Characterizing Quantum Operations

Shelby Kimmel

Joint work with Marcus Silva,
Raytheon BBN Technologies

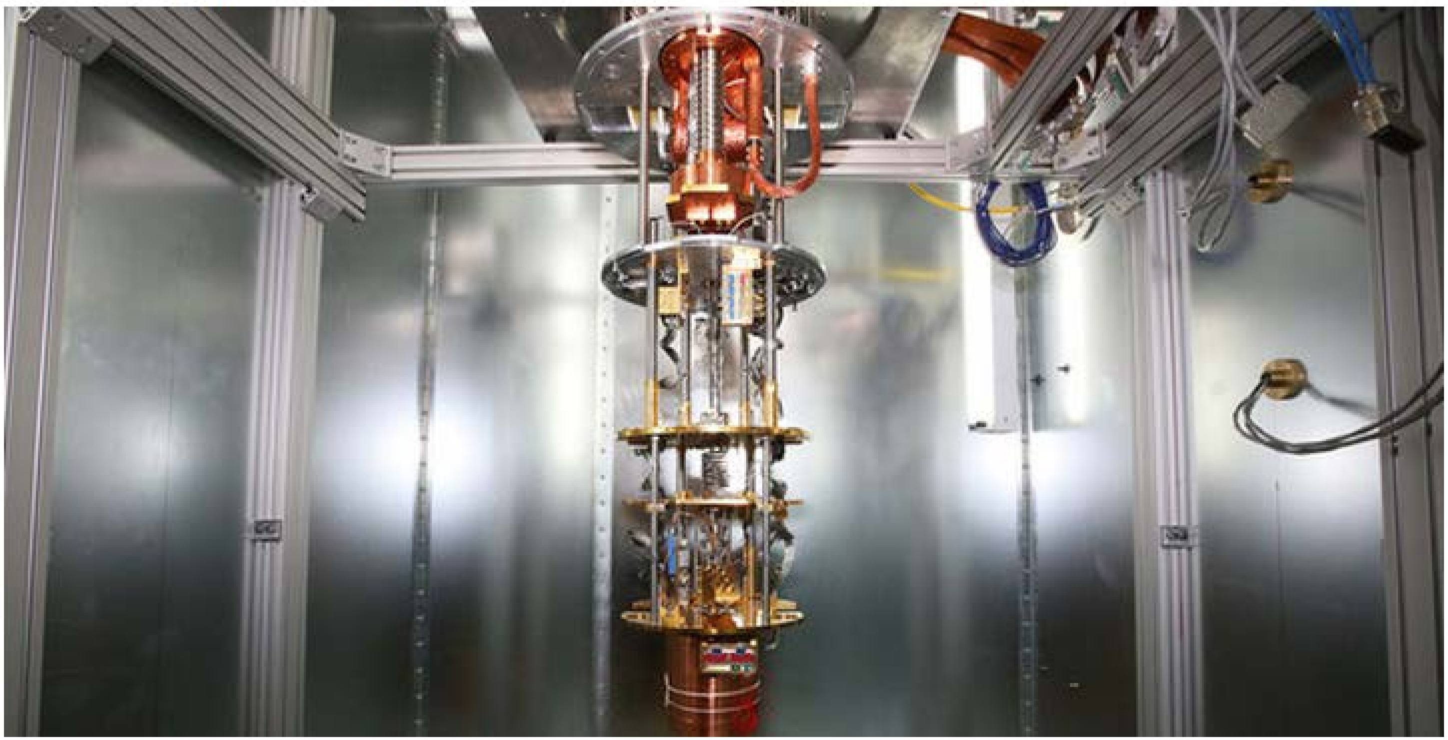
Quantum Computers Will Be Cool

The New York Times

Business Day

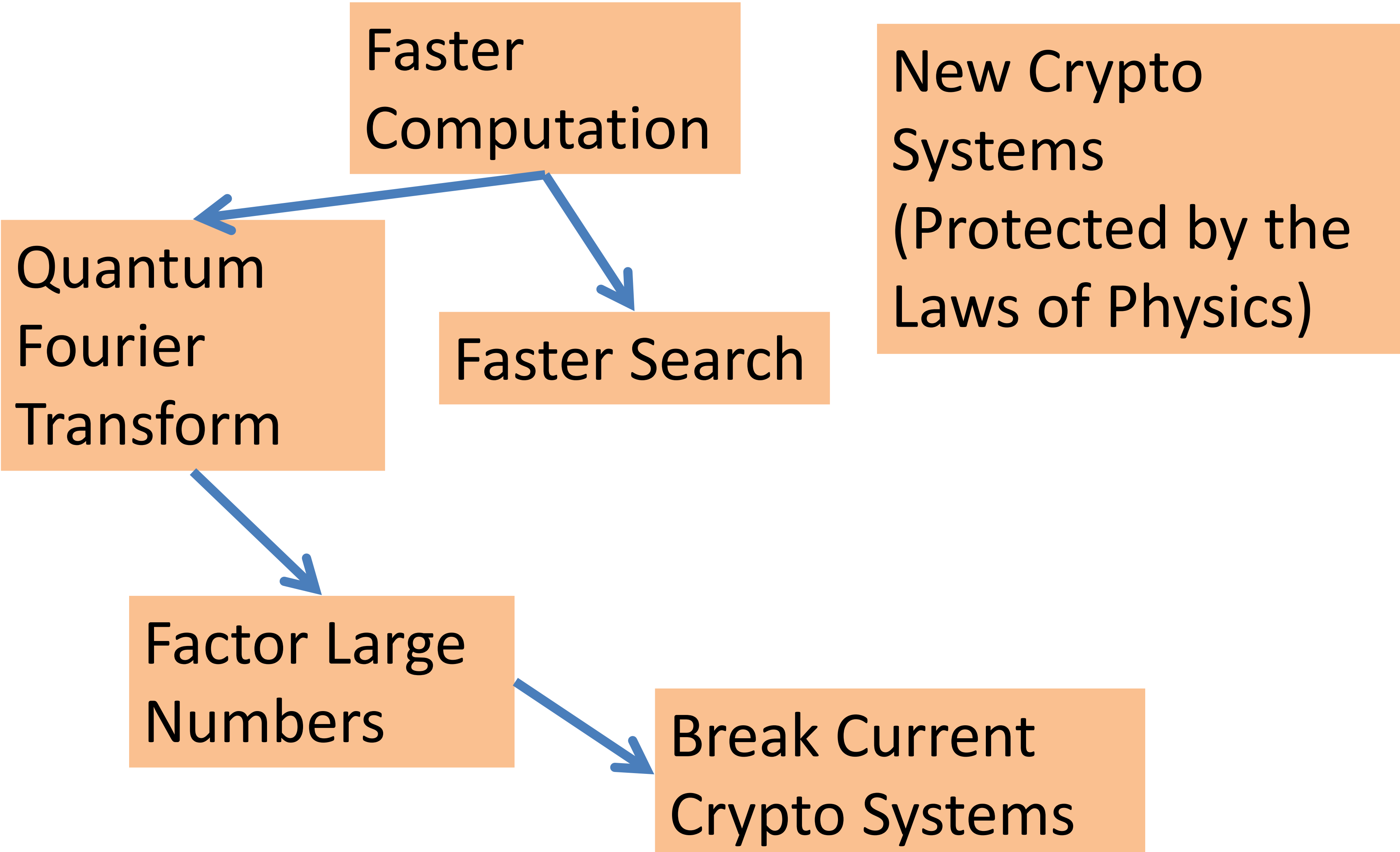
Technology

A Strange Computer Promises Great Speed



By QUENTIN HARDY, Published: March 21, 2013

