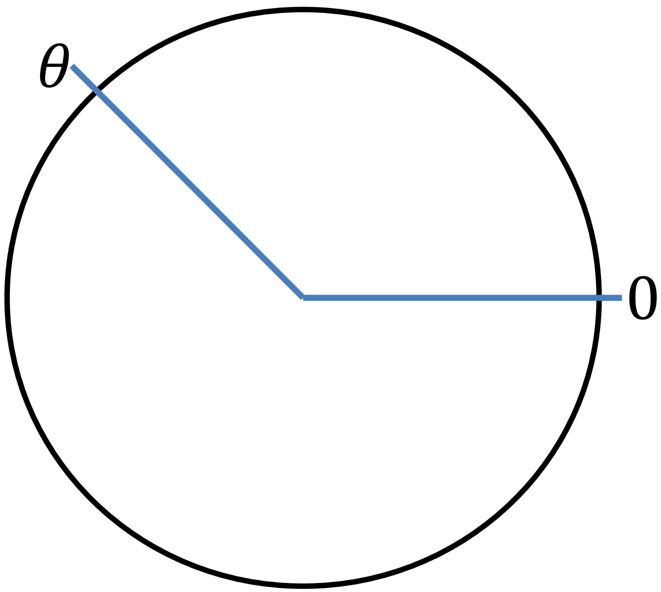
For k in $\mathbb{Z}$, each in time k

$k=1 \quad k=2 \quad k=4 \quad k=8$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$

Optimal – by information theory.

Robust Phase Estimation

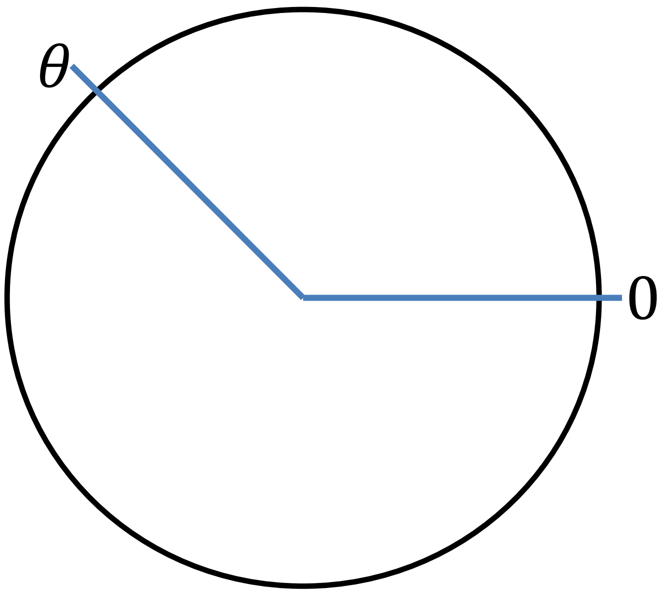


Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

For k in $\mathbb{Z}$, each in time k

Robust Phase Estimation



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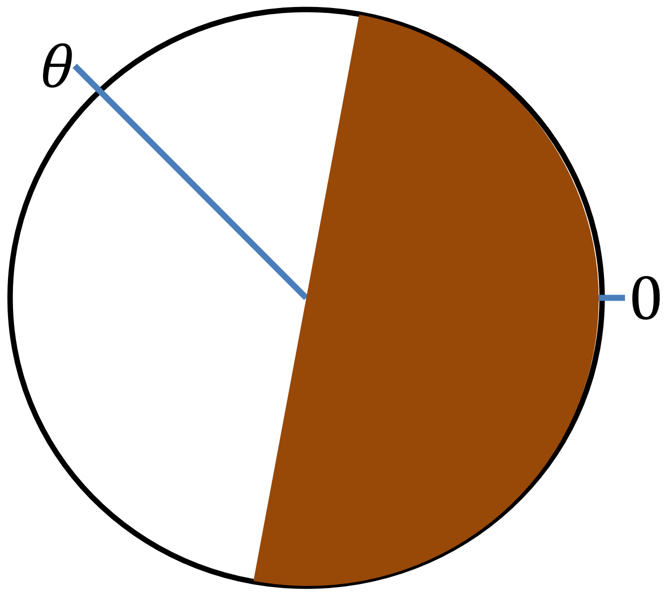
$$\frac{1+\sin\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos\theta}{2}+\delta_{k2}$$

Using only $k=1$ can't get an accurate estimate!

