Robust Characterization of Quantum Processes

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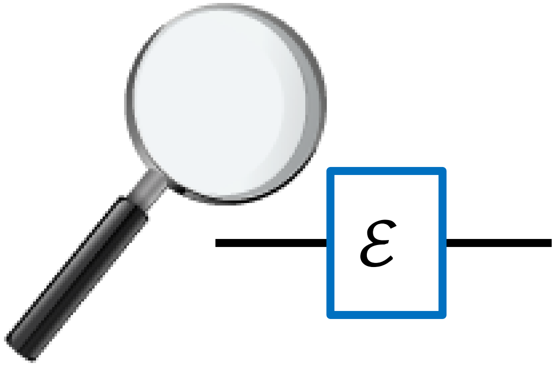
Why don’t we have a working quantum computer?

Too Many Errors

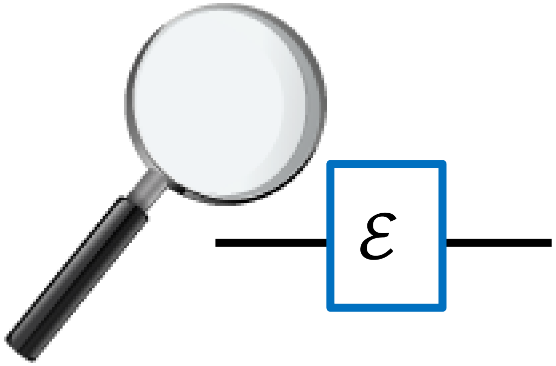
Can Improve Operations with Better Characterization of Errors



"Depolarizing error"

Improvement to Computer

Cooling

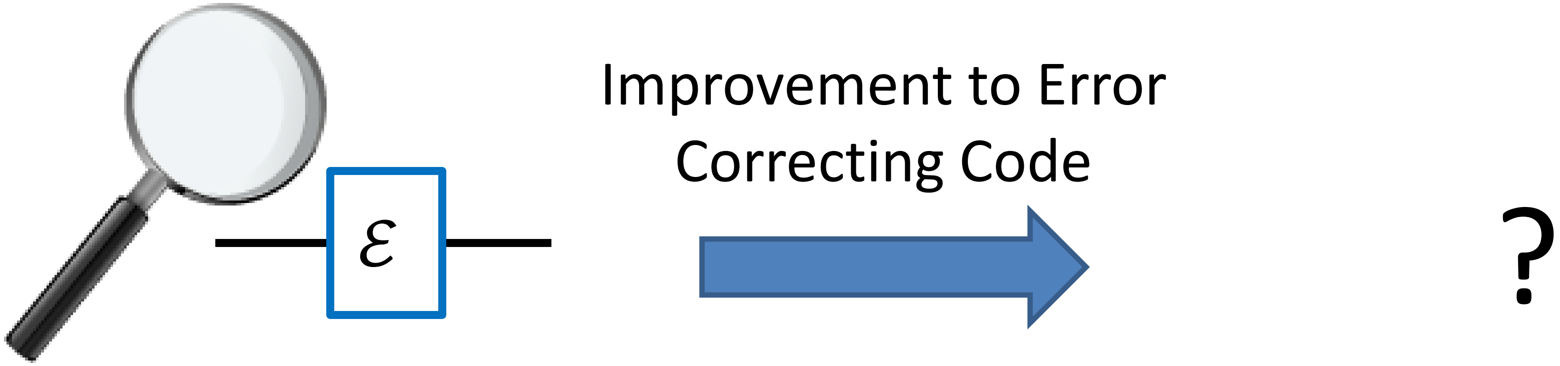


"Extra rotation around z-axis"

Improvement to Computer

Magnetic Shielding

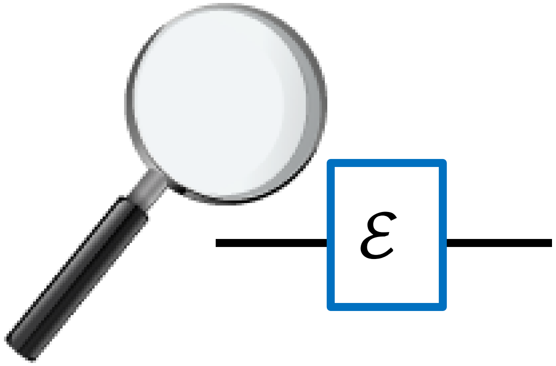
Can Improve Error Correcting Codes with Better Characterization of Errors



Improvement to Error Correcting Code

“Non local, correlated error”

Standard Techniques Have Problems



Need nearly perfect state preparation, measurement and other operations. Otherwise systematic errors give inaccurate or even invalid results.

Not "robust"

Robust Techniques

- Gate Set Tomography Procedures [Stark ‘13, Blume-Kohout et al. ’13, Merkel et al. ‘12]
  - Characterizes many processes at once
- Randomized Benchmarking (RB) [Emerson et al. ‘05, Knill et al. ‘08, Magesan et al. ‘11, ‘12]
  - Can only characterize 1 parameter of 1 type of process.

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  - Can only characterize ~~1~~ parameter of ~~1~~ type of process.

    almost all any
  - Can efficiently test performance of a universal gate set.

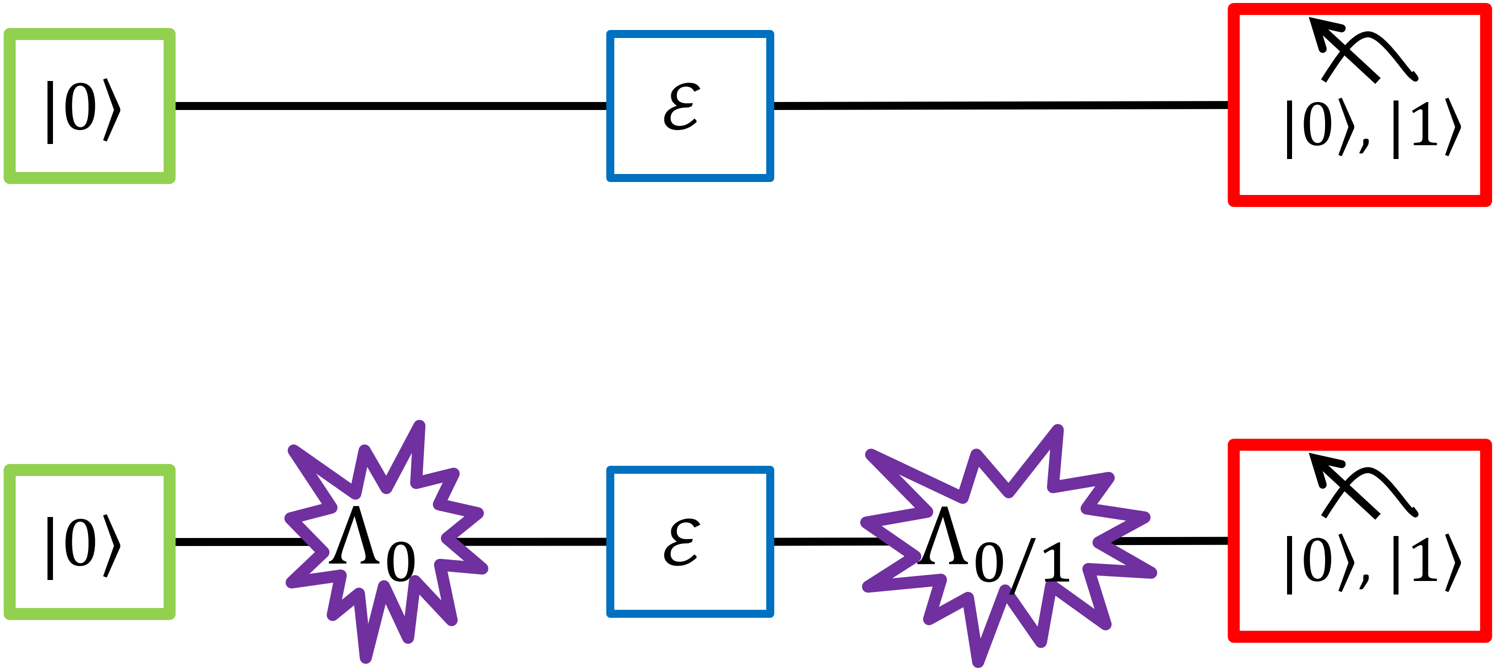
Outline

- **Background**:
  - Issues with standard process characterization
  - Randomized benchmarking framework, challenges of current implementation
- **Our Results**:
  - Robust characterization of unital part of any process
  - Efficient bound on average fidelity of universal gate set.