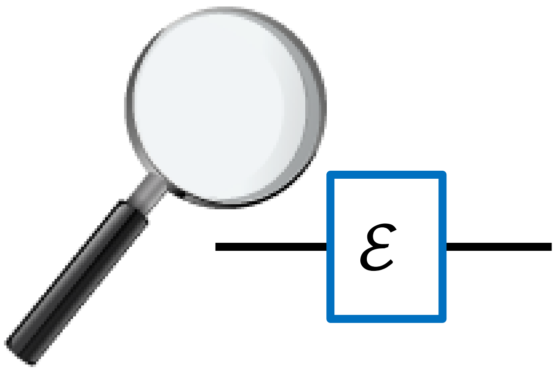
Quantum Computers Will Be Cool



Why Don’t We All Have One?

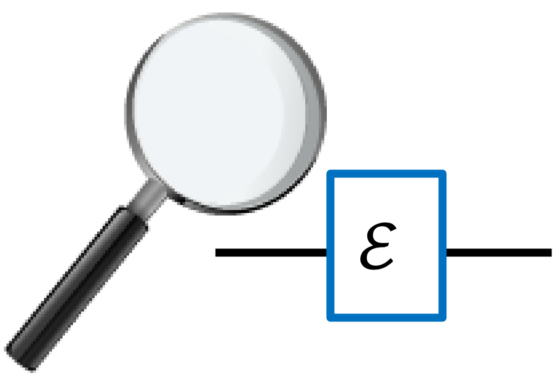
Too Many Errors

Can Improve Operations with Better Characterization of Errors



"Depolarizing error"

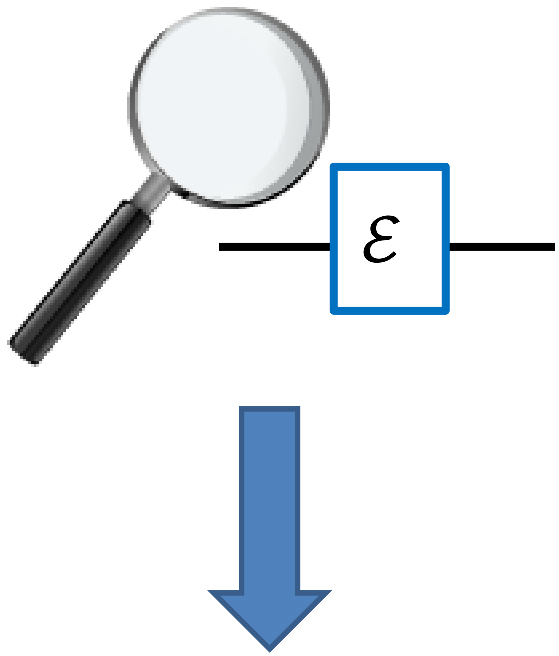
Improvement to Computer



"Extra rotation around z-axis"