Robust Phase Estimation



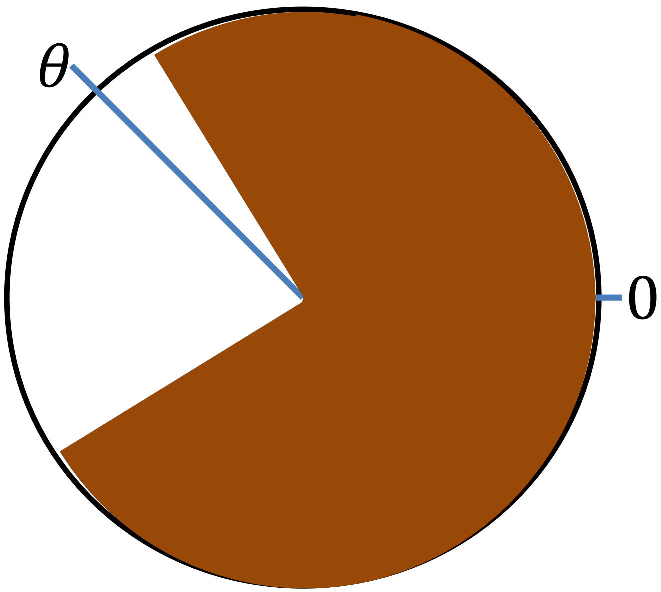
Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1 + \sin k\theta}{2} + \delta_{k1}, \qquad \frac{1 + \cos k\theta}{2} + \delta_{k2}$$

For k in $\mathbb{Z}$, each in time k

$k = 1$

Robust Phase Estimation



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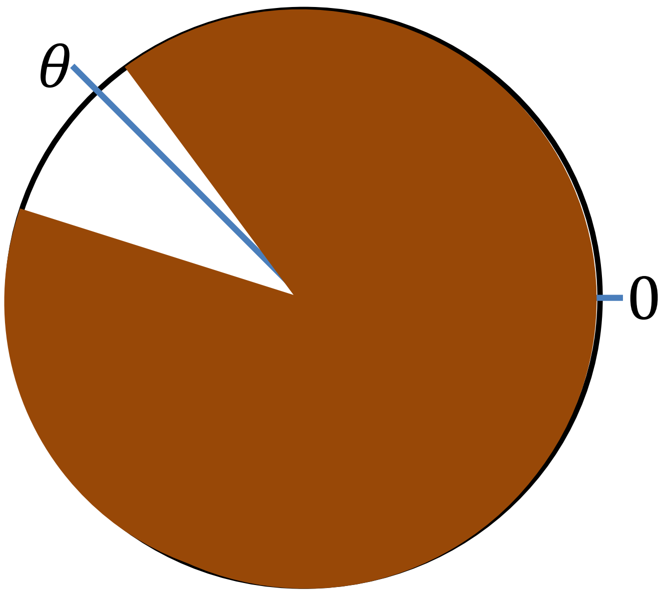

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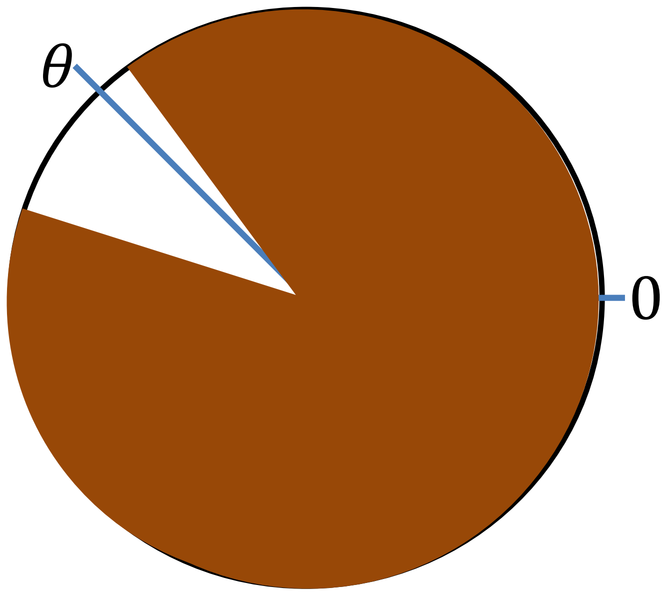
For k in $\mathbb{Z}$, each in time k

$k=1 \qquad k=2 \qquad k=4$

Can estimate θ with standard deviation $\sigma(\theta)\sim\frac{1}{T}$,

as long as $|\delta_k|<\frac{1}{\sqrt{8}}\approx .35$ for all k.