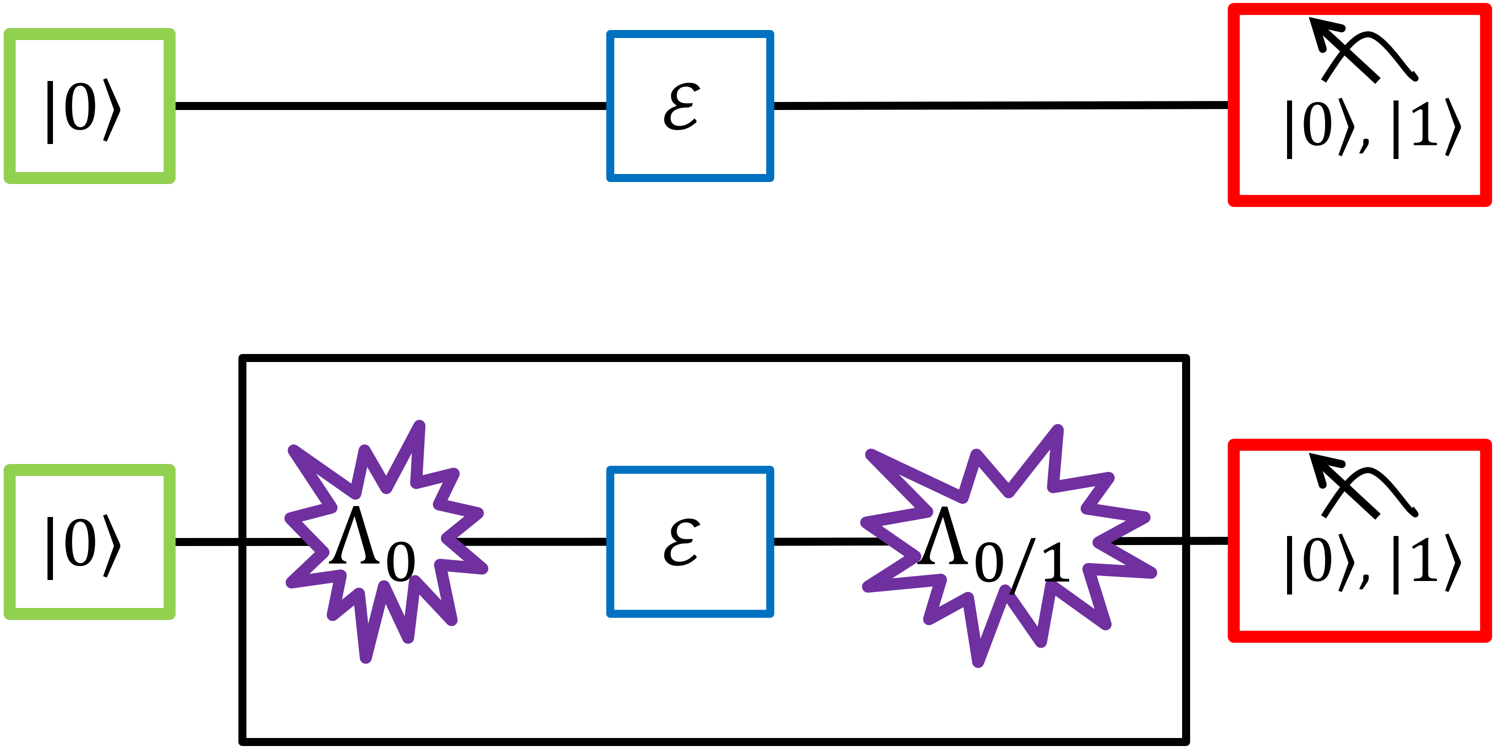
Quantum Process (Map)

- Completely positive trace preserving (CPTP) map = any process that takes valid quantum states to valid quantum states.
- E.g. unitary, depolarizing process, dephasing process, amplitude damping process
- n qubits, $O(16^n)$ free parameters

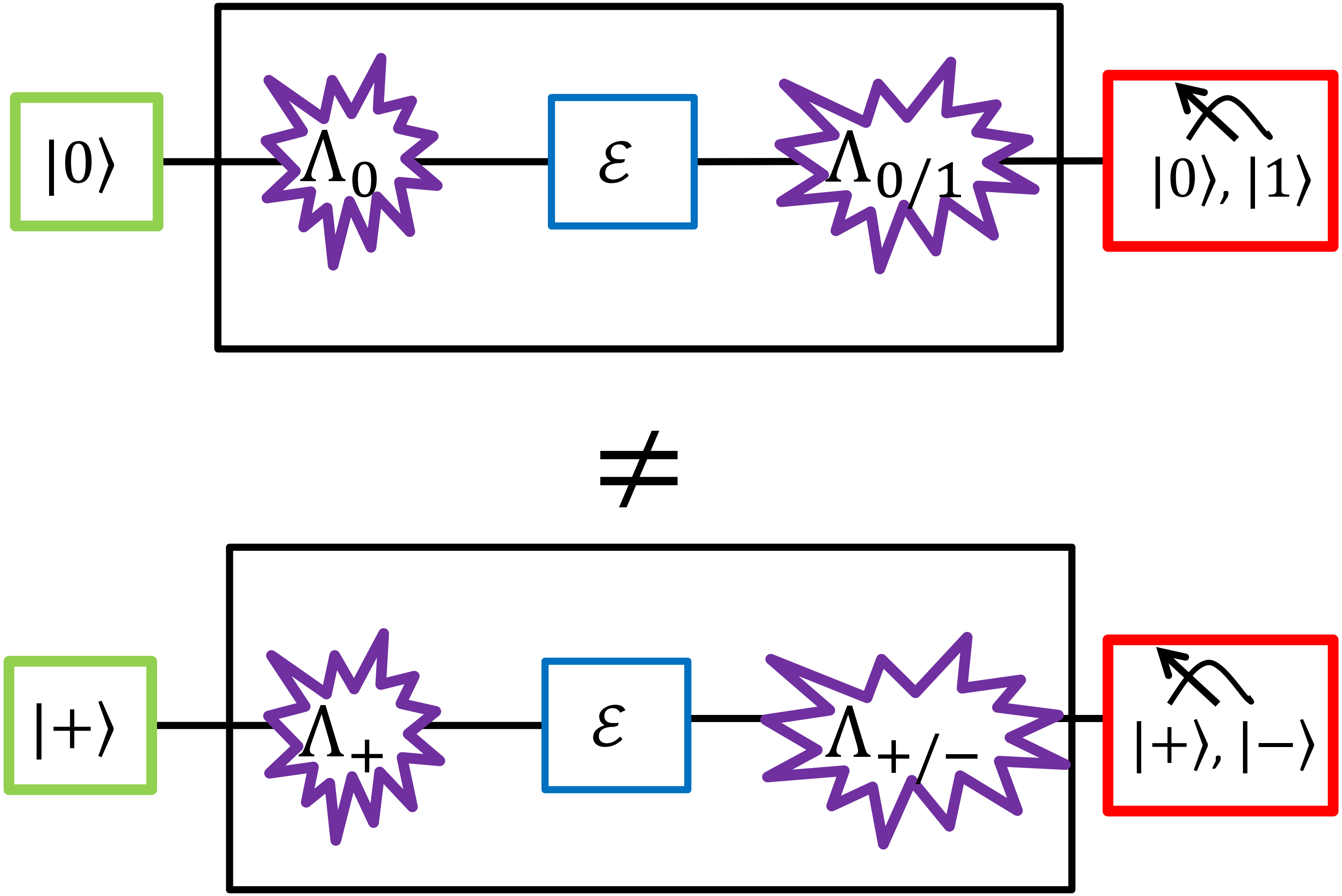
Problem with Standard Process Tomography