Improvement to Computer

Standard Techniques Have Problems

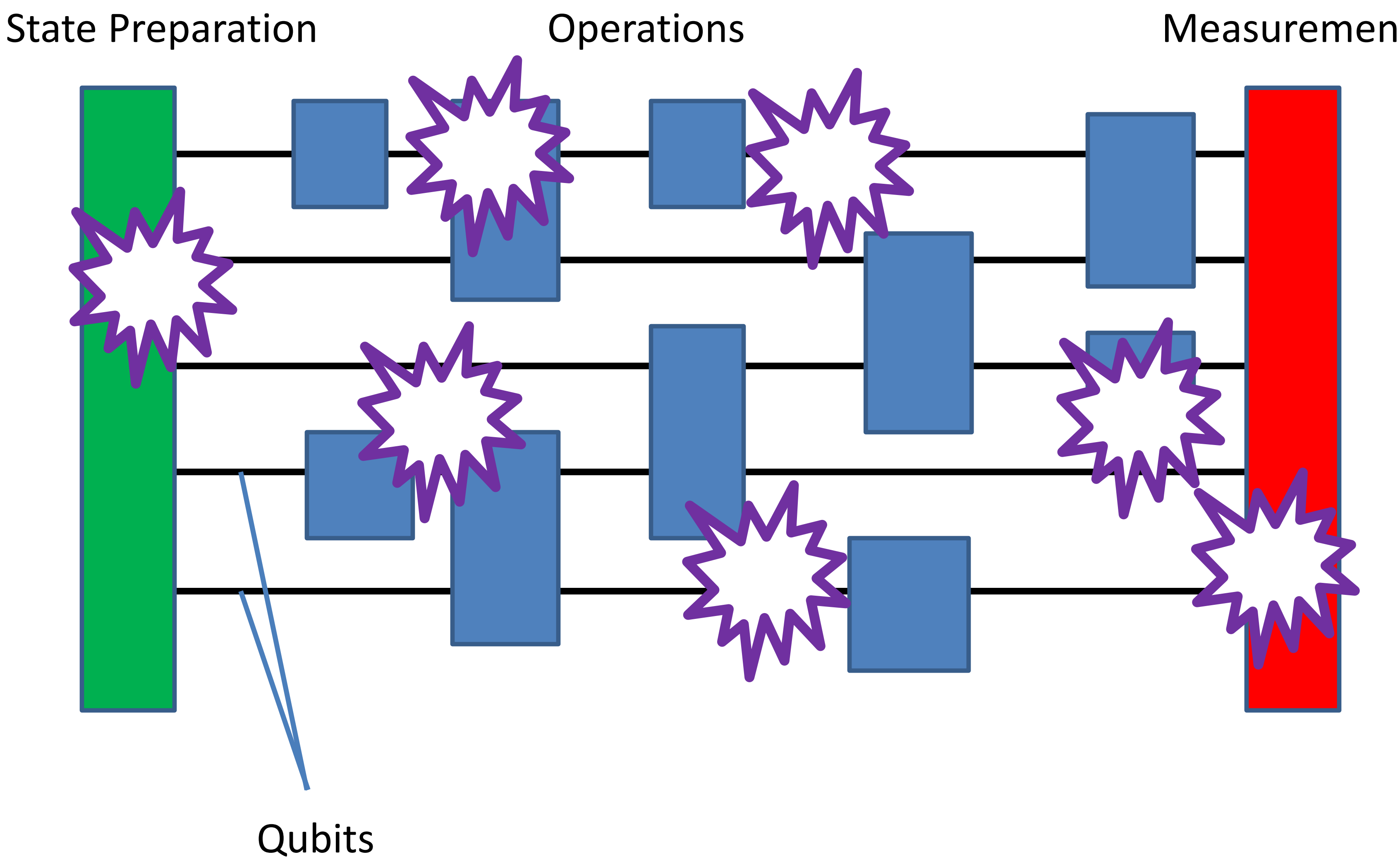


Often, outcome is not a valid quantum operation!

Outline: Improving Quantum Process Characterization

- **Motivation**: Process Characterization can improve errors and thus lead towards the realization of a quantum computer
- **Problem**: Standard techniques for process characterization have systematic errors
- **Solution**: We provide a way to get around these systematic errors and get lots of useful information about quantum operations. (Magesan et al. ‘11, ’12, Kimmel and Silva…soon!)