Robust Phase Estimation



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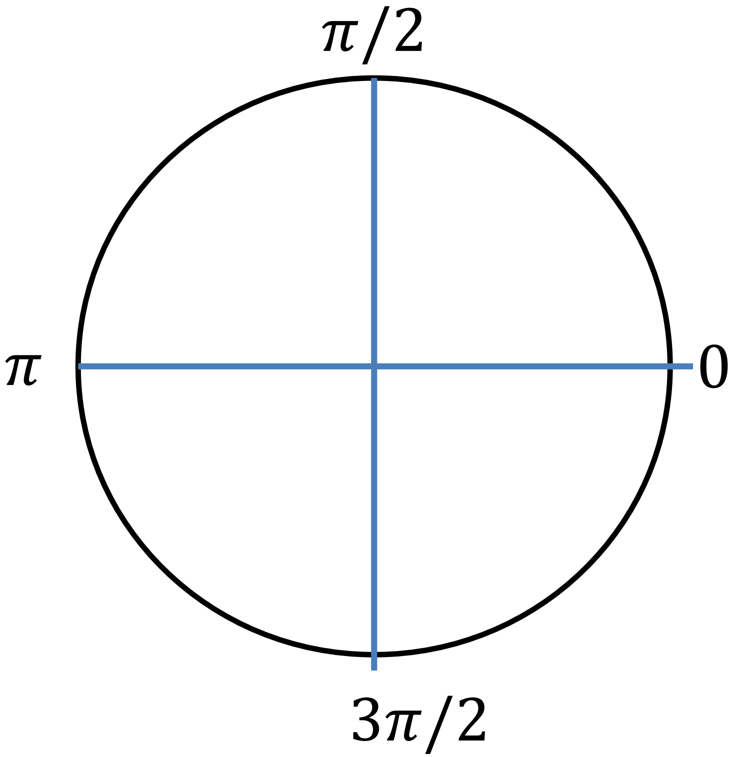
Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$,

as long as $|\delta_k| < \frac{1}{\sqrt{8}} \approx .35$ for all k.

…but need upper bound on size of δ to know how many extra samples to take.

Proof Sketch



Binomial variable variance: $np(1-p)$

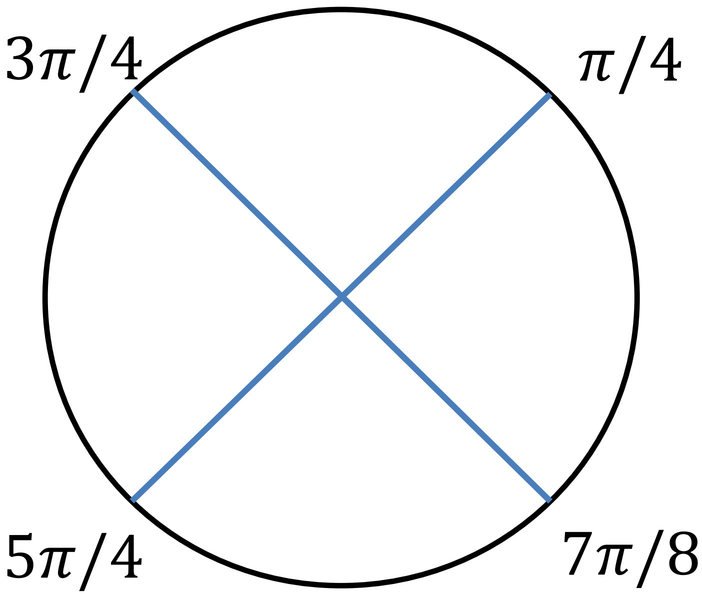
Variance small when $p = 0,1$

When $k\theta \sim \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$,

$\frac{1+\sin k\theta}{2}$ or $\frac{1+\cos k\theta}{2}$ equals 1 or 0

Even with δ errors, “heads” probability still close to 1 or 0

Proof Sketch



Binomial variable variance: $np(1-p)$

Variance largest when $p \approx 1/2$

When $k\theta \sim \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$,

$\frac{1+\sin k\theta}{2}$, $\frac{1+\cos k\theta}{2}$ equals $\frac{1}{2} \pm \frac{1}{\sqrt{8}}$

If δ error is $> \frac{1}{\sqrt{8}}$, can trick you into excluding the wrong half.