Problem with Standard Process Tomography



Problem with Standard Process Tomography

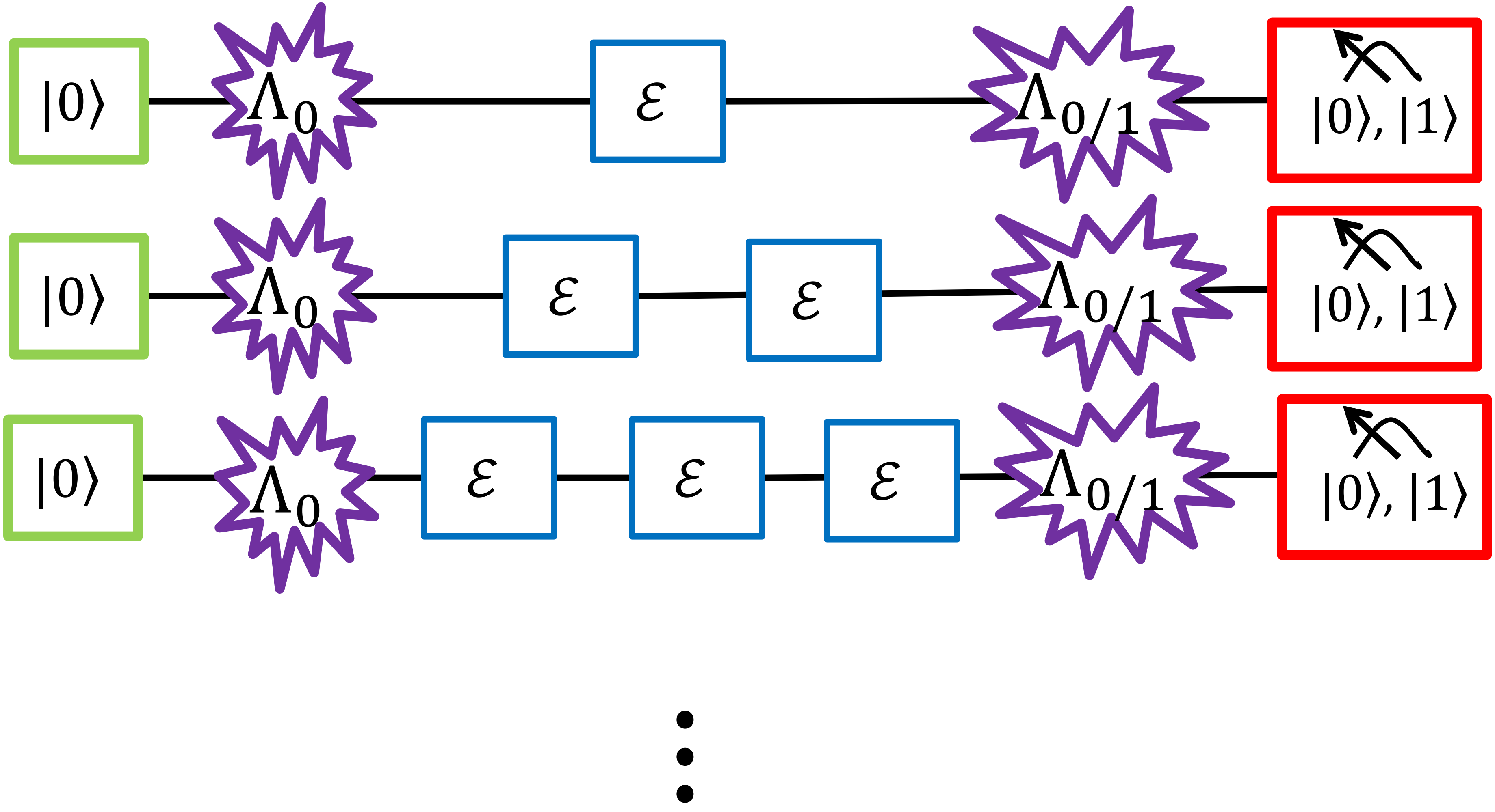


$|0\rangle$ Λ_0 $\mathcal{E}$ $\Lambda_{0/1}$ $|0\rangle, |1\rangle$

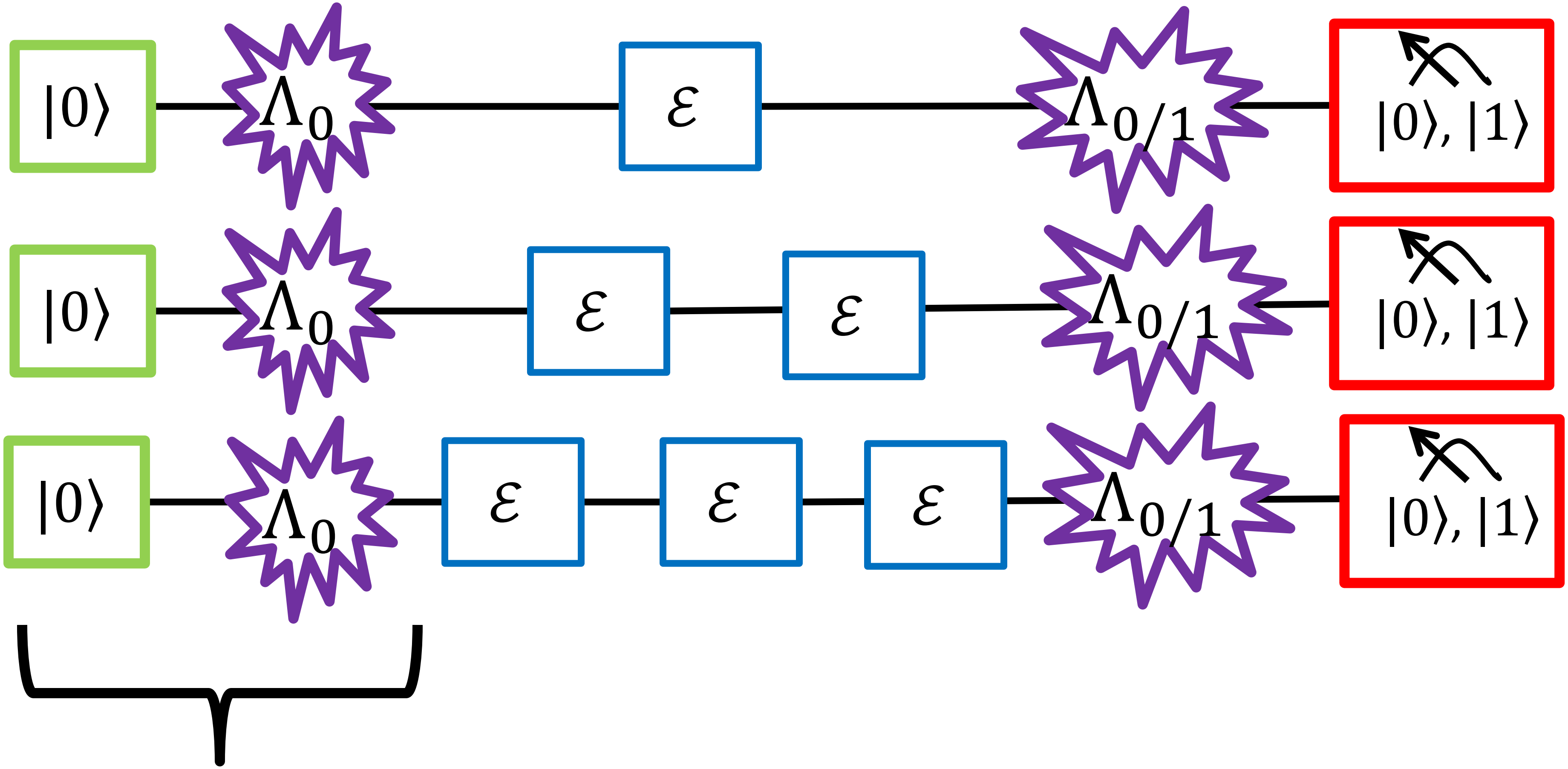
$\neq$

$|+\rangle$ Λ_+ $\mathcal{E}$ $\Lambda_{+/-}$ $|+\rangle, |-\rangle$

Repeated Application

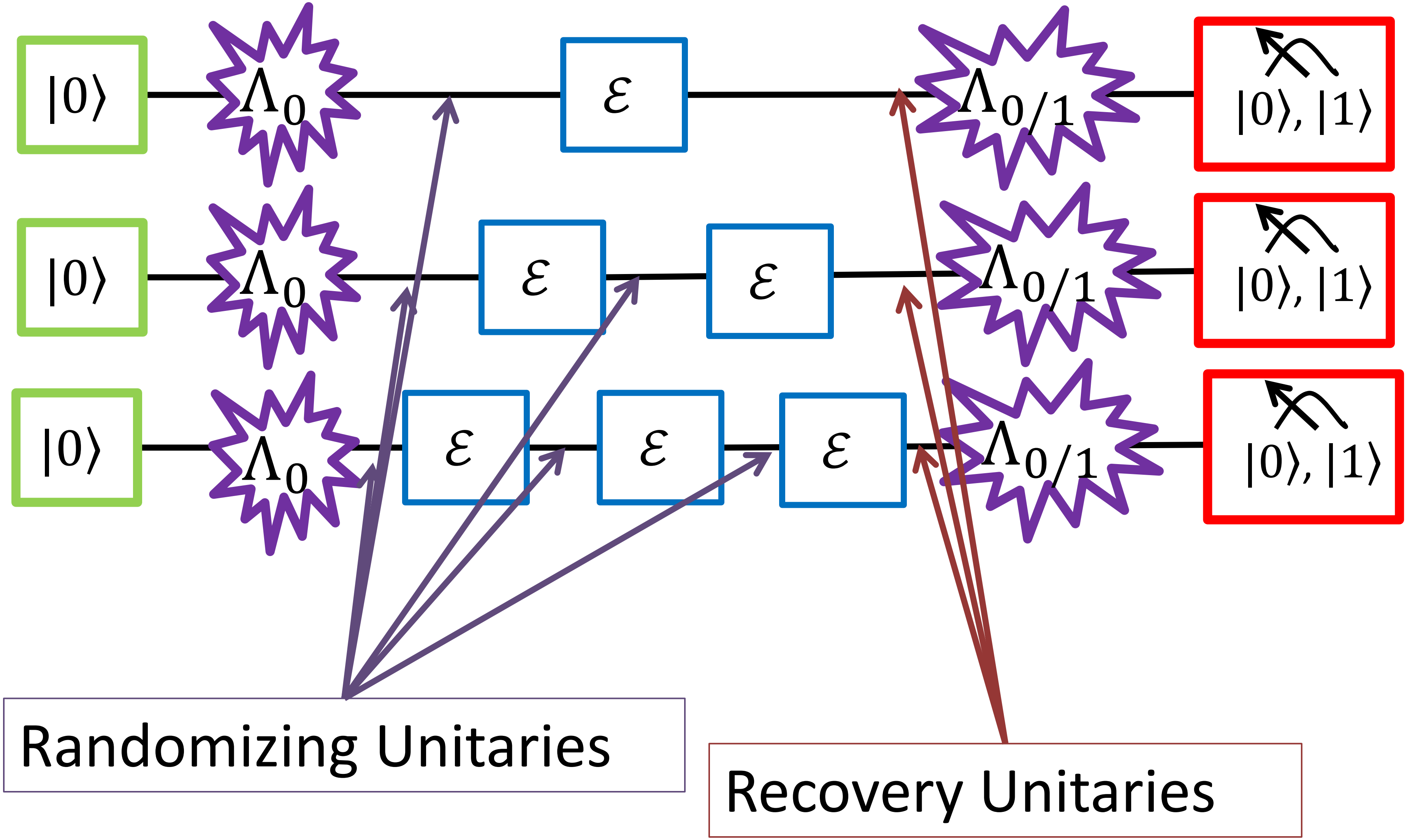


Repeated Application



If eigenstate of $\mathcal{E}$, will only see how $\mathcal{E}$ acts on *this* state