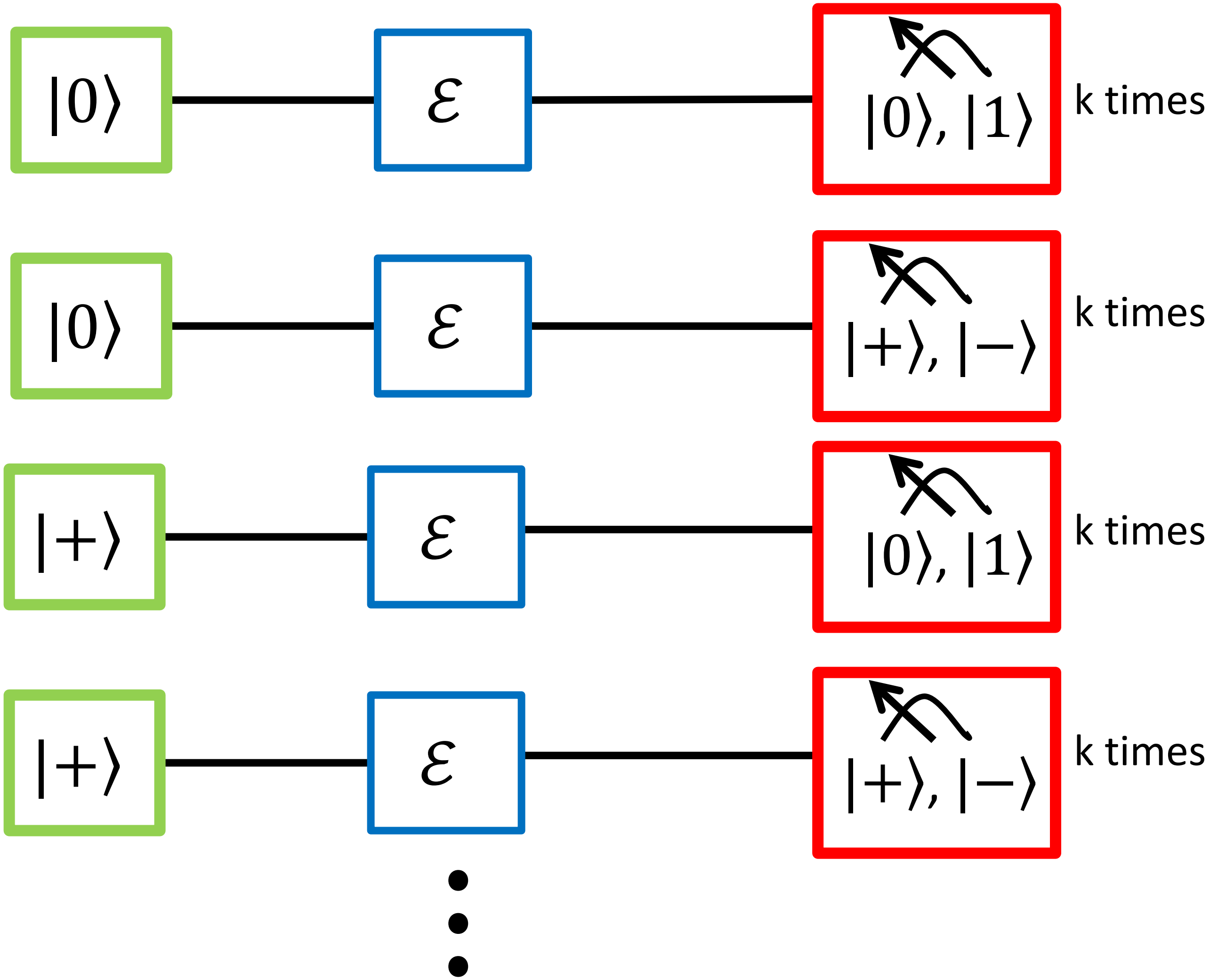
Quantum Computation (Circuit Model)