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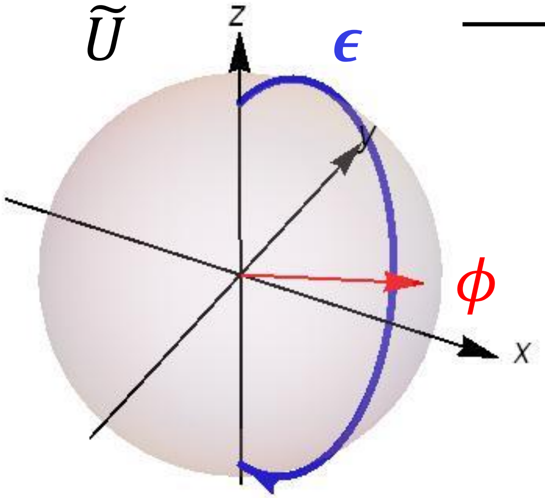
Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities like:



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Robust Phase Estimation for Gate Estimation

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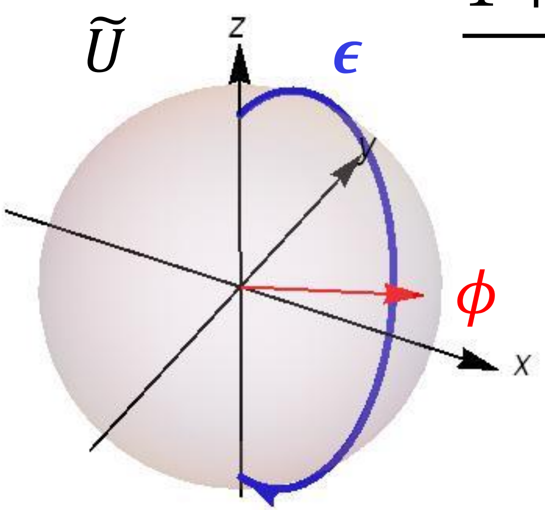


$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

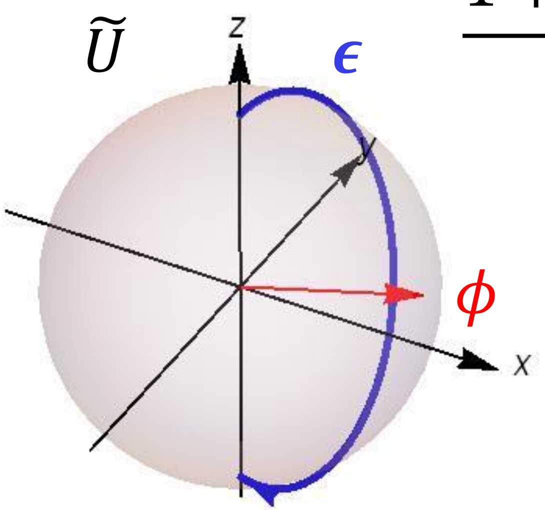
$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

$A=\pi(1+\epsilon)$ is total amplitude of rotation

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1},\qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

$A=\pi(1+\epsilon)$ is total amplitude of rotation

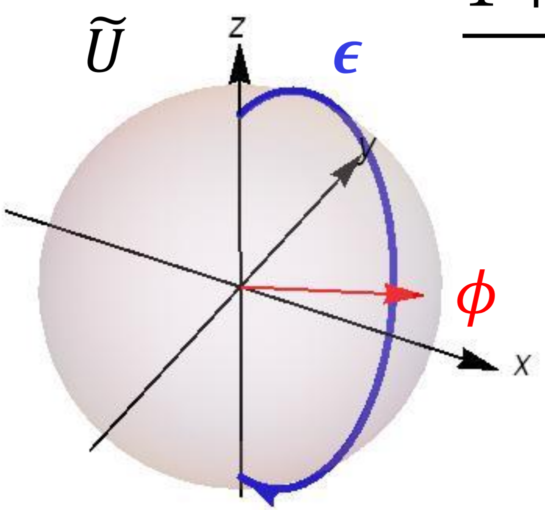
Size less < ϕ^2

Don’t need to know details!

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



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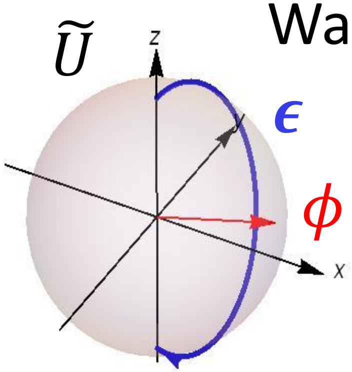
Heisenberg limited! Estimate of ϵ with standard deviation $\sigma(\epsilon)\sim\frac{1}{N}$, where N is the number of times $\widetilde{U}$ is applied.

