Robust Single-Qubit Process Calibration via Robust Phase Estimation

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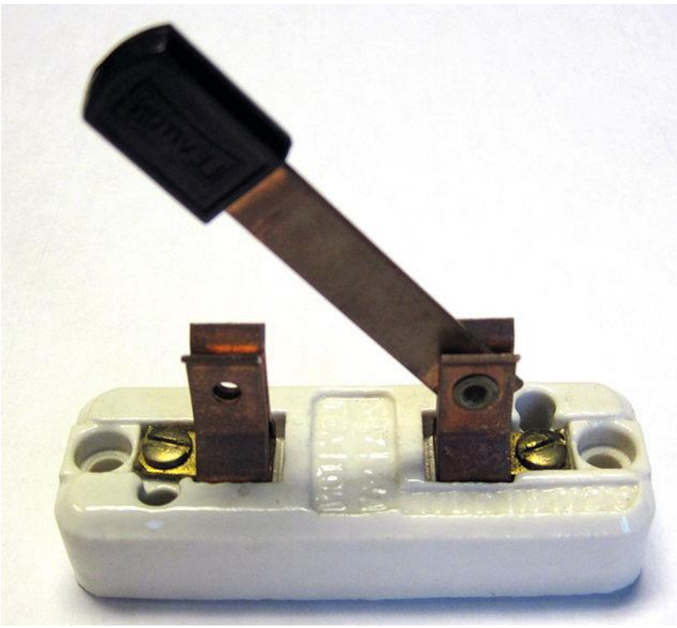
Arxiv: 1502.02677

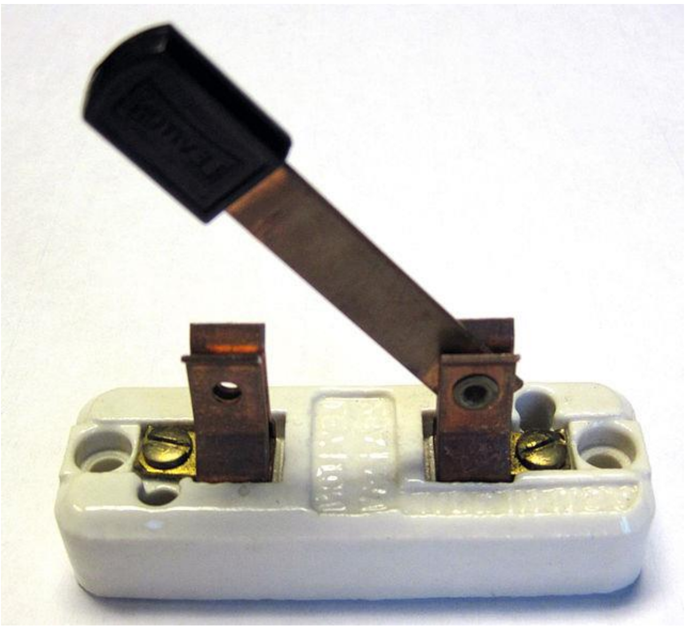
Joint Center for Quantum Information and Computer Science

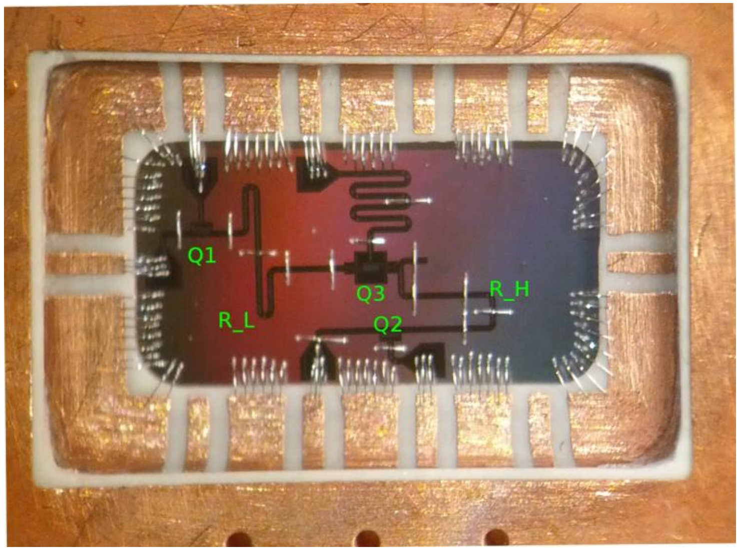
UMIACS
University of Maryland Institute for Advanced Computer Studies

Imagine…



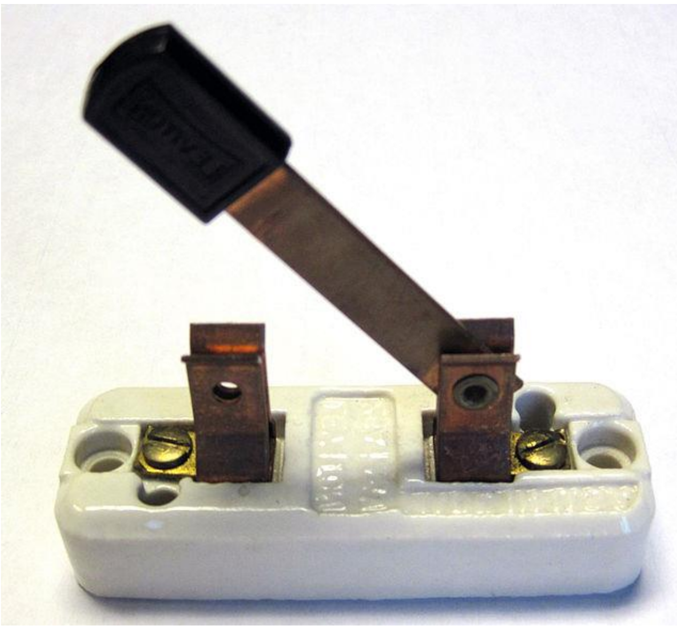
Imagine…

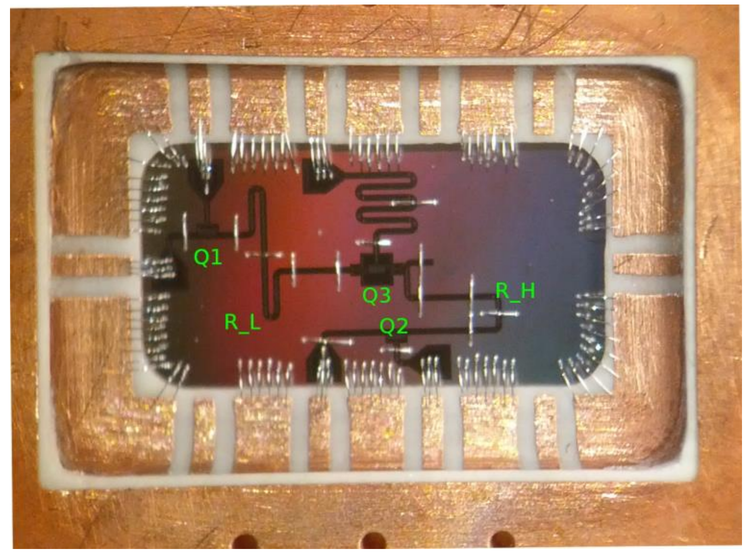




[BBN]