Randomized Benchmarking

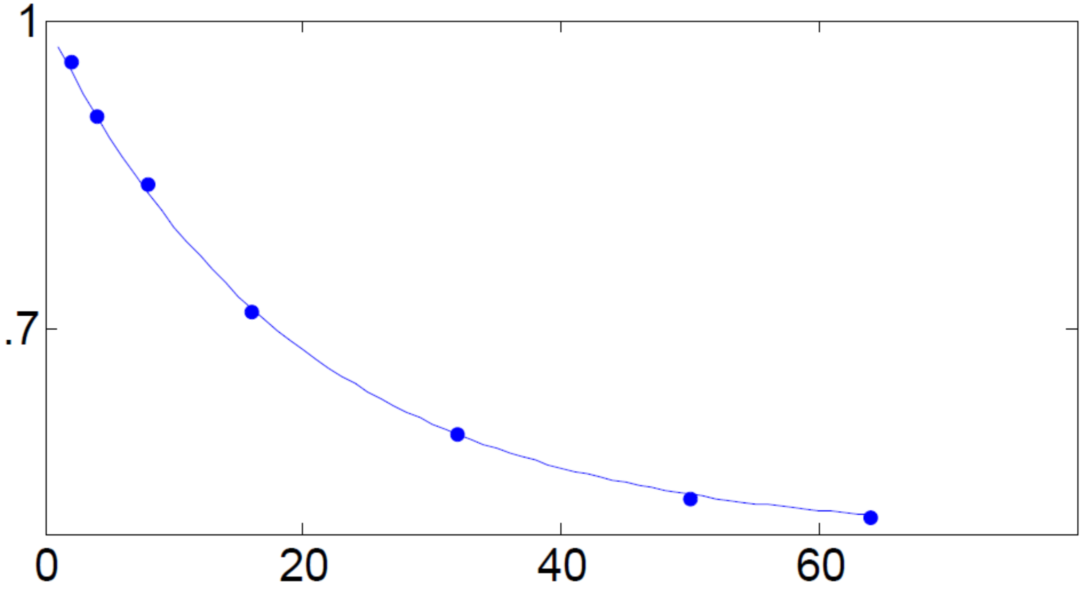


Randomizing Unitaries

Recovery Unitaries

Randomized Benchmarking

Simulated Randomized Benchmarking Experiment

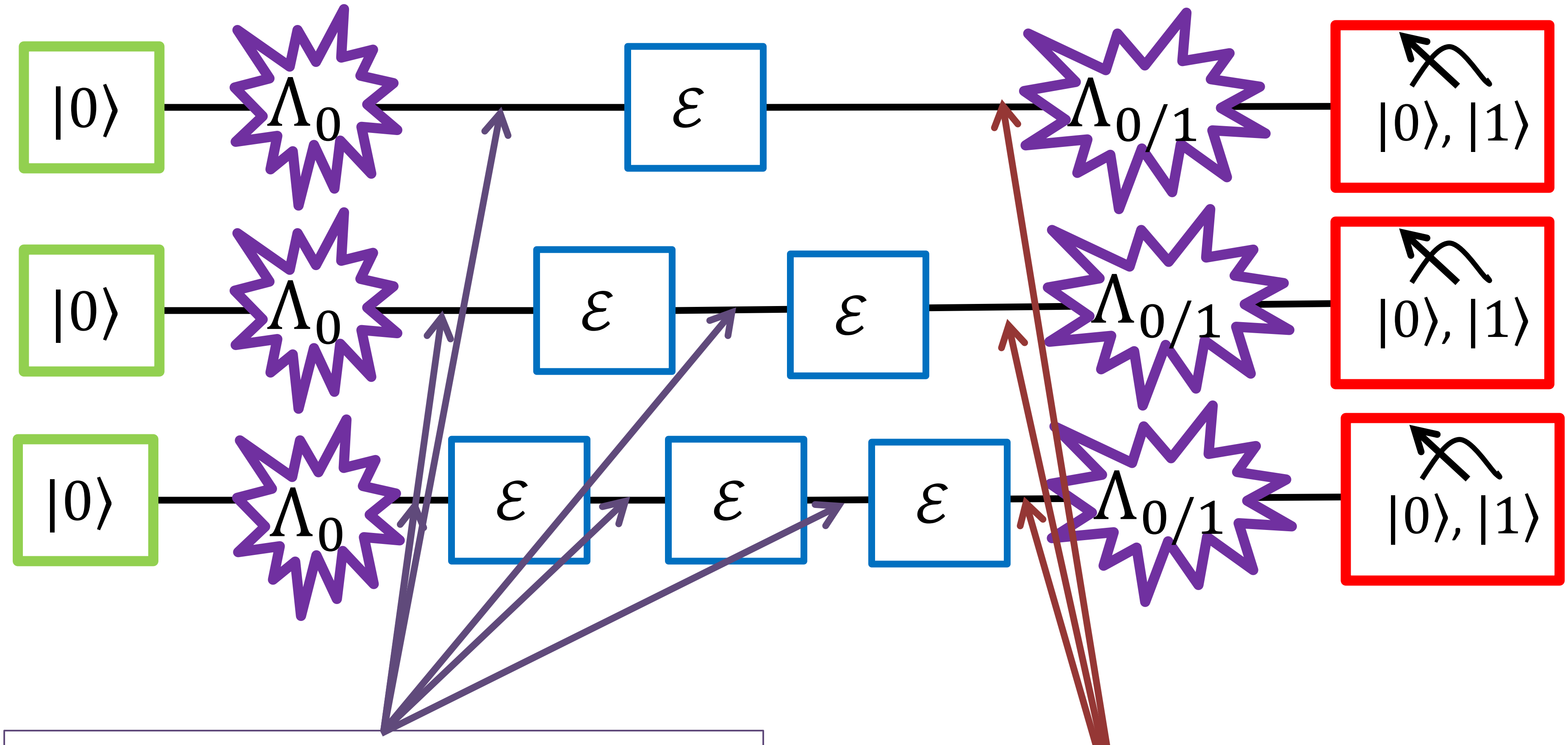


Value of
Measurement
Observable

Sequence Length

Decay constant depends on one parameter of $\mathcal{E}$

Randomized Benchmarking



Randomizing Unitaries Have Errors!

Recovery Unitaries

Two Issues with RB

1. How can we extract more than just 1 parameter?
2. How can we deal with errors on the randomizing operations?

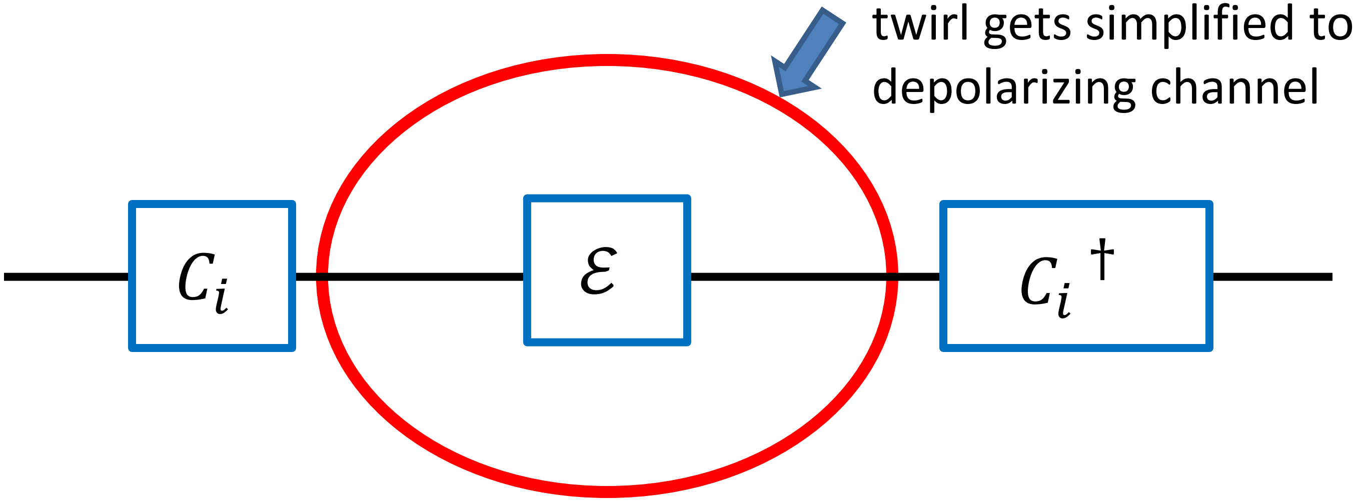
Randomizing Operation: Clifford Twirl

$$\frac{1}{|C_i|} \sum_{c_i \text{ in Cliffords}} {C_i}^{\dagger} \circ \mathcal{E} \circ C_i\,(\rho) = (1-q)\rho + q\,\frac{\mathbb{I}}{d}$$