Robust Phase Estimation for Gate Estimation



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$$\left|\langle 0|(Z_{-\pi/2}\widetilde{U}Z_{\pi}\widetilde{U}Z_{-\pi/2})^{k}|0\rangle\right|^{2}=\frac{1+\cos m_{\epsilon}k\phi}{2}+O(\epsilon^{2})$$

$$\left|\langle 0|(Z_{-\pi/2}\widetilde{U}Z_{\pi}\widetilde{U}Z_{-\pi/2})^{k}|\rightarrow\rangle\right|^{2}=\frac{1+\sin m_{\epsilon}k\phi}{2}+O(\epsilon^{2})$$

Heisenberg limited! Estimate of ϕ with standard deviation $\sigma(\phi)\sim\frac{1}{N}$, where N is the number of times $\widetilde{U}$ is applied.

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Looks like need perfect $\left|\langle 0|\widetilde{U}^k| \rightarrow\rangle\right|^2$

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- Imperfect measurement
- Additional errors like depolarizing errors
- Imperfect Z rotation

Example: State Preparation Errors Add to δ errors

Want: $\left|\langle 0|\widetilde{U}^k| \rightarrow\rangle\right|^2$

Suppose can only prepare $|0\rangle$, measure $|0\rangle$.
No perfect X-rotation, so can't prepare $|\rightarrow\rangle$.
Instead prepare $\rho'_{\rightarrow}$

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Trace Distance: $\mathrm{D}(\rho,\sigma)$ = maximum difference in probability between any two experiments on states ρ,σ.

Thus if use $\rho'_{\rightarrow}$ instead of $|\rightarrow\rangle$, δ error changes by at most $\mathrm{D}(\rho'_{\rightarrow}, |\rightarrow\rangle\langle\rightarrow|)$

Example: State Preparation Errors Add to δ errors

Want experiment with outcome probability:

$$\left|\langle 0|\tilde{U}^k|\rightarrow\rangle\right|^2 = \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho_\rightarrow)\right)$$

Have experiment with outcome probability:

$$\mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_\rightarrow)\right)$$

$$\mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_\rightarrow)\right) = \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho_\rightarrow)\right) - \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_\rightarrow - \rho_\rightarrow)\right)$$

Have

Want

$< \mathrm{D}(\rho_\rightarrow, \rho'_\rightarrow)$

Example: Depolarizing Errors

$$\Lambda(\rho) = \gamma\rho + (1-\gamma)\mathbb{I}/2$$

$$\frac{1+\sin k\theta}{2} + \delta_{k1} \rightarrow \frac{1}{2} + \gamma^k \left(\frac{\sin k\theta}{2} + \delta_{k1} \right)$$

$$\delta_{k1} \rightarrow \left(1-\gamma^k\right)\frac{\sin k\theta}{2} + \gamma^k \delta_{k1}$$

Depolarizing Errors Cause δ errors to increase with increasing k
→ eventually will overwhelm .35 bound.

Robust phase estimation will give most accurate estimate possible, up to the k where passes .35 bound. (No longer efficient).

Additional Errors

~~Looks like need perfect $\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2$~~

All of the following errors simply contribute to δ errors

- Imperfect state preparation
- Imperfect measurement
- Additional errors like depolarizing errors
- Imperfect Z rotation

Can incorporate any error, as long as can bound how much that error will shift your outcome probability (and total shift is less than .35)

Bounding δ Errors

Need upper bounds on

- Size of ϕ, ϵ
- Trace distance between ideal and true state preparation
- "Trace distance" between ideal and true measurement

We provide simple (length-0/1) sequences to upper bound these quantities.

Sample Procedure

1. Bound δ errors
2. Choose # of samples to take each round based on size of δ errors and desired precision
3. Robust phase estimation on ϵ.
4. Robust phase estimation on ϕ.
5. Use controls to correct errors, repeat.