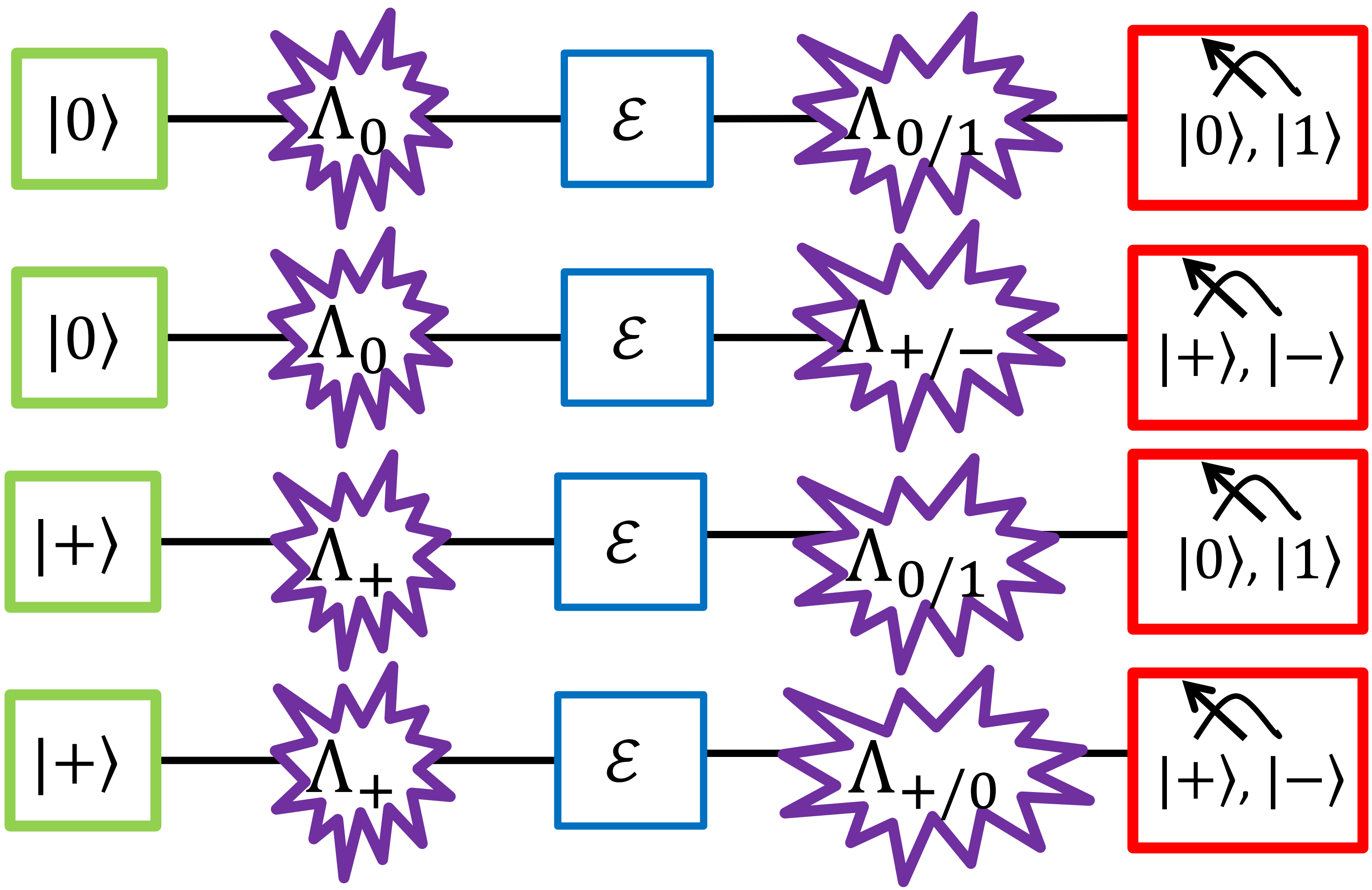
Quantum Process

- Completely positive trace preserving (CPTP) map = any process that takes valid quantum states to valid quantum states.
- E.g. unitary, depolarizing process, dephasing process, amplitude damping process
- n qubits, $16^n - 4^n$ free parameters