Imagine…





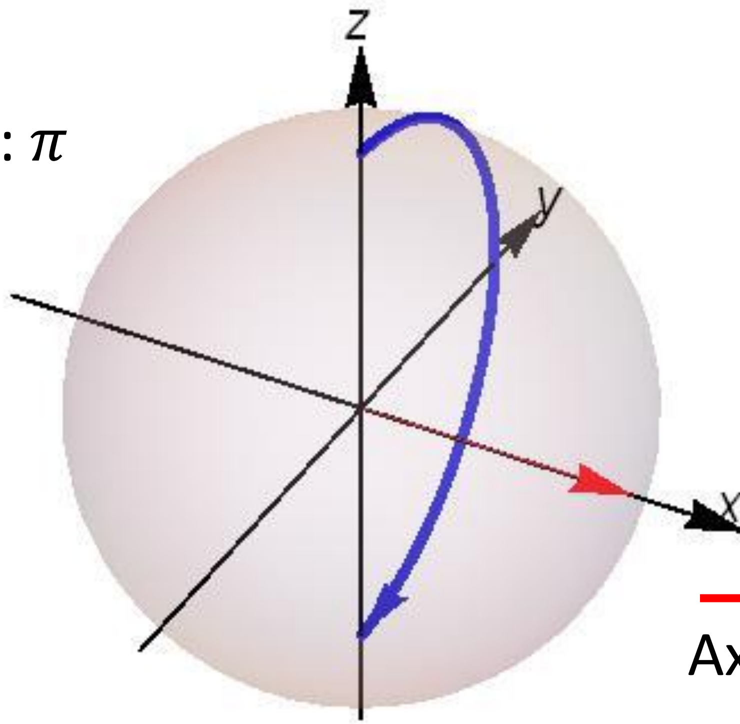
[BBN]

- All gates need to be tuned
- State preparation is off
- Measurements are off

→ Want to quickly determine imperfections in gate controls and then tune to fix.

Need to Calibrate Operations

Amplitude of Rotation: π

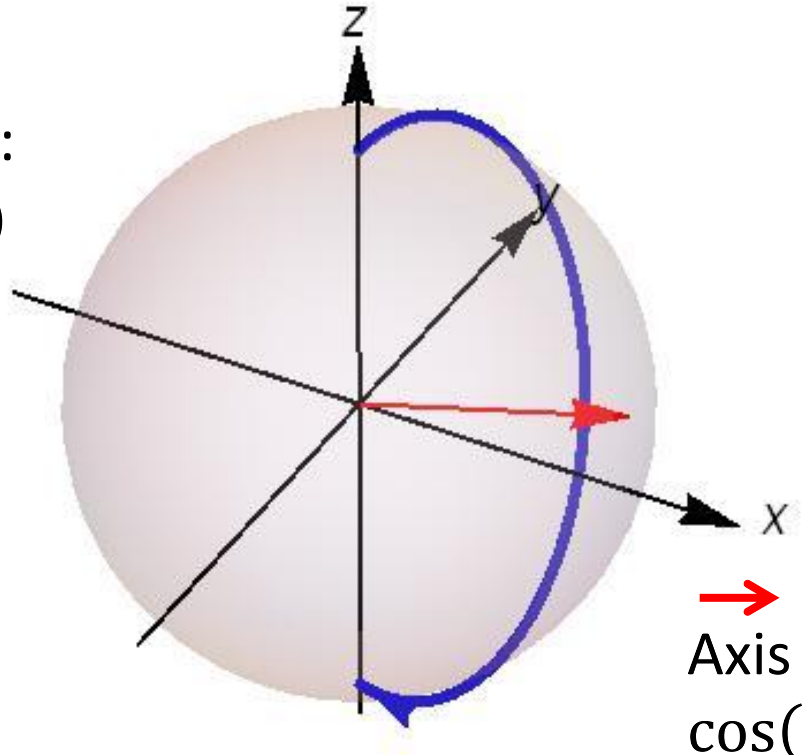


Axis of Rotation: $\hat{x}$

Ideal Unitary U

Need to Calibrate Operations

Amplitude of Rotation: $\vec{}$
$A = \pi(1+\epsilon)$



Axis of Rotation: $\vec{}$
$\cos(\phi)\hat{x} + \sin(\phi)\hat{z}$

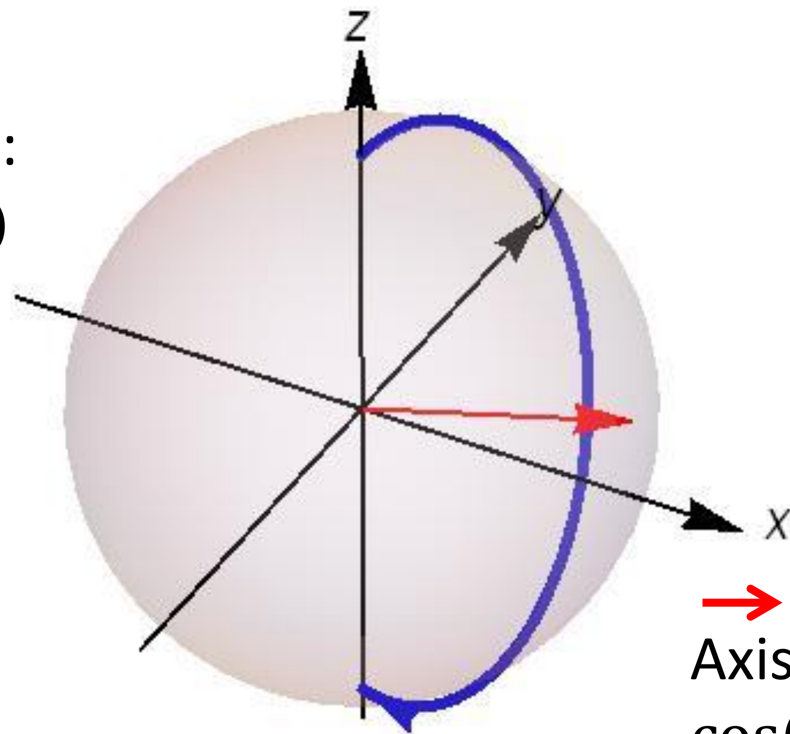
Unitary $\widetilde{U}$

Need to Calibrate Operations

Amplitude of Rotation:

$A = \pi(1+\epsilon)$

ϵ is an "Amplitude Error"



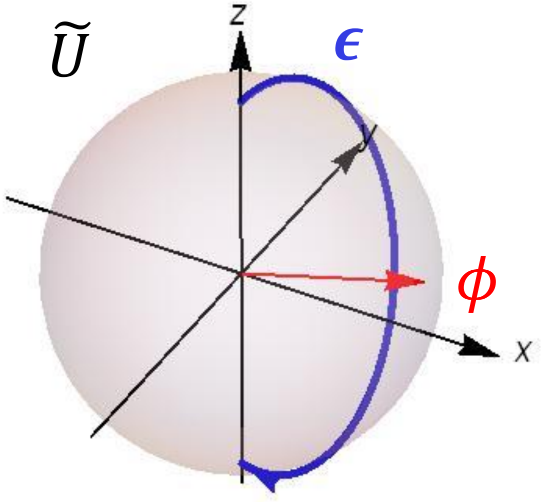
Unitary $\widetilde{U}$

Axis of Rotation:

$\cos(\phi)|x\rangle + \sin(\phi)\,|z\rangle$

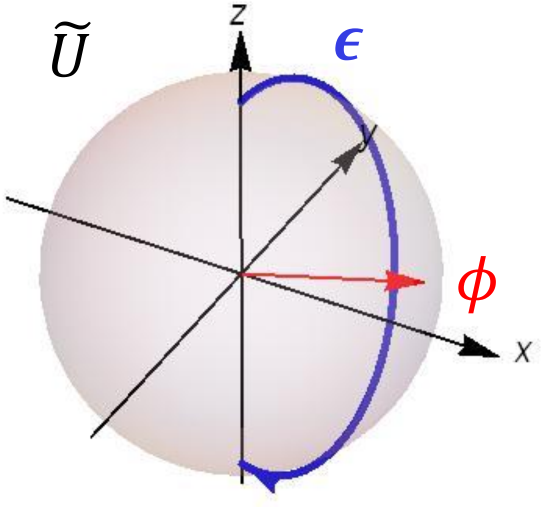
ϕ is an "Off-Resonance Error"

How to Estimate Control Errors



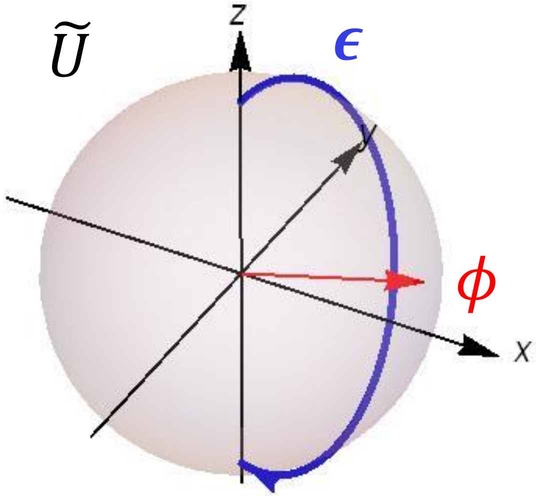
Ad hoc Rabi – Ramsey Sequences.