Sample Procedure

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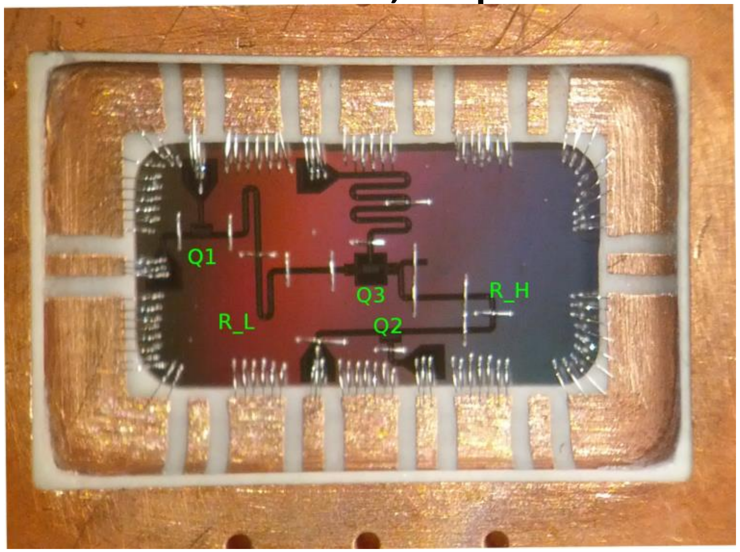
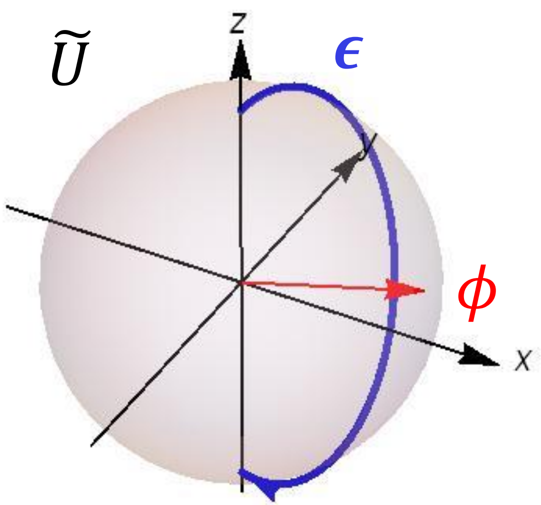


[BBN]

Recap:



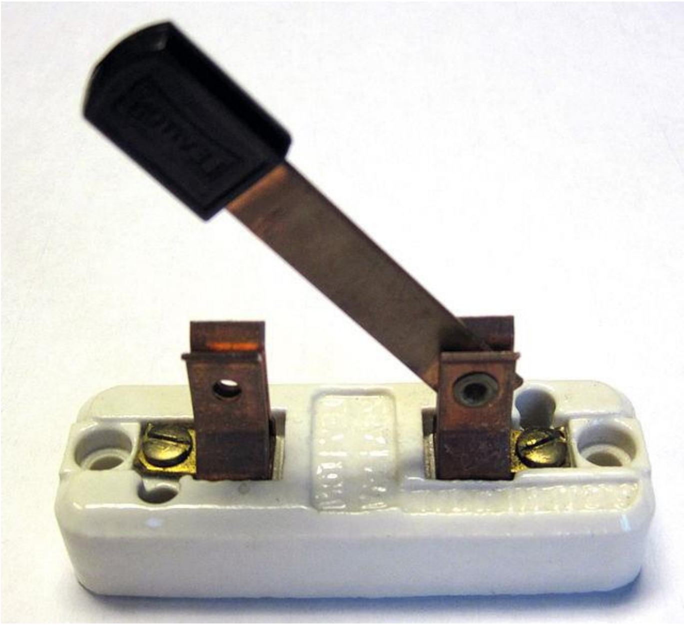
Robust Phase Estimation

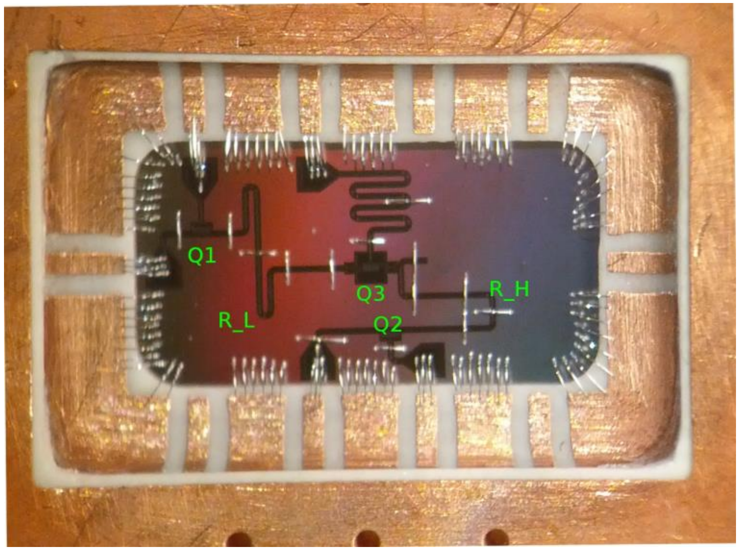
- Don’t need perfect state preparation and measurement
- No additional gates*
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Open Questions

- Multi-Qubit Operations?
- Connection to Randomized Benchmarking?
- Connection to Gate Set Tomography?
- What if figure of merit is number of experiments?