Result is depolarizing channel (very simple process) that depends on only one parameter of $\mathcal{E}$: Average fidelity of $\mathcal{E}$ to the identity

Average fidelity of $\mathcal{E}$ = $\int d\,|\psi\rangle\ \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle$

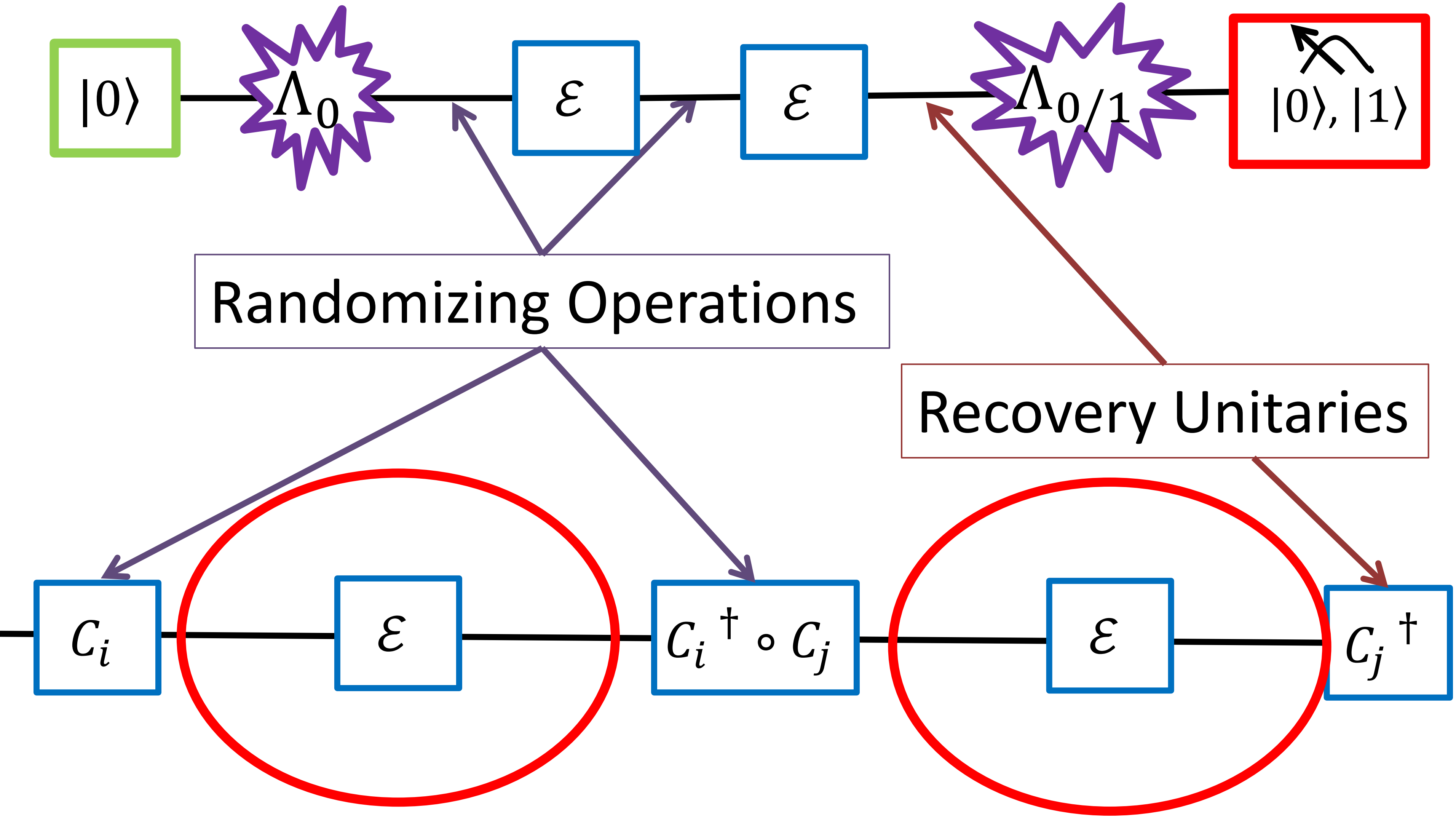
Randomizing Operation: Clifford Twirl



Everything inside the Clifford twirl gets simplified to a depolarizing channel

To implement (approximately), repeat many times, each time randomly choosing C_i, and average results

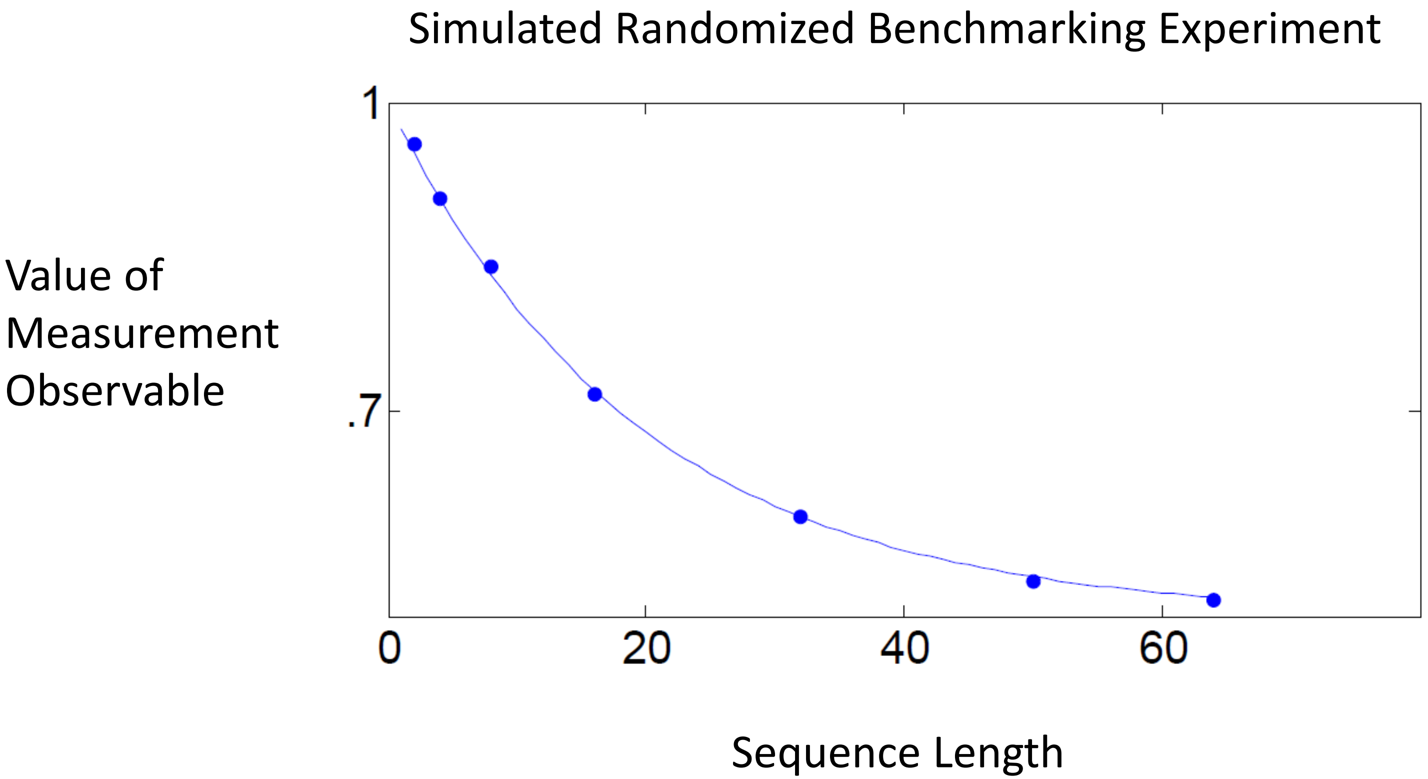
Randomizing Operation: Clifford Twirl



Randomizing Operations

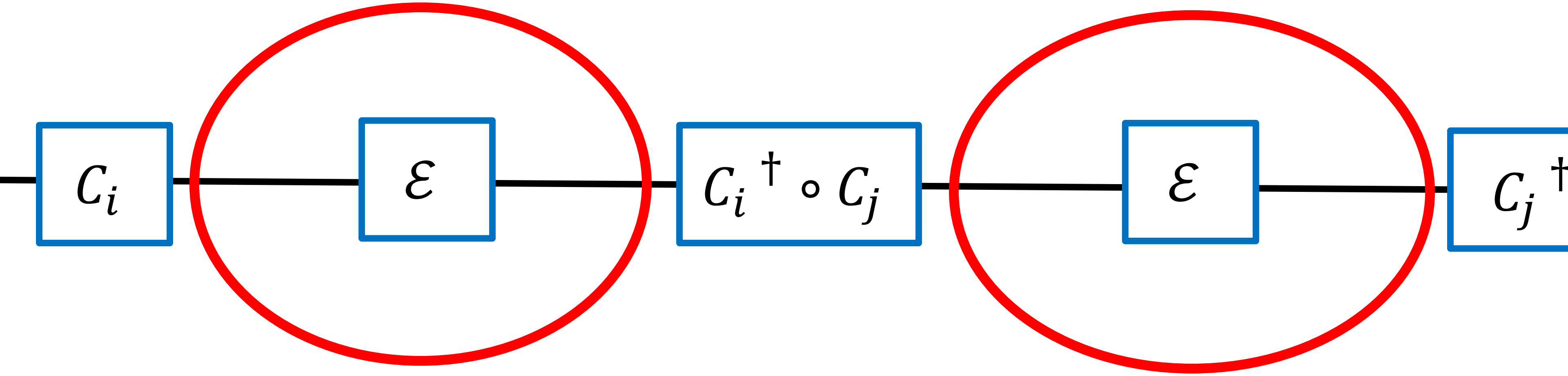
Recovery Unitaries

Randomizing Operations



Decay constant depends on 1 parameter of $\mathcal{E}$:
Average fidelity of $\mathcal{E}$ to the identity.