How to Estimate Control Errors



Process Tomography [Chuang & Nielsen '97]

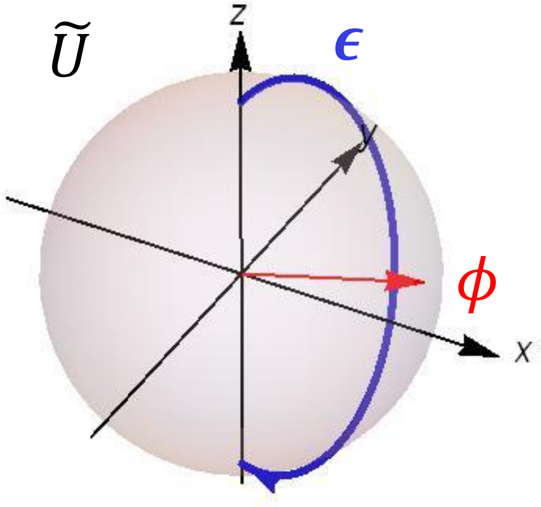
How to Estimate Control Errors



Process Tomography

- Need perfect state preparation and measurement
- Need perfect additional gates
- Time consuming: need to learn 12 parameters to extract ϕ and ϵ

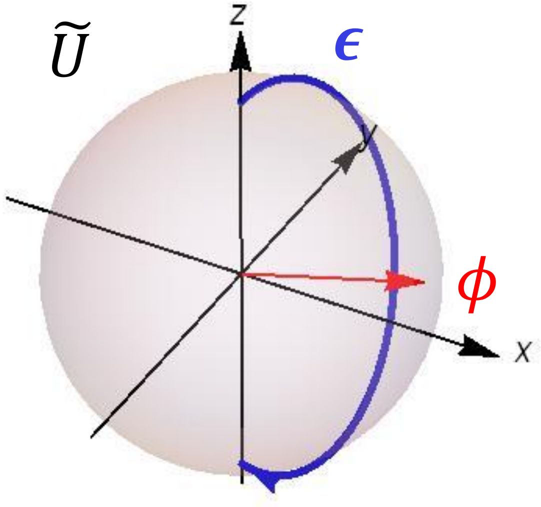
How to Estimate Control Errors



~~Process Tomography~~

Randomized Benchmarking Tomography

How to Estimate Control Errors

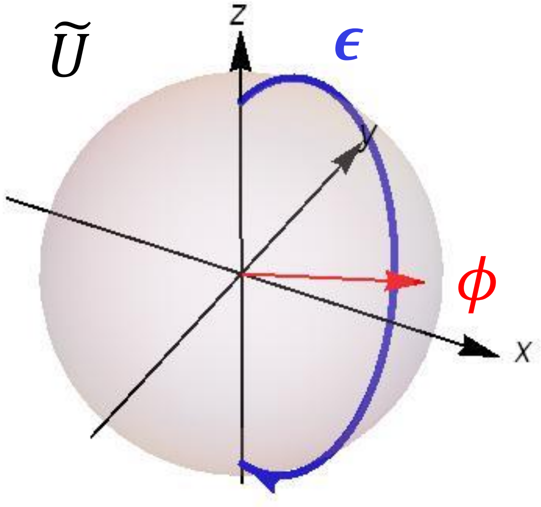


~~Process Tomography~~

Randomized Benchmarking

- Don’t need perfect state preparation and measurement
- Must be able to perform single-qubit Cliffords (although not perfectly)
- Time consuming: need to learn 9 parameters to extract ϕ and ϵ

How to Estimate Control Errors



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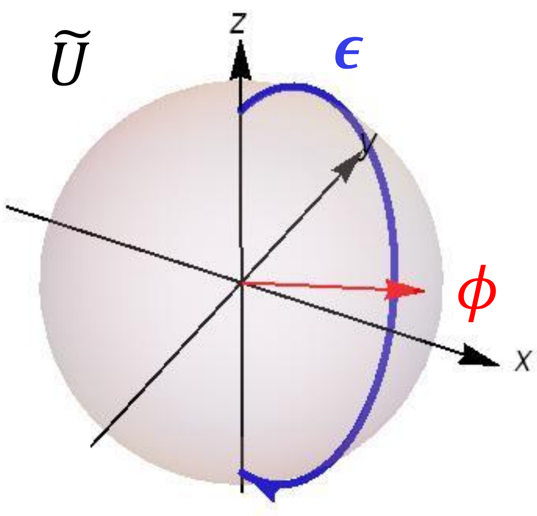


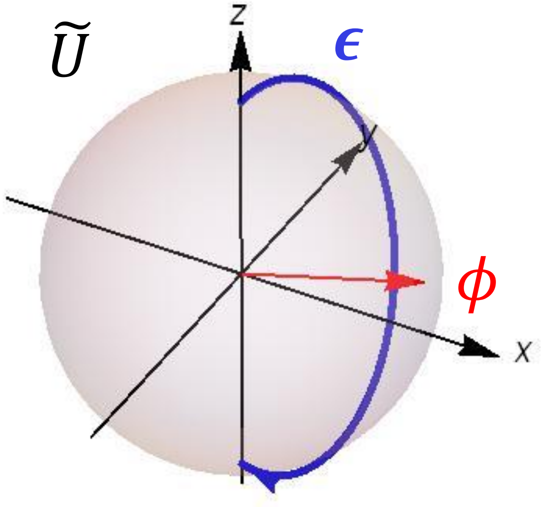
Process Tomography

Randomized Benchmarking

GST

[Blume-Kohout et al '13]

How to Estimate Control Errors



- ~~Process Tomography~~
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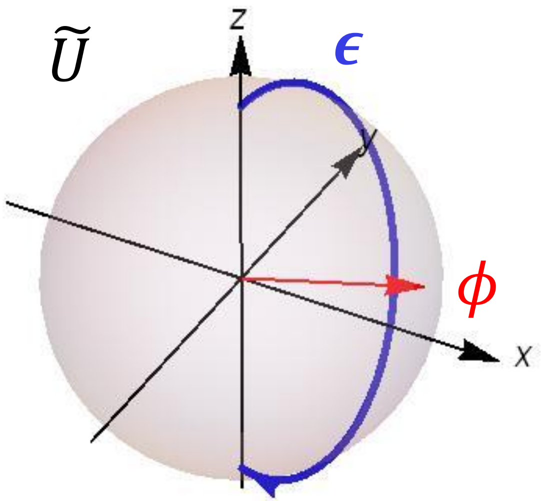