Think this might be useful?





[BBN]

Arxiv: 1502.02677

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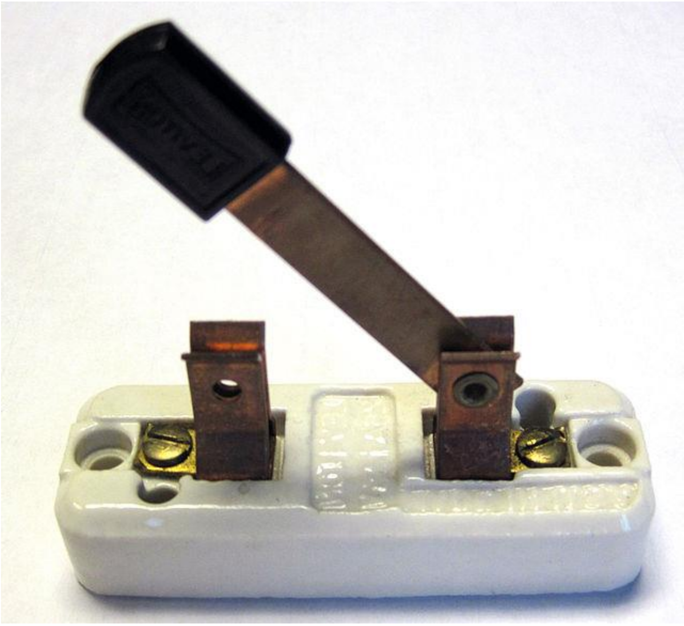
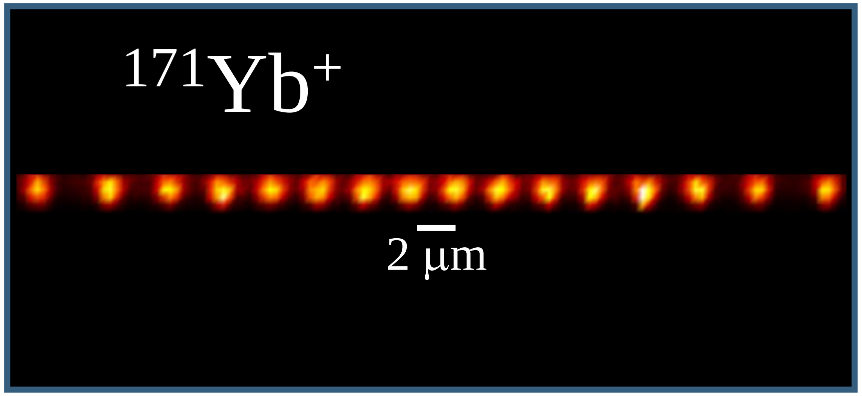
e.g. Controls have 5 digits of precision. Estimate ϵ, θ to 5 digits of precision, but after correcting still inaccurate at 3 digits of precision. δ errors could be cause.