Standard Quantum Process Characterization

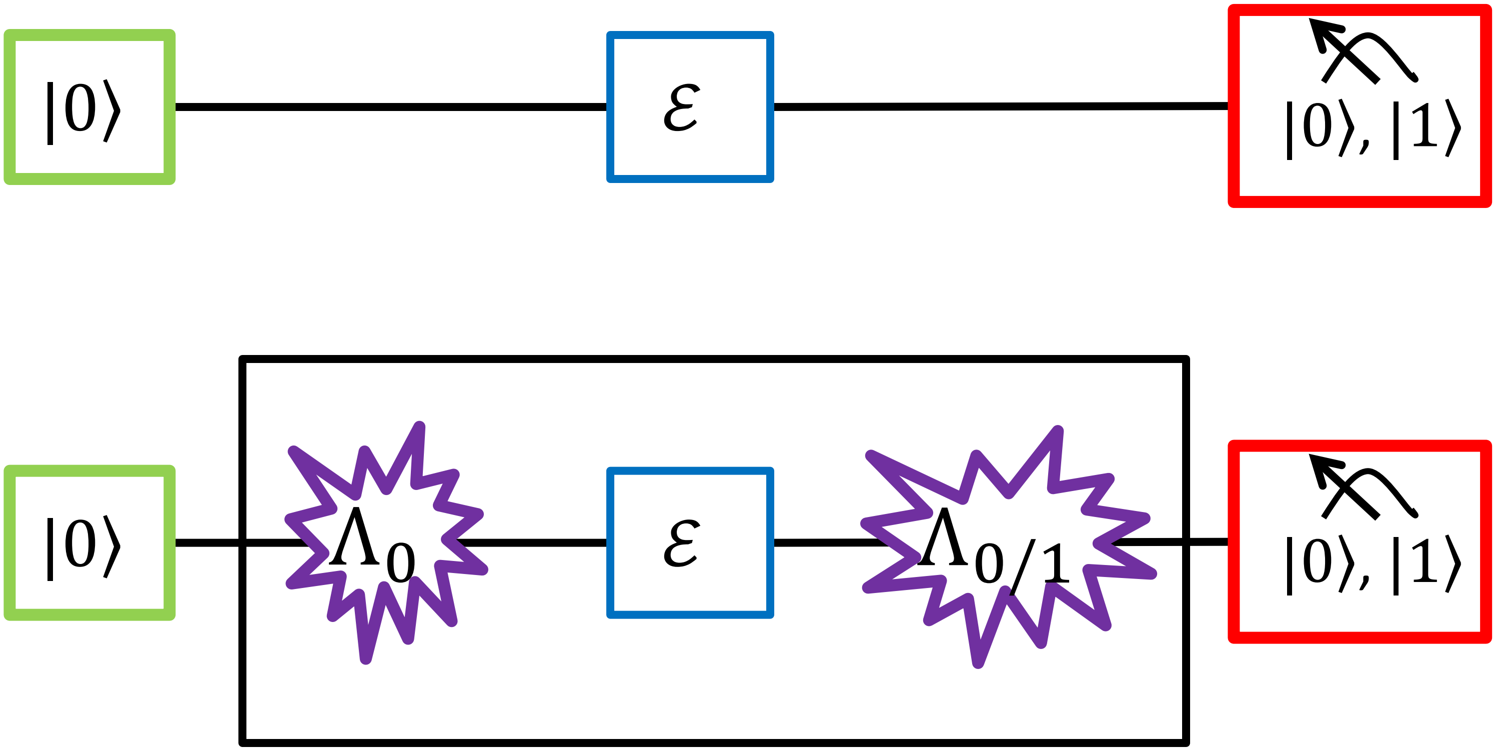


The Problem



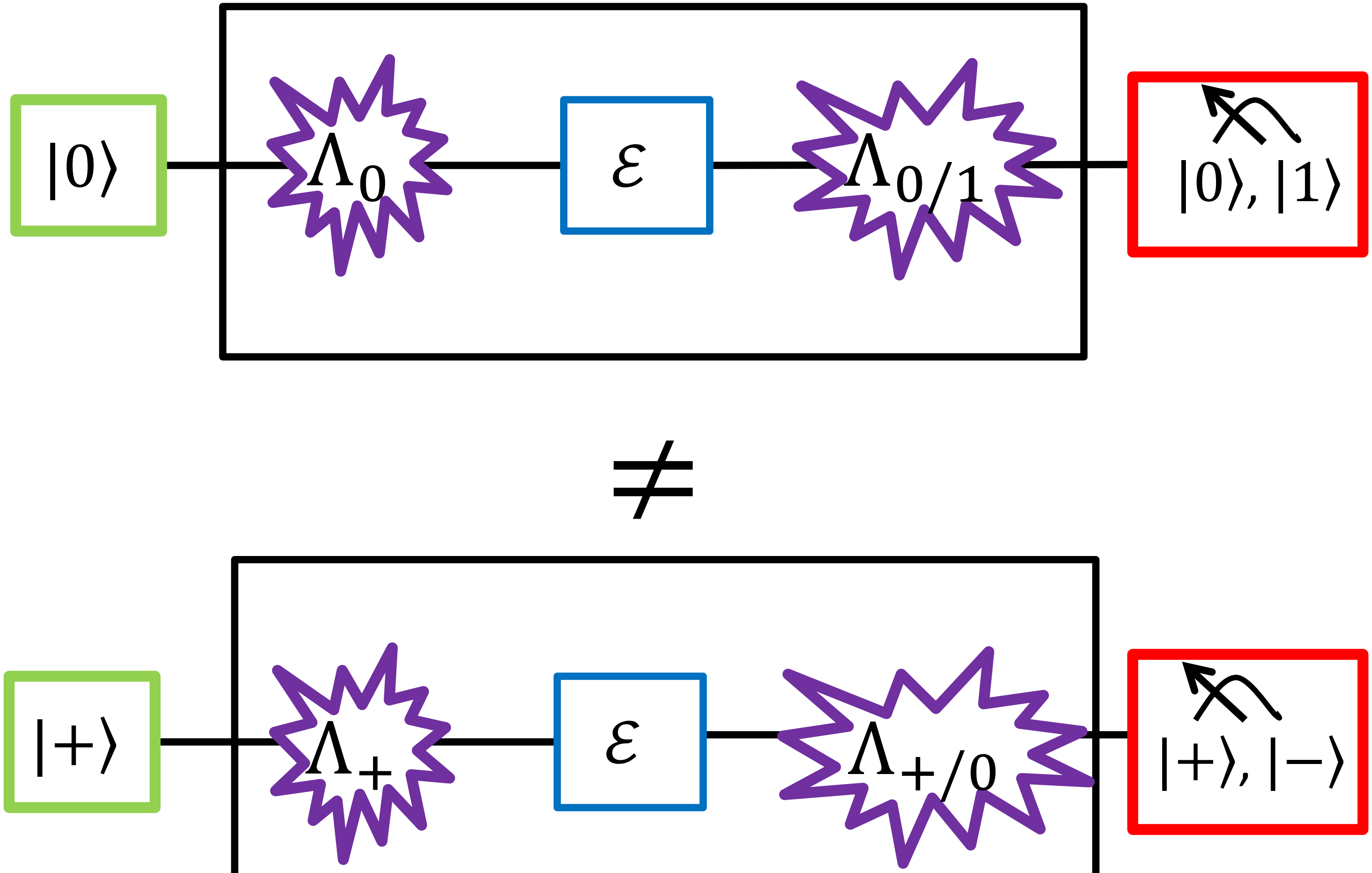
$|0\rangle$
Λ_0
$\mathcal{E}$
$\Lambda_{0/1}$
$|0\rangle, |1\rangle$
$|0\rangle$
Λ_0
$\mathcal{E}$
$\Lambda_{+/-}$
$|+\rangle, |-\rangle$
$|+\rangle$
Λ_+
$\mathcal{E}$
$\Lambda_{0/1}$
$|0\rangle, |1\rangle$
$|+\rangle$
Λ_+
$\mathcal{E}$
$\Lambda_{+/0}$
$|+\rangle, |-\rangle$