~~Process Tomography~~ ~~GST~~

~~Randomized Benchmarking~~

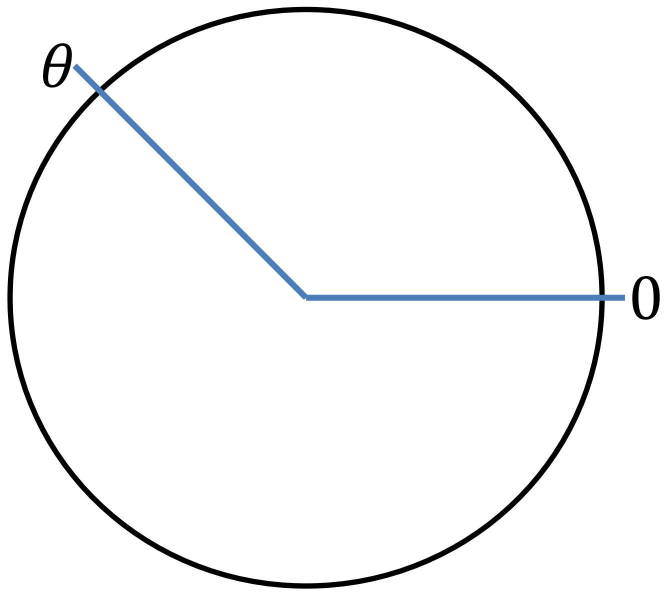
Robust Phase Estimation

- Don’t need perfect state preparation and measurement
- No additional gates, except Z-rotations
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Outline

- Motivation for Robust Phase Estimation
- Robust phase estimation
- Application to Parameter Estimation

Phase Estimation [Higgins et al. ‘09]

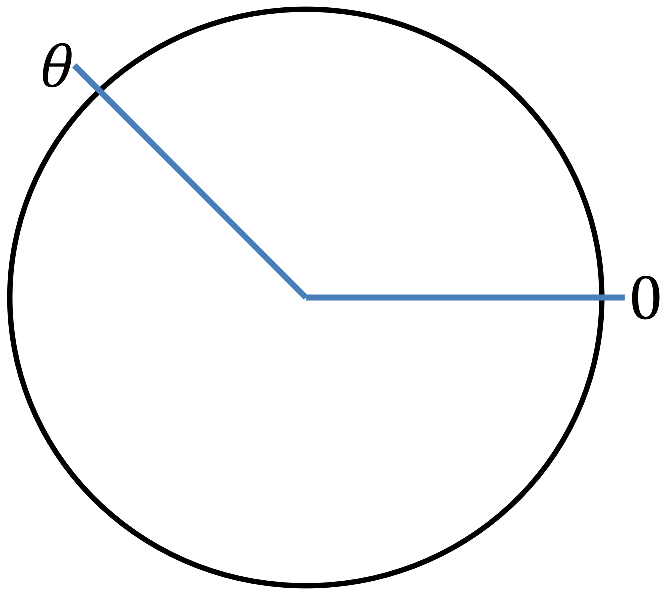


Can sample from 2 binomial random variables with probability of “heads”

$$\frac{1+\sin k\theta}{2}, \qquad \frac{1+\cos k\theta}{2}$$

For k in $\mathbb{Z}$, each in time k

Phase Estimation [Higgins et al. '09]



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$k = 1$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{\sqrt{T}}$

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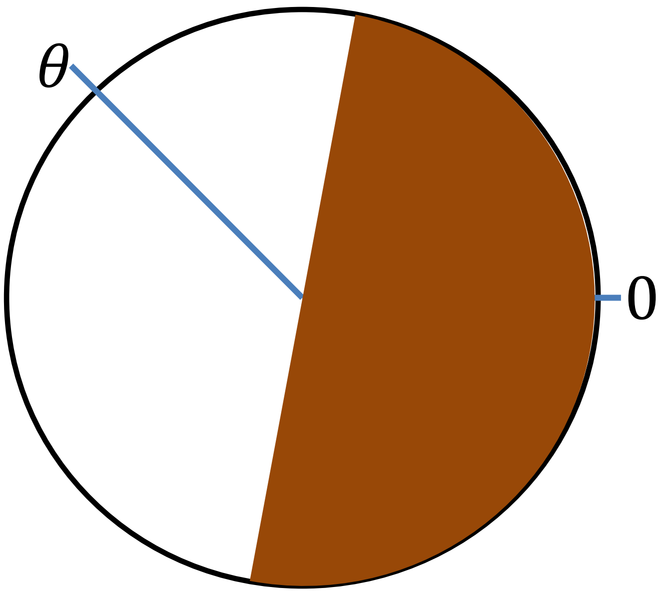

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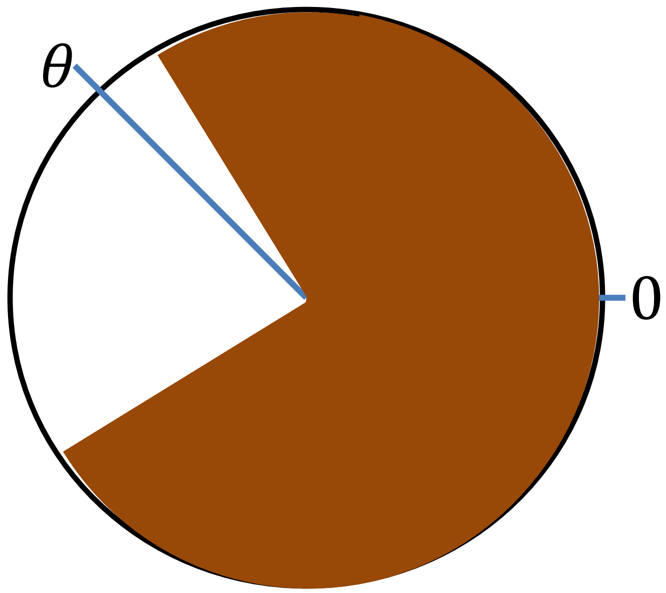

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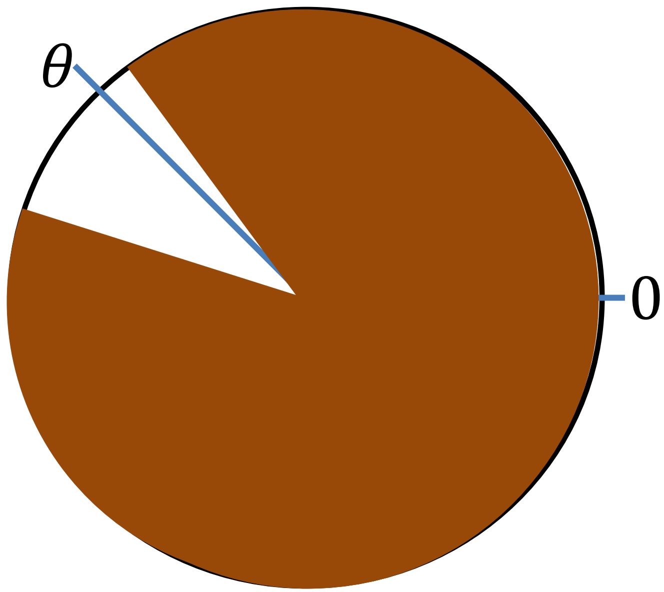
For k in $\mathbb{Z}$, each in time k

$k = 1 \quad k = 2$

Phase Estimation [Higgins et al. ‘09]



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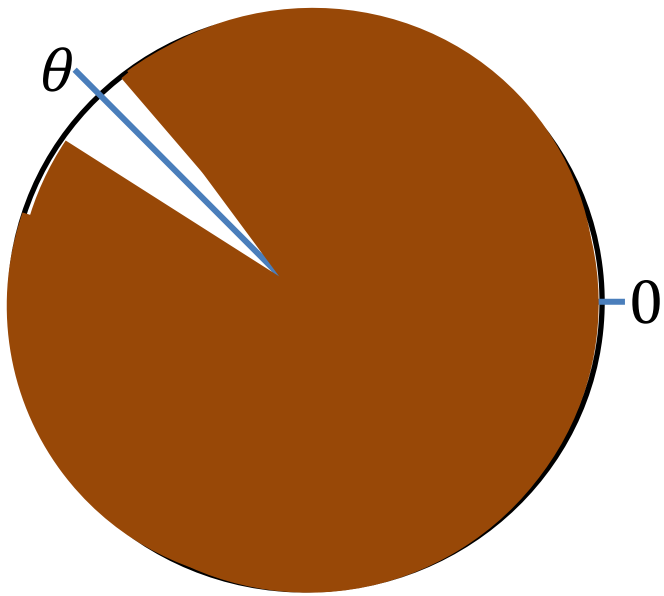