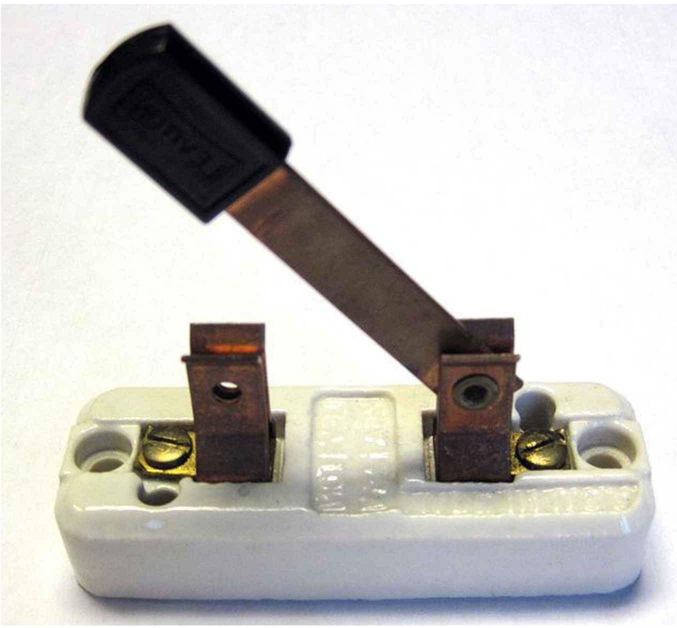
Imagine…

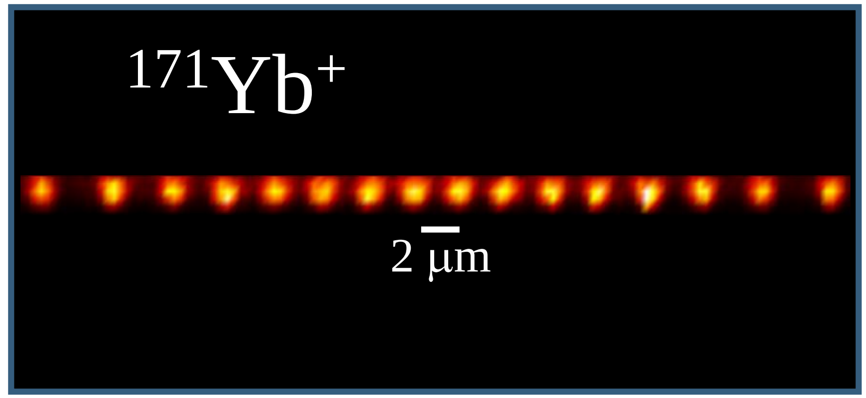




[Monroe Lab]

Imagine…





[Monroe Lab]

- All gates are off
- State preparation is off
- Measurements are off

Want to quickly determine imperfections in gate controls and then tune to fix.

Proof Sketch

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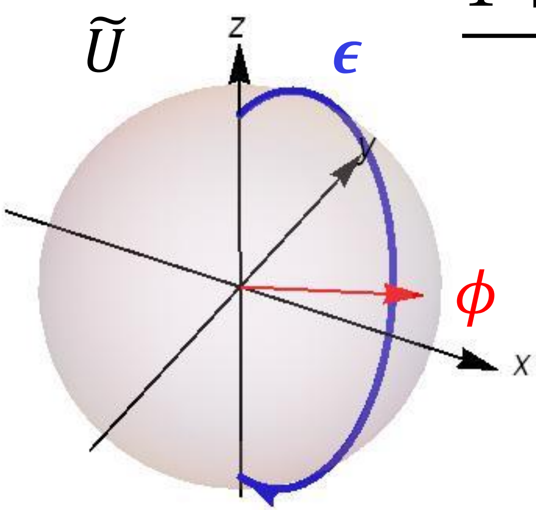

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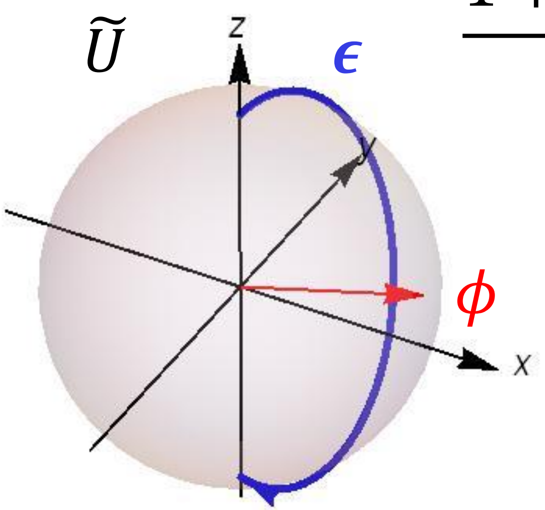

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Have depolarizing errors? OK! Just add to δ errors.

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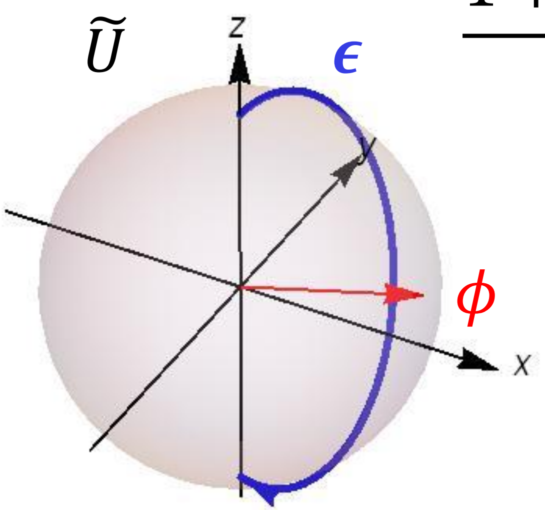

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