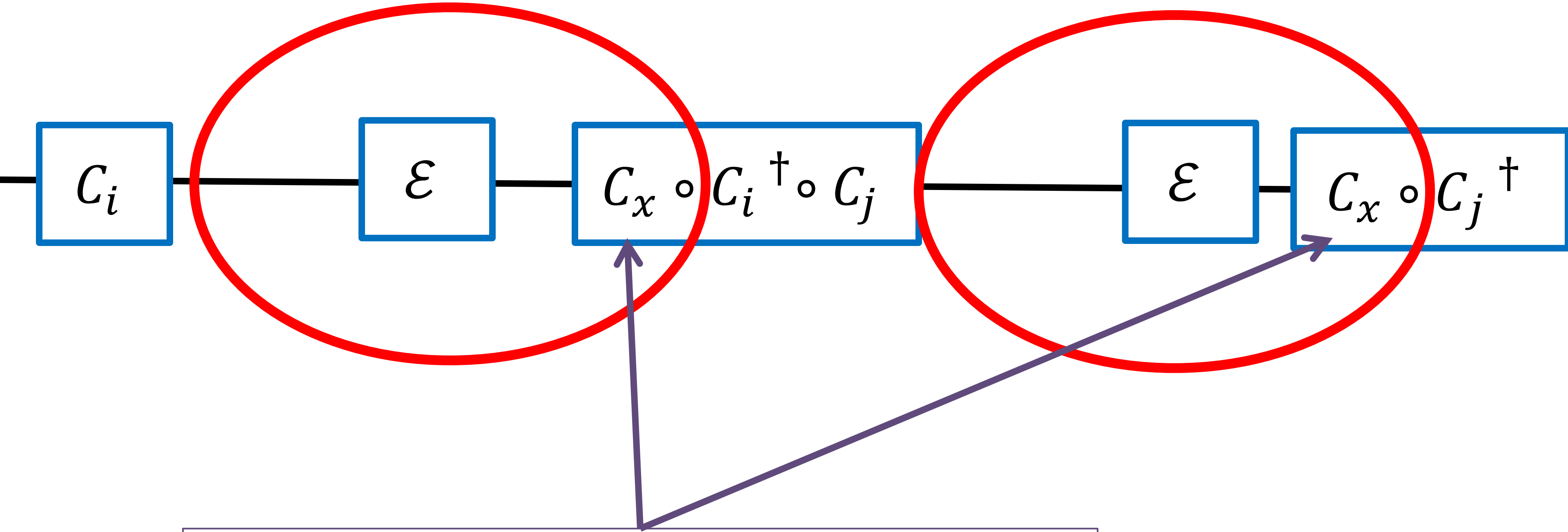
1. Extracting More Information



Twirl simplifies too much!

- no twirl
- stick additional information inside twirl

1. Extracting More Information



C_x is fixed – not random. The same C_x is applied in each twirl.

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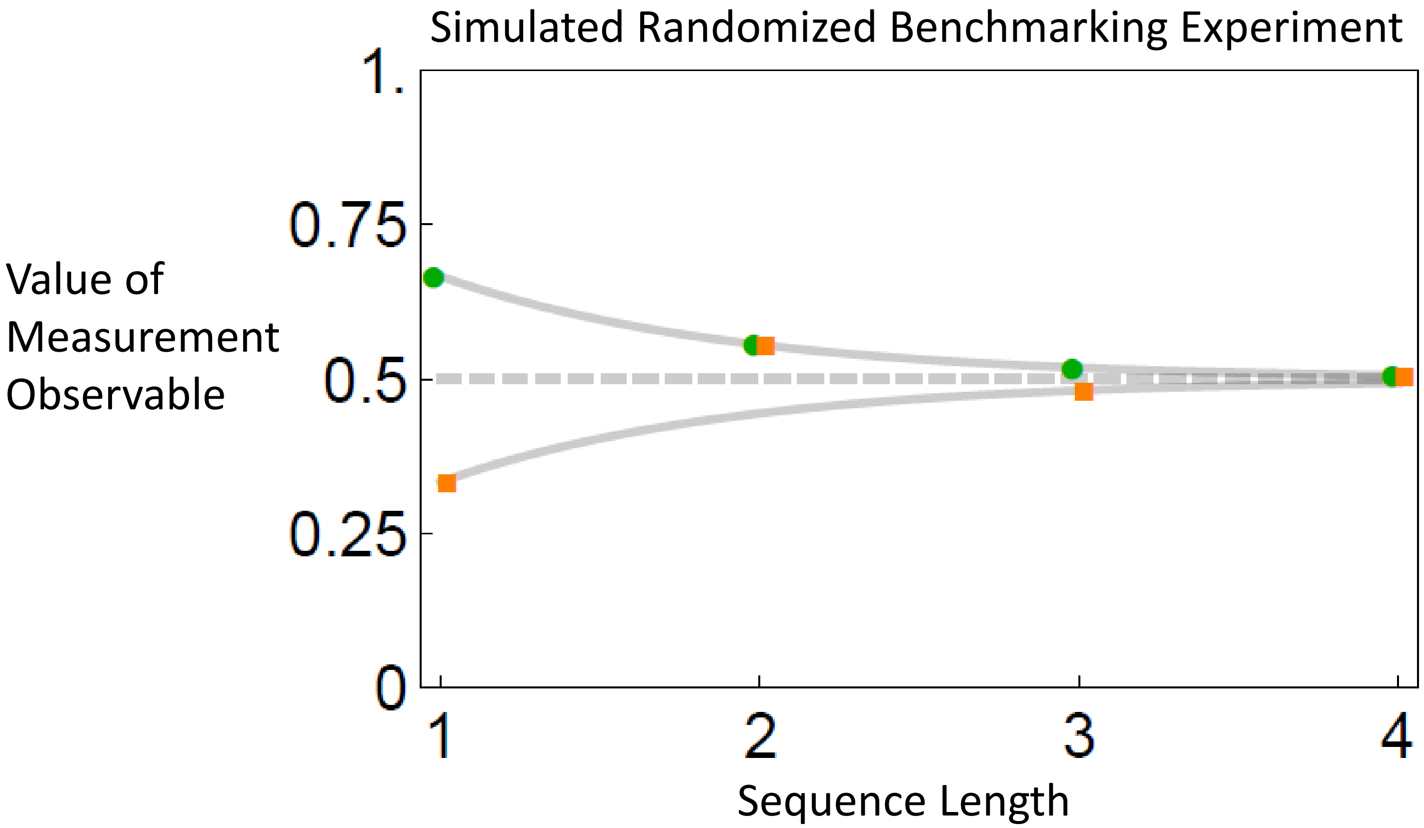

Decay constant depends on 1 parameter of $\mathcal{E}$:

Average Fidelity of $\boldsymbol{\mathcal{E}}$ ***to*** $\boldsymbol{C_x}^{\dagger}$ (can have fast decays)

1. Extracting More Information

CPTP map: $16^n - 4^n$ parameters for n-qubit map

$$\begin{array}{cc}
 & 4^n \\
4^n & \left[\begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline & & & \\ & & & \\ & & & \end{array}\right]
\end{array}$$

To compose two maps, just multiply matrices!