The Problem

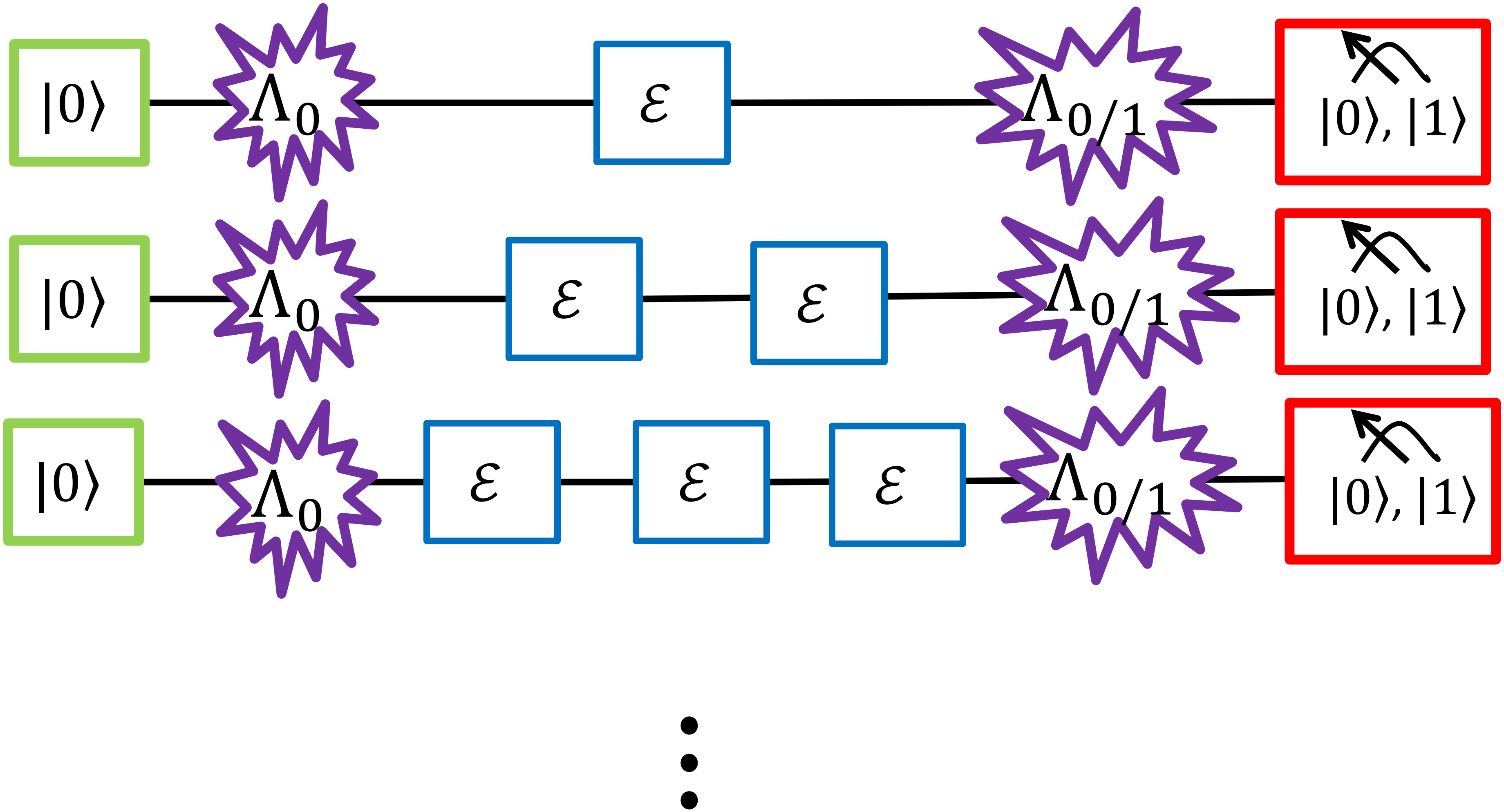


$|0\rangle$
$\mathcal{E}$
$|0\rangle, |1\rangle$
$|0\rangle$
Λ_0
$\mathcal{E}$
$\Lambda_{0/1}$
$|0\rangle, |1\rangle$