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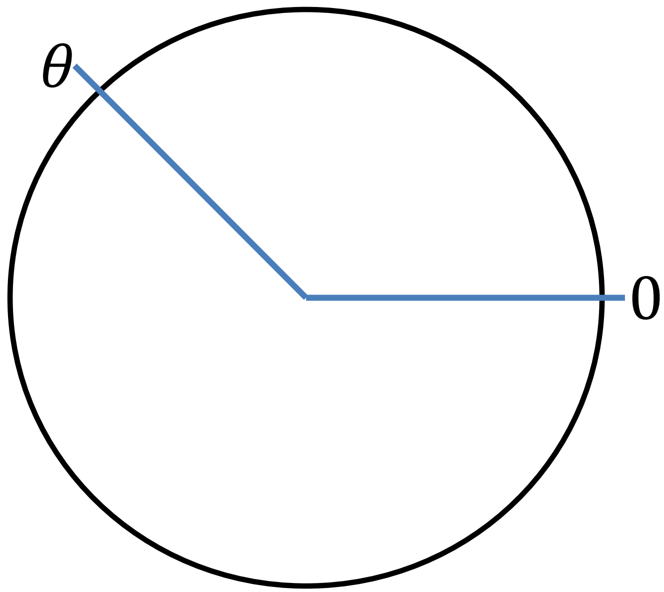
For k in $\mathbb{Z}$, each in time k

$k=1$ $k=2$ $k=4$ $k=8$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$

Optimal – by information theory.

Robust Phase Estimation

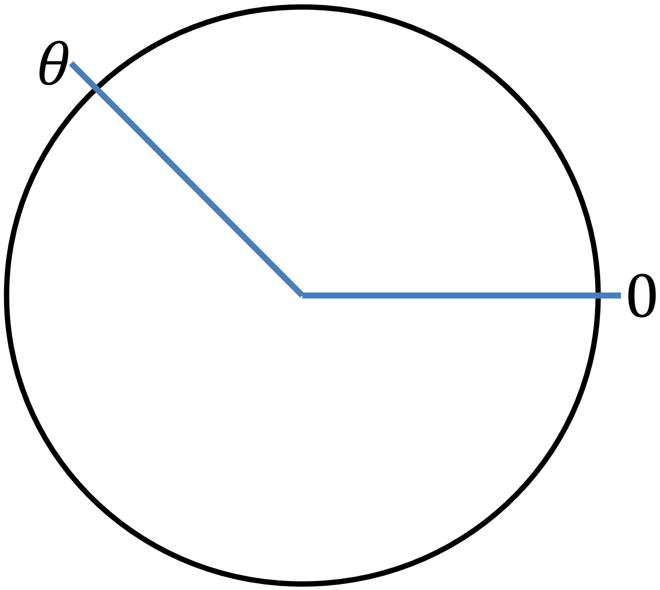


Can sample from 2 binomial random variables with probability of "heads"

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

For k in $\mathbb{Z}$, each in time k

Robust Phase Estimation



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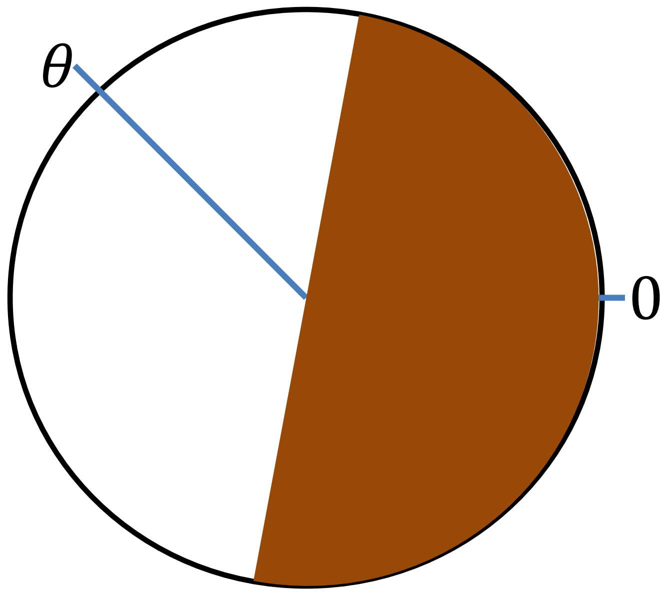
Using only $k = 1$ can't get an accurate estimate!

Robust Phase Estimation



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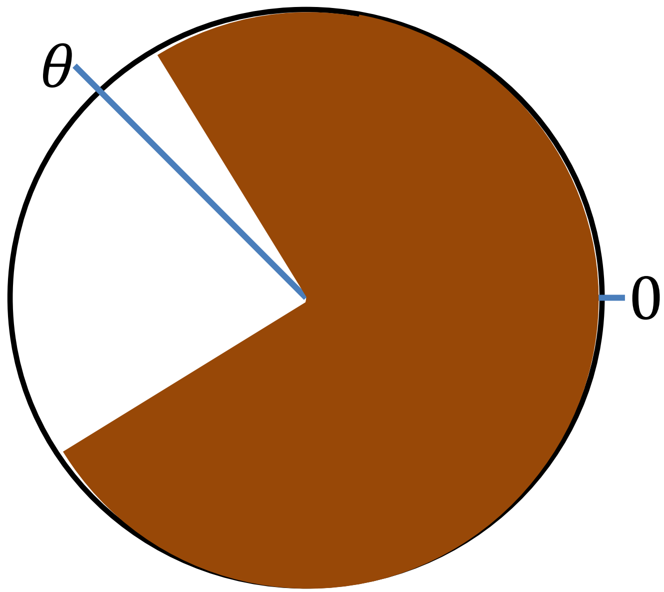
Robust Phase Estimation



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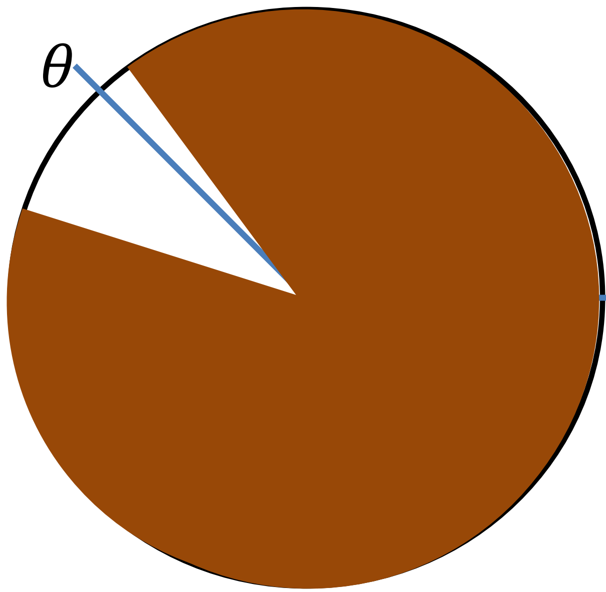

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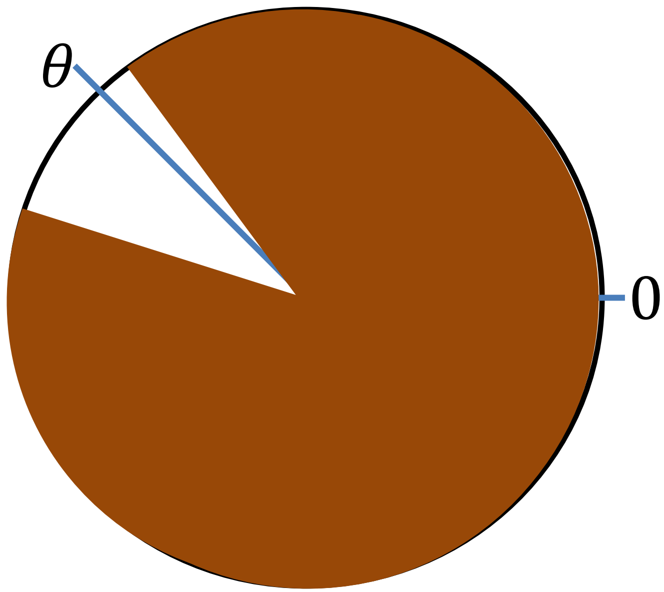
For k in $\mathbb{Z}$, each in time k

$k=1 \quad k=2 \quad k=4$

Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$,
as long as $|\delta_k| < .35$ for all k.