1. Extracting More Information

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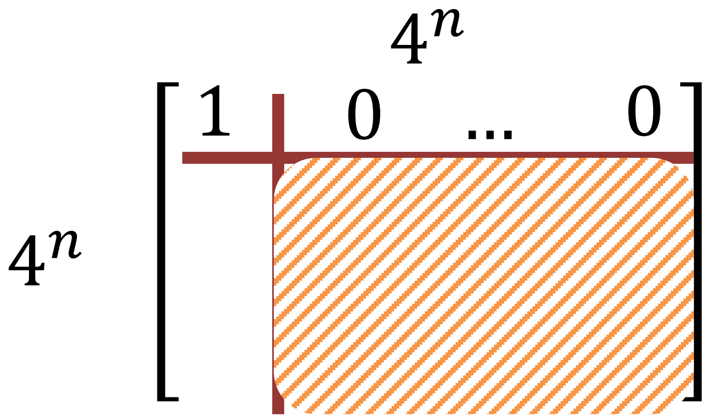
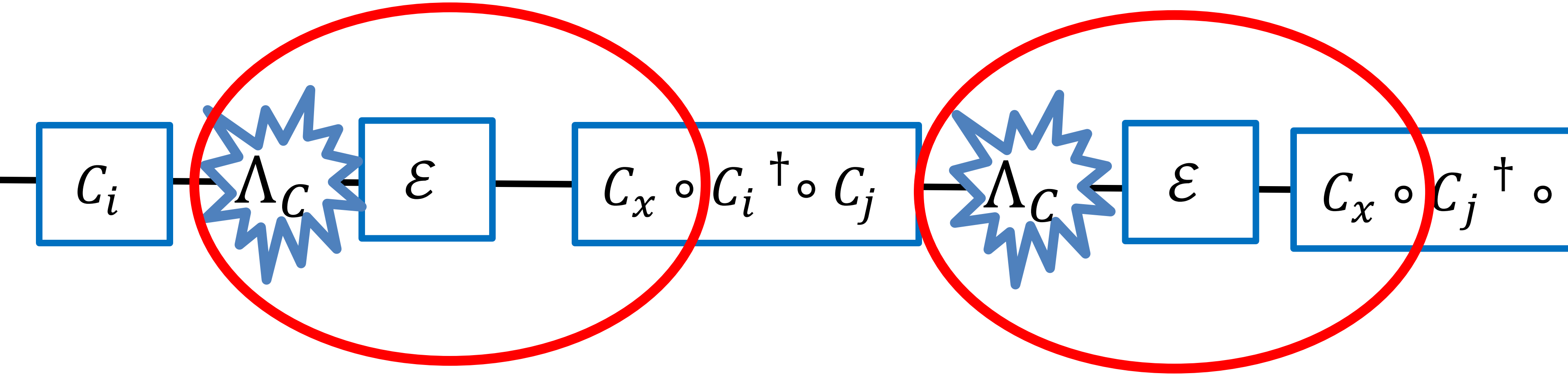


- Vectors V span a subspace S
- Learn inner product between V and unknown vector u
- Can learn projection of u onto S

- Cliffords span unital part
- Learn inner product between Cliffords and $\mathcal{E}$
- Learn projection of $\mathcal{E}$ onto unital subspace

2. Dealing with Errors



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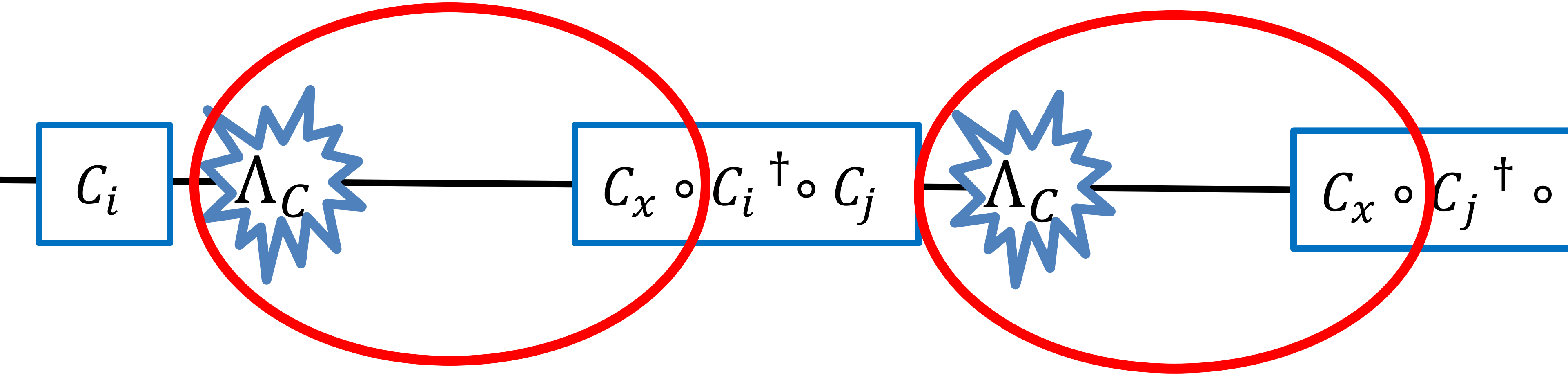
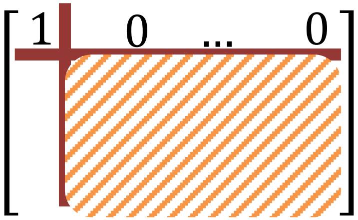


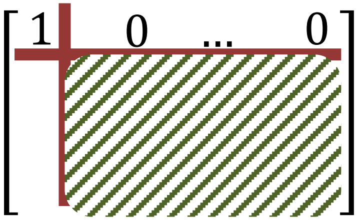
2. Dealing with Errors

almost complete characterization of Λ_C



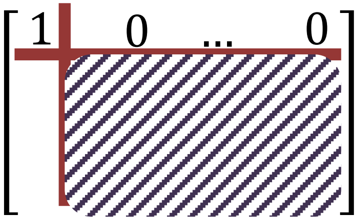
$+$

almost complete characterization of $\Lambda_C \circ \mathcal{E}$



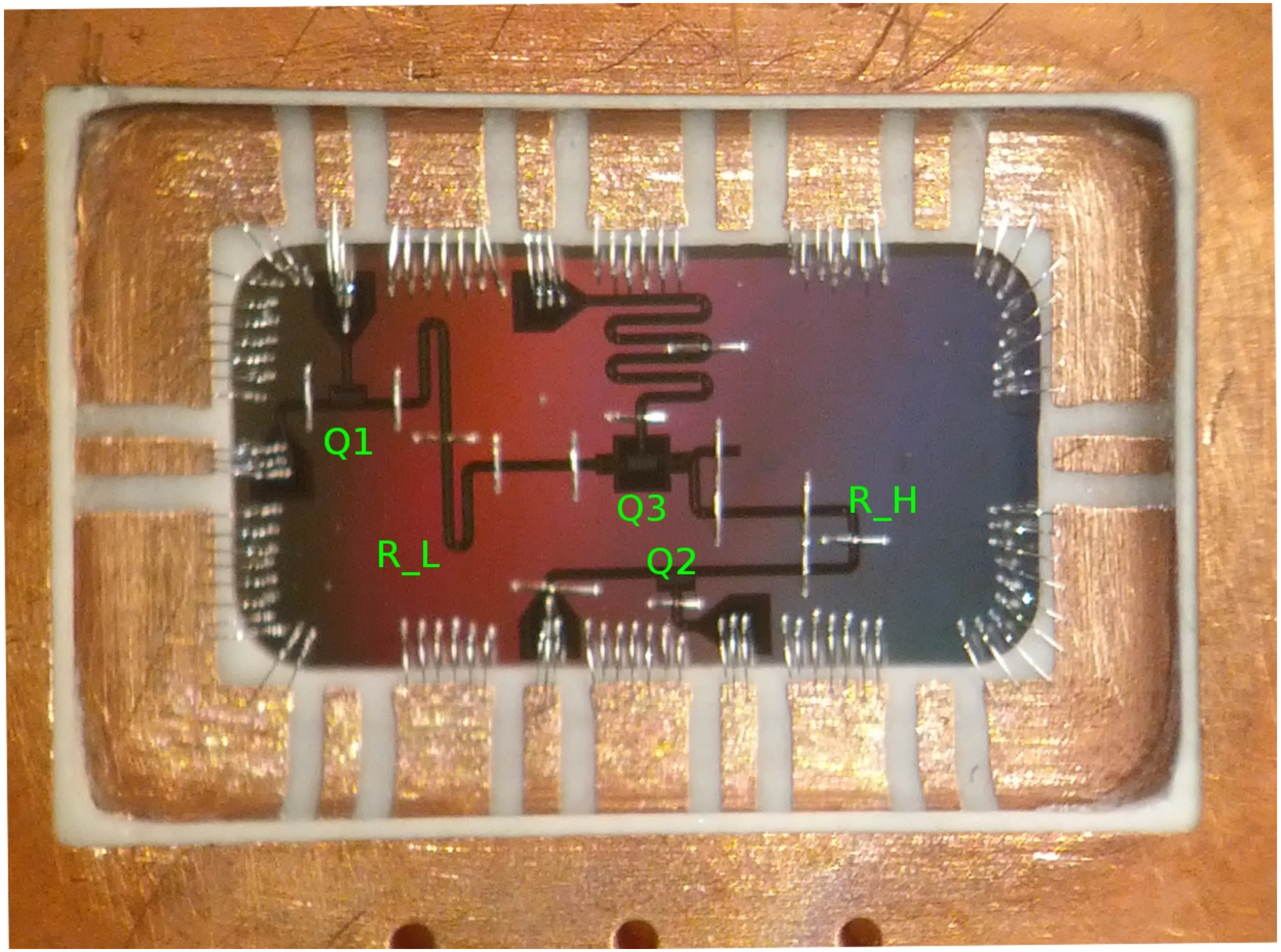
$=$

almost complete characterization of $\mathcal{E}$



All without the systematic errors of previous procedures!

Experimental Implementation

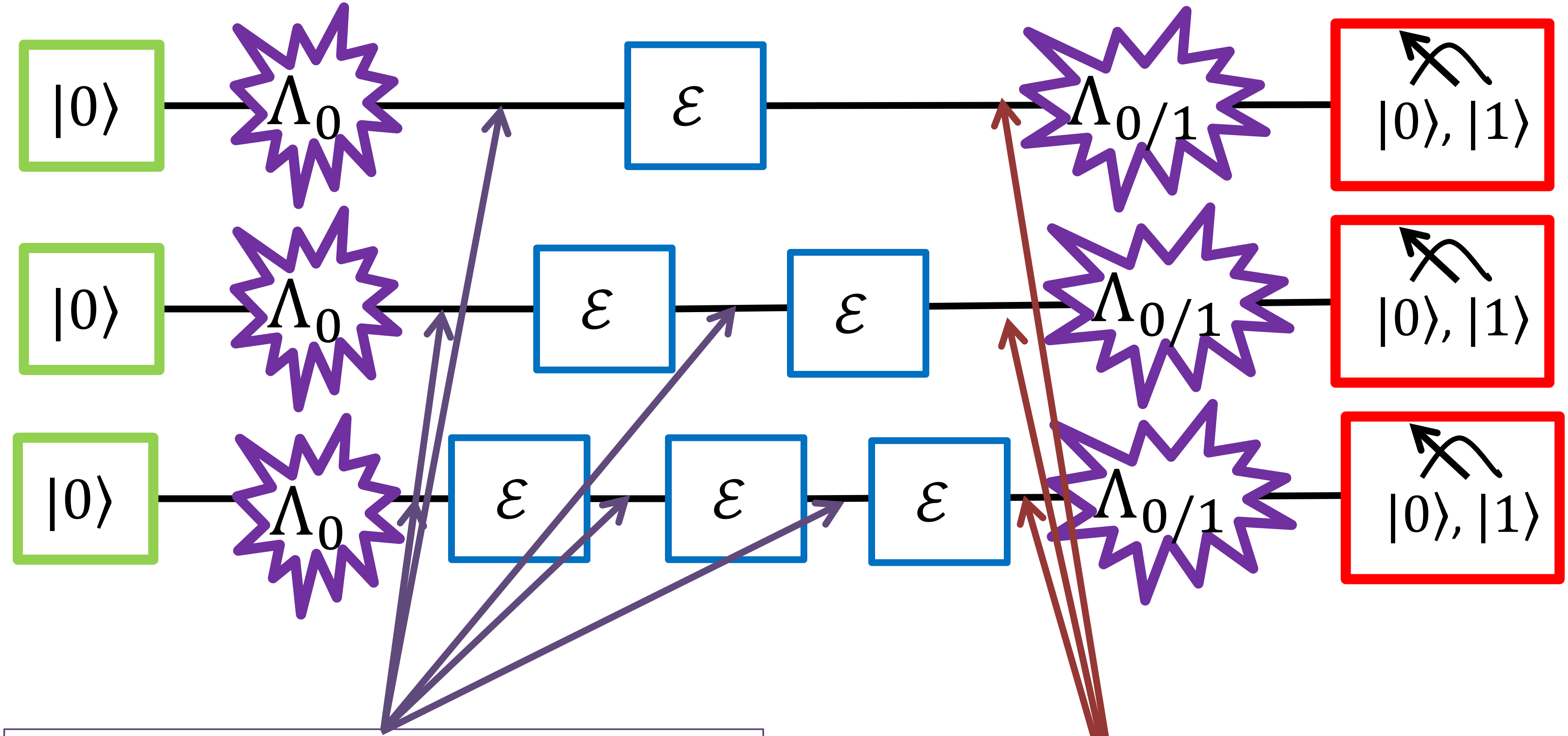


Negative Witness Test [Moroder et al. '13]

- To be a valid quantum process, must be trace preserving and completely positive
- Complete positivity = in Choi representation, all eigenvalues must be positive

- Negative witness test:
  - Look at value of smallest eigenvalues of reconstructed map in Choi representation.
  - If negative, BAD!

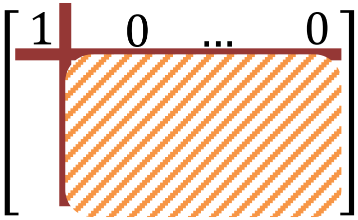
Randomized Benchmarking



Randomizing Unitaries Have Errors!

Recovery Unitaries

Efficient Fidelity Estimate

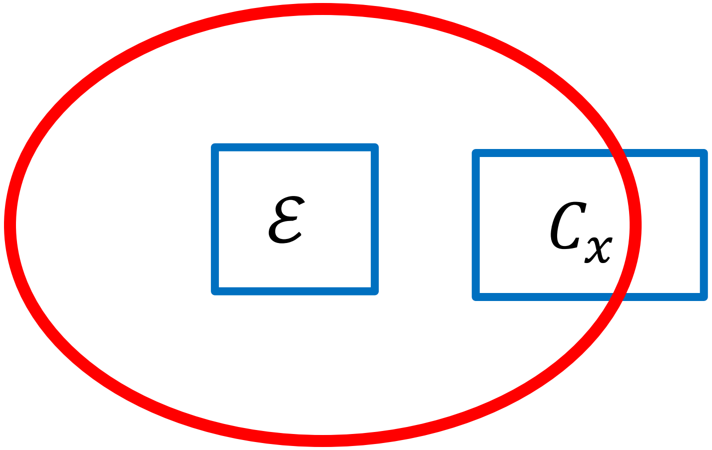


Requires an exponential number of measurement settings with different C_x

Instead, only want to check that your operations are good enough.

Want to check implementation of Clifford Gates and T gates
= universal gate set

Efficient Fidelity Estimate

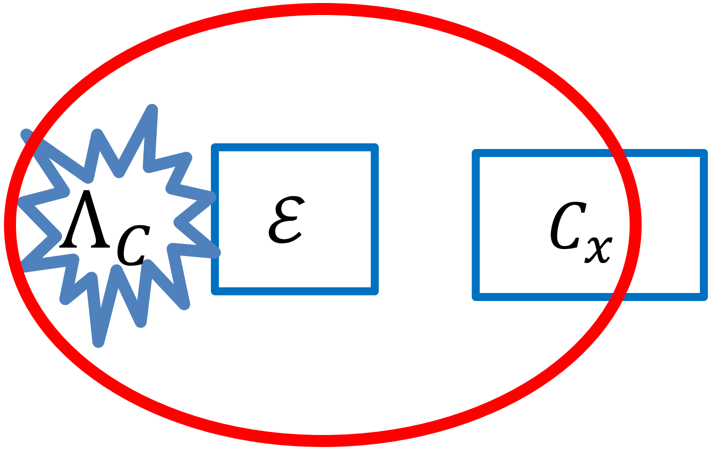


Average fidelity to any unitary $\mathcal{U}$ of

- $\mathrm{O}(\log n)$ T gates
- $\mathrm{O}(\mathrm{poly}\, n)$ Cliffords

only need to repeat for $\mathrm{O}(\mathrm{poly}\, n)$ different C_x.

Efficient Fidelity Estimate



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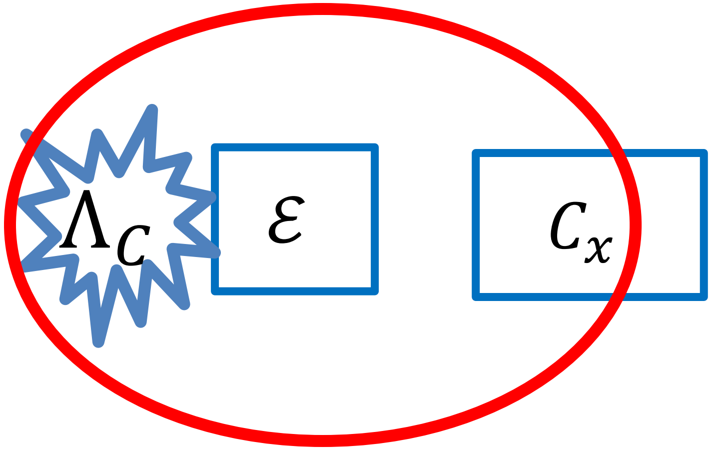

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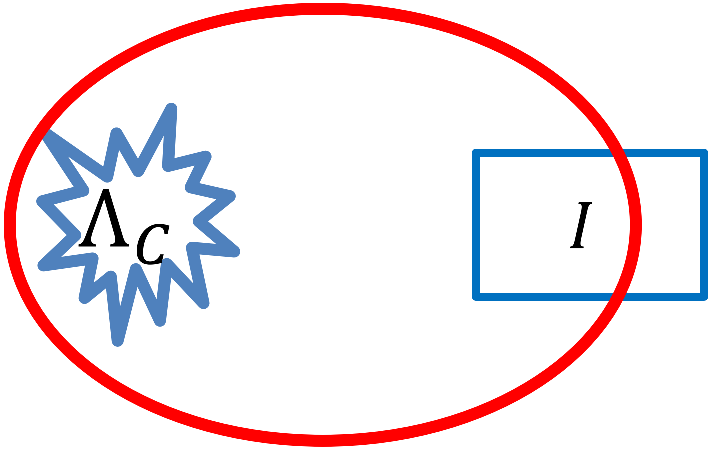

If Λ_C is close to Identity, can closely bound the average fidelity of $\mathcal{E}$ to $\mathcal{U}$.

Can test a universal gate set!

Conclusions and Open Questions

- Can robustly measure unital part of any quantum process
- Can efficiently and robustly test fidelity of universal quantum gate set operations.
- Experimentally implemented with superconducting qubit system at BBN

- What about the non-unital part?
- Can we extract other information efficiently and robustly (compressed sensing?)
- How does RB compare to Gate Set Tomography methods?