Robust Phase Estimation



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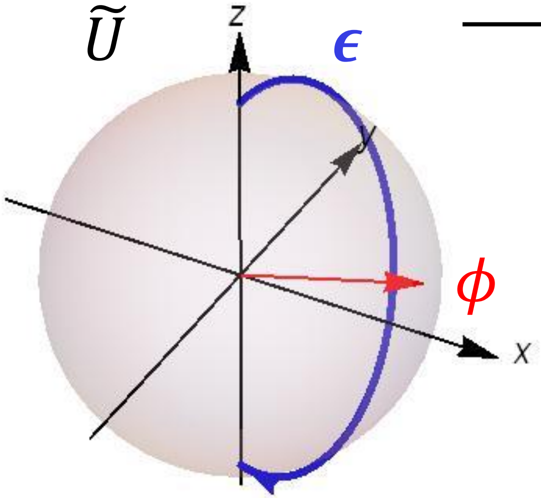
Can estimate θ with standard deviation $\sigma(\theta) \sim \frac{1}{T}$, as long as $|\delta_k| < .35$ for all k.

…but need upper bound on size of δ to know how many extra samples to take.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities like:



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Robust Phase Estimation for Gate Estimation

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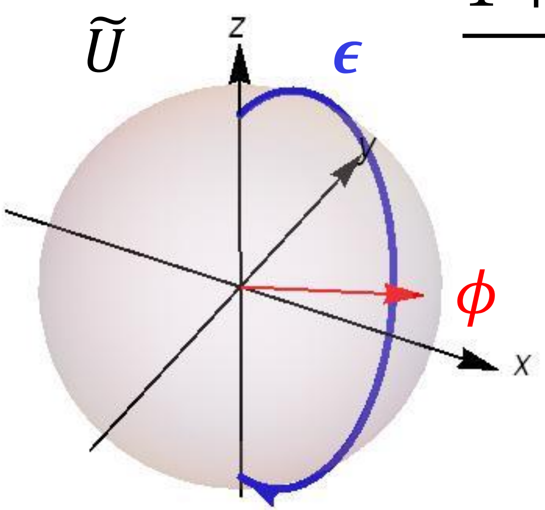

$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

$A=\pi(1+\epsilon)$ is total amplitude of rotation

Robust Phase Estimation for Gate Estimation

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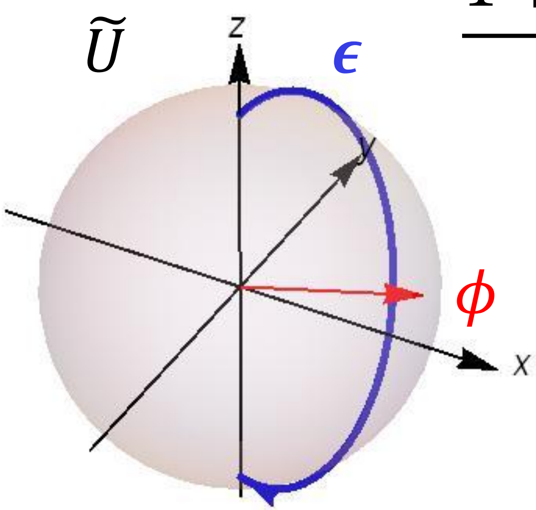

$$\left|\langle 0|\tilde{U}^k|0\rangle\right|^2 = \frac{1+\cos kA}{2} + \sin^2\frac{kA}{2}\sin^2\phi$$

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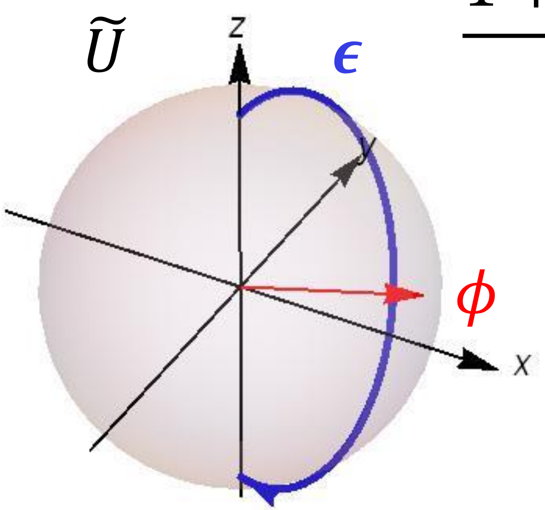
$A=\pi(1+\epsilon)$ is total amplitude of rotation

Size less < ϕ^2

Don't need to know details!

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:



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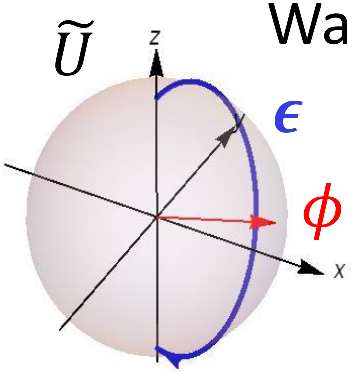
Heisenberg limited! Estimate of ϵ with standard deviation $\sigma(\epsilon)\sim\frac{1}{N}$, where N is the number of times $\tilde{U}$ is applied.

Robust Phase Estimation for Gate Estimation



Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$

$$\left|\langle 0|(Z_{-\pi/2}\widetilde{U}Z_{\pi}\widetilde{U}Z_{-\pi/2})^k|0\rangle\right|^2 \approx \frac{1+\cos m_\epsilon k\phi}{2}+O(\epsilon^2)$$

$$\left|\langle 0|(Z_{-\pi/2}\widetilde{U}Z_{\pi}\widetilde{U}Z_{-\pi/2})^k|\rightarrow\rangle\right|^2 \approx \frac{1+\sin m_\epsilon k\phi}{2}+O(\epsilon^2)$$

Heisenberg limited! Estimate of ϕ with standard deviation $\sigma(\phi) \sim \frac{1}{N}$, where N is the number of times $\widetilde{U}$ is applied.

Additional Errors

Looks like need perfect $\left|\langle 0|\widetilde{U}^k| \rightarrow\rangle\right|^2$

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All of the following errors simply contribute to δ errors

- Imperfect state preparation
- Imperfect measurement
- Additional errors like depolarizing errors
- Imperfect Z rotation

Example: State Preparation Errors Add to δ errors

Want: $\left|\langle 0|\widetilde{U}^k| \rightarrow\rangle\right|^2$

Suppose can only prepare $|0\rangle$, measure $|0\rangle$.
No perfect X-rotation, so can't prepare $|\rightarrow\rangle$.
Instead prepare $\rho'_{\rightarrow}$

Example: State Preparation Errors Add to δ errors

Want: $\left|\langle 0\middle|\widetilde{U}^k\middle| \rightarrow\rangle\right|^2$

Suppose can only prepare $|0\rangle$, measure $|0\rangle$.
No perfect X-rotation, so can't prepare $|\rightarrow\rangle$.
Instead prepare $\rho'_{\rightarrow}$

Trace Distance: $\mathrm{D}(\rho, \sigma)$ = maximum difference in probability between any two experiments on states ρ, σ.

Thus if use $\rho_{\rightarrow}$ instead of $|\rightarrow\rangle$, δ error changes by at most $\mathrm{D}(\rho'_{\rightarrow}, |\rightarrow\rangle\langle\rightarrow|)$

Example: State Preparation Errors Add to δ errors

Want experiment with outcome probability:

$$\left|\langle 0|\tilde{U}^k| \rightarrow\rangle\right|^2 = \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho_{\rightarrow})\right)$$

Have experiment with outcome probability:

$$\mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_{\rightarrow})\right)$$

$$\mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_{\rightarrow})\right) = \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho_{\rightarrow})\right) - \mathrm{tr}\left(M_0 \tilde{\mathcal{U}}^k(\rho'_{\rightarrow} - \rho_{\rightarrow})\right)$$

Have

Want

$< \mathrm{D}(\rho_{\rightarrow}, \rho'_{\rightarrow})$

Additional Errors

~~Looks like need perfect $\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2$~~

All of the following errors simply contribute to δ errors

- Imperfect state preparation
- Imperfect measurement
- Additional errors like depolarizing errors
- Imperfect Z rotation

Can incorporate any error, as long as can bound how much that error will shift your outcome probability (and total shift is less than .35)

Bounding δ Errors

Need upper bounds on

- Size of ϕ, ϵ
- Trace distance between ideal and true state preparation
- "Trace distance" between ideal and true measurement

We provide simple (length-0/1) sequences to upper bound these quantities.

Sample Procedure

1. Bound δ errors
2. Choose # of samples to take each round based on size of δ errors and desired precision
3. Robust phase estimation on ϵ.
4. Robust phase estimation on θ.
5. Use controls to correct errors, repeat.

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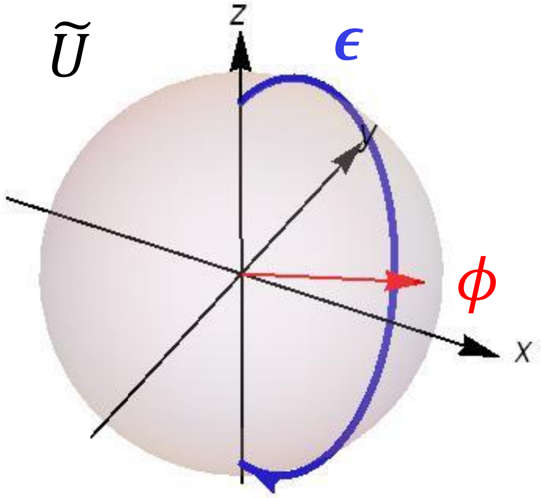
If bad δ bounds, ϵ, θ estimates accurate, just not as precise.

e.g. Controls have 5 digits of precision. Estimate ϵ, θ to 5 digits of precision, but after correcting still inaccurate at 3 digits of precision. δ errors could be cause.

Recap:



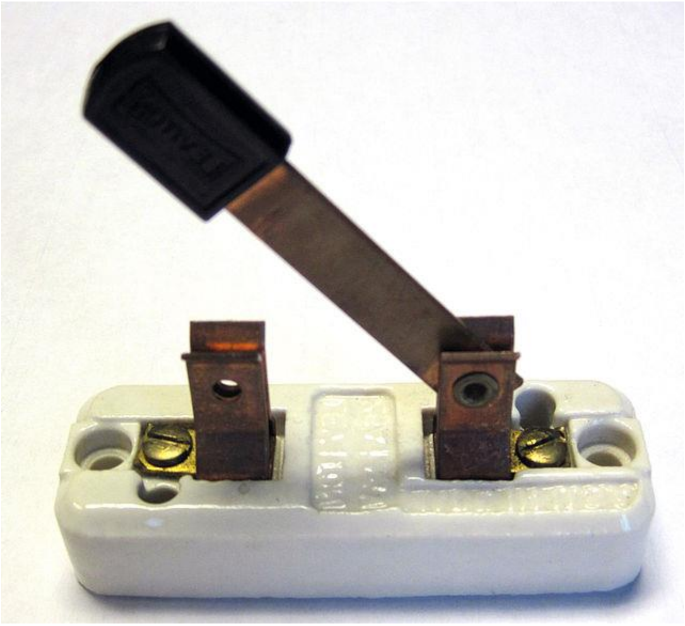
Robust Phase Estimation

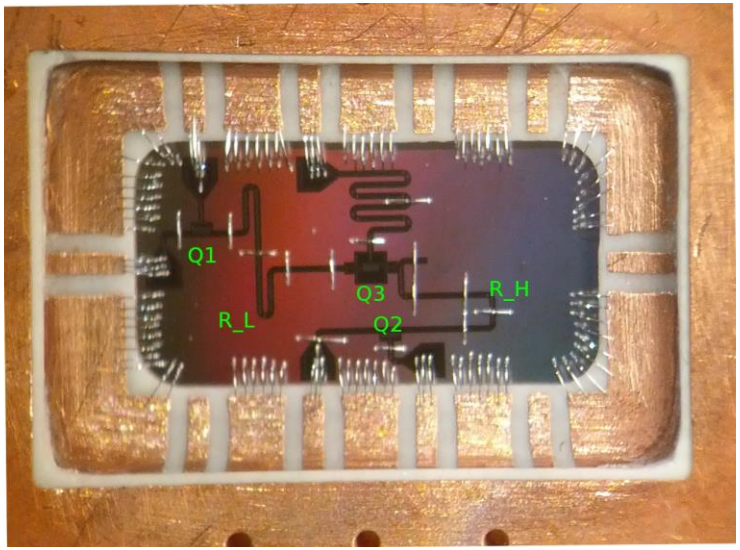
- Don't need perfect state preparation and measurement
- No additional gates, except Z-rotations
- Learn ϕ and ϵ with optimal efficiency
- Non-adaptive
- Accommodates additional errors like depolarizing noise.

Open Questions

- Multi-Qubit Operations?
- No perfect Z-rotation?
- Connection to Randomized Benchmarking

Think this might be useful?





[BBN]

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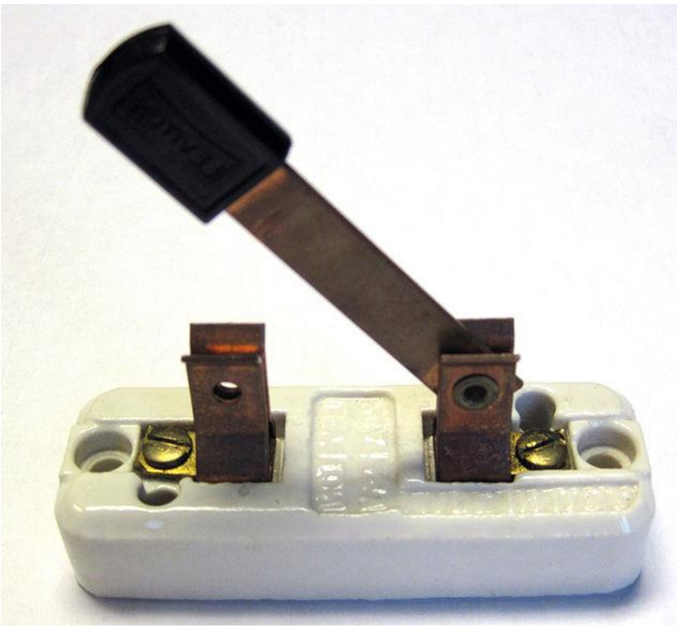

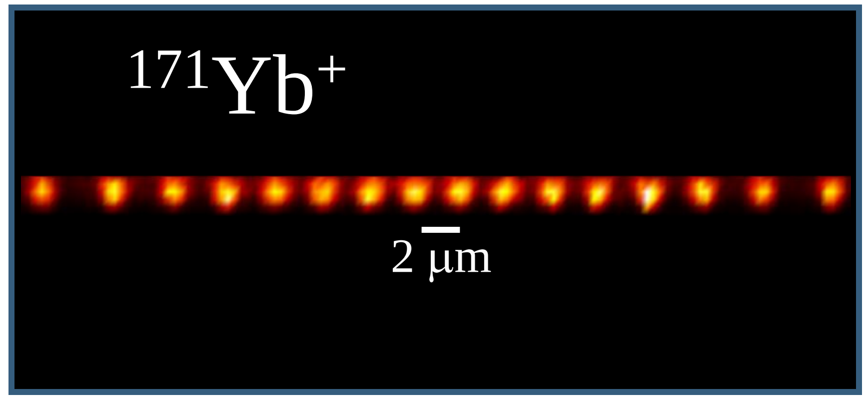

[Monroe Lab]

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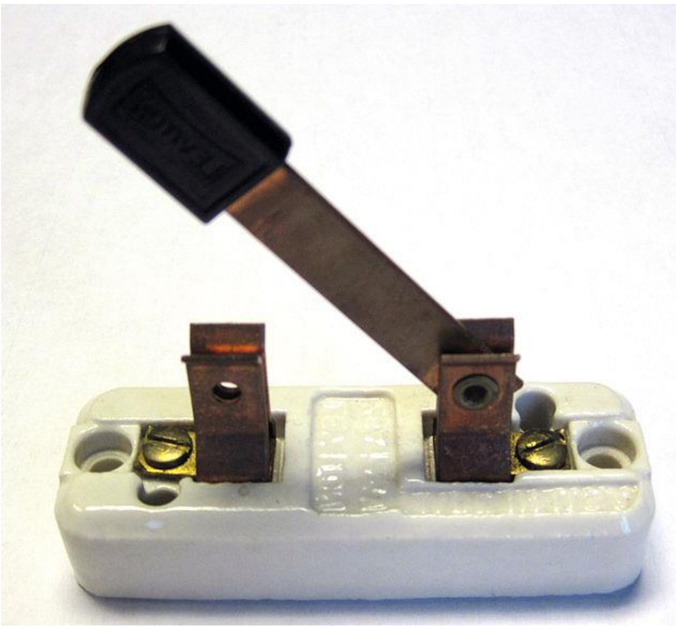

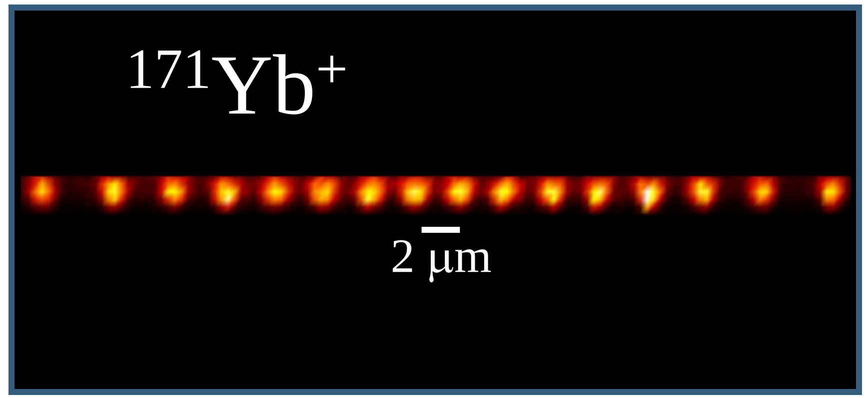

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- State preparation is off
- Measurements are off

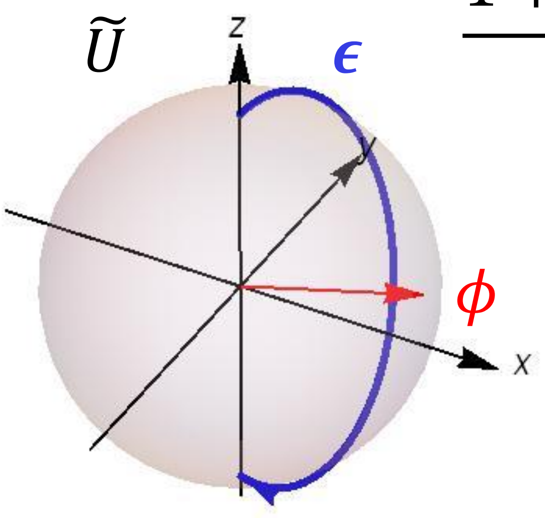
→ Want to quickly determine imperfections in gate controls and then tune to fix.

Proof Sketch

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

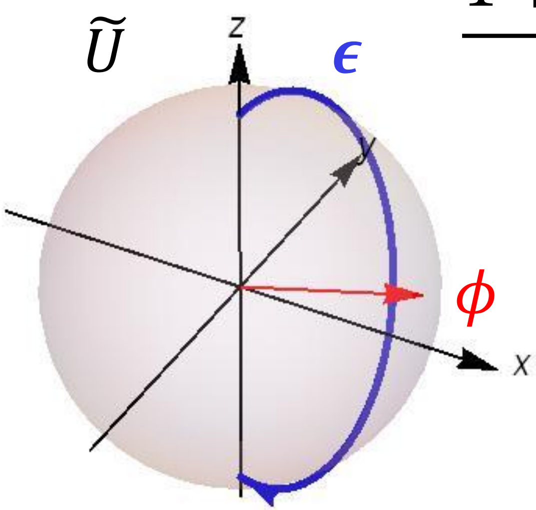
$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

Don’t have perfect state prep and measurement? OK! Just add to δ error.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\tilde{U}^k|0\rangle\right|^2 = \frac{1+\cos kA}{2} + \sin^2\frac{kA}{2}\sin^2\phi$$

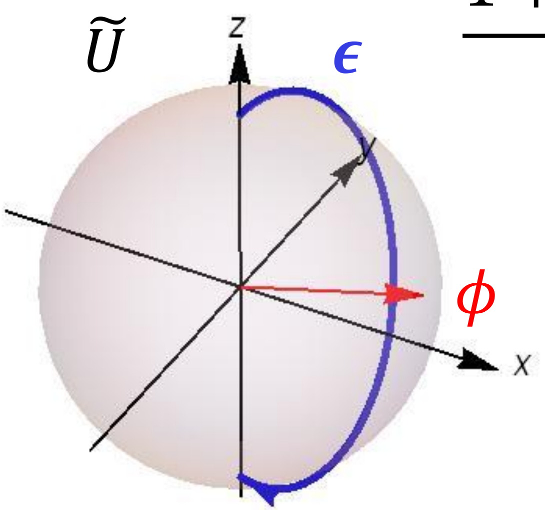
$$\left|\langle 0|\tilde{U}^k|\rightarrow\rangle\right|^2 = \frac{1+\sin kA}{2} - \sin kA\sin^2\frac{\phi}{2}$$

Have depolarizing errors? OK! Just add to δ errors.

Robust Phase Estimation for Gate Estimation

Want 2-outcome experiments with probabilities:

$$\frac{1+\sin k\theta}{2}+\delta_{k1}, \qquad \frac{1+\cos k\theta}{2}+\delta_{k2}$$



$$\left|\langle 0|\widetilde{U}^k|0\rangle\right|^2=\frac{1+\cos kA}{2}+\sin^2\frac{kA}{2}\sin^2\phi$$

$$\left|\langle 0|\widetilde{U}^k|\rightarrow\rangle\right|^2=\frac{1+\sin kA}{2}-\sin kA\sin^2\frac{\phi}{2}$$

Heisenberg limited! Estimate of ϵ with standard deviation $\sigma(\epsilon)\sim\frac{1}{N}$, where N is the number of times $\widetilde{U}$ is applied.