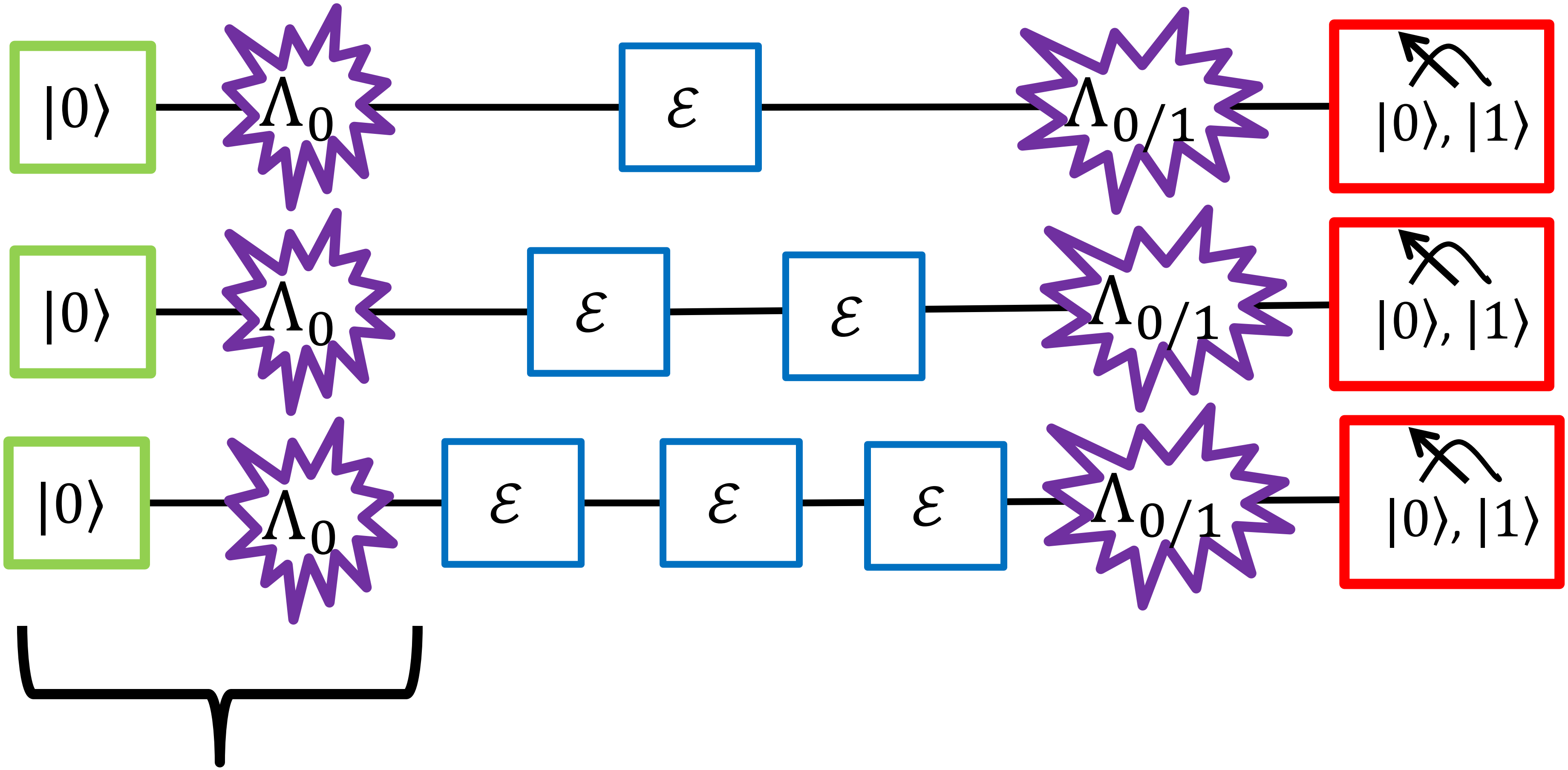
The Problem



Repeated Application

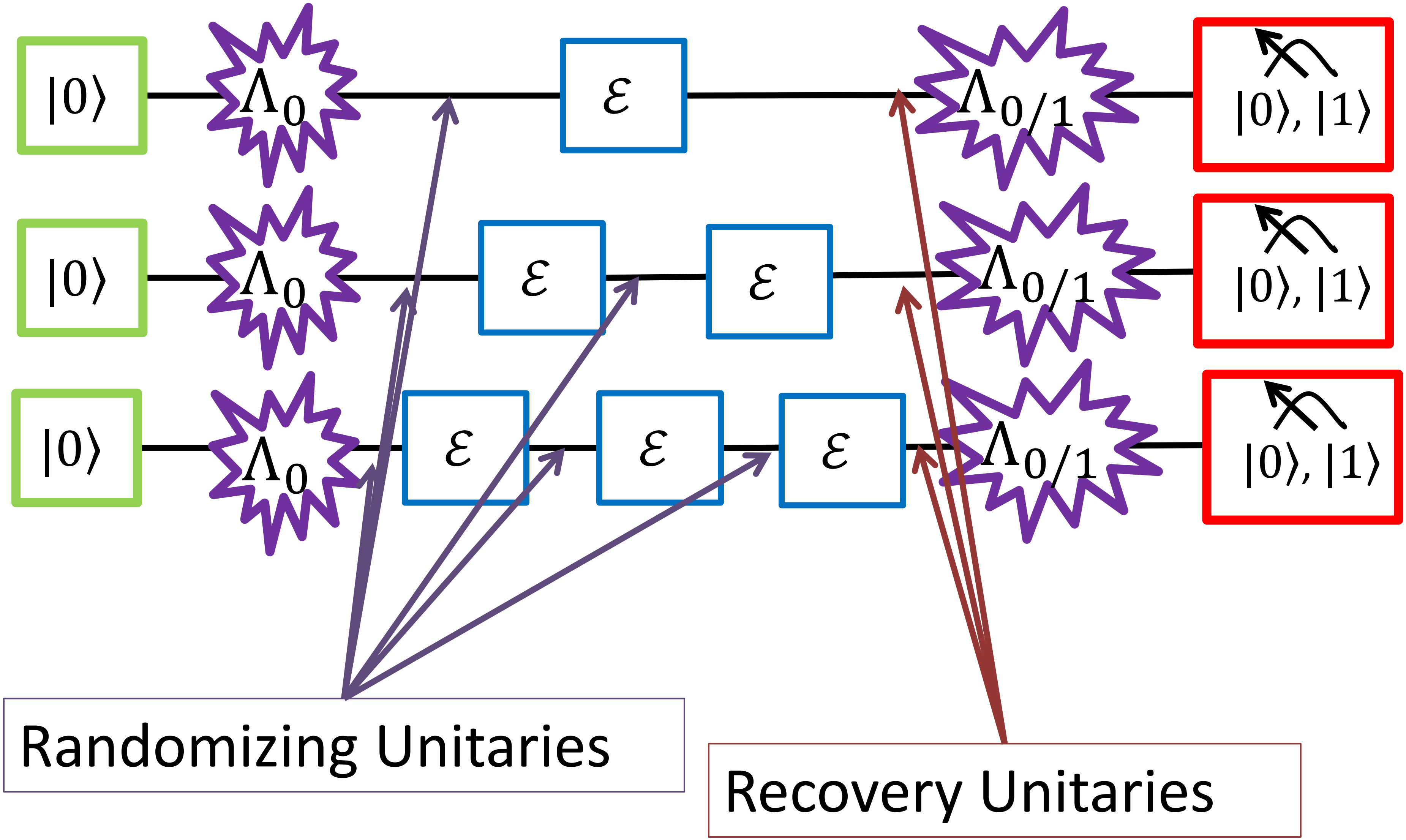


Repeated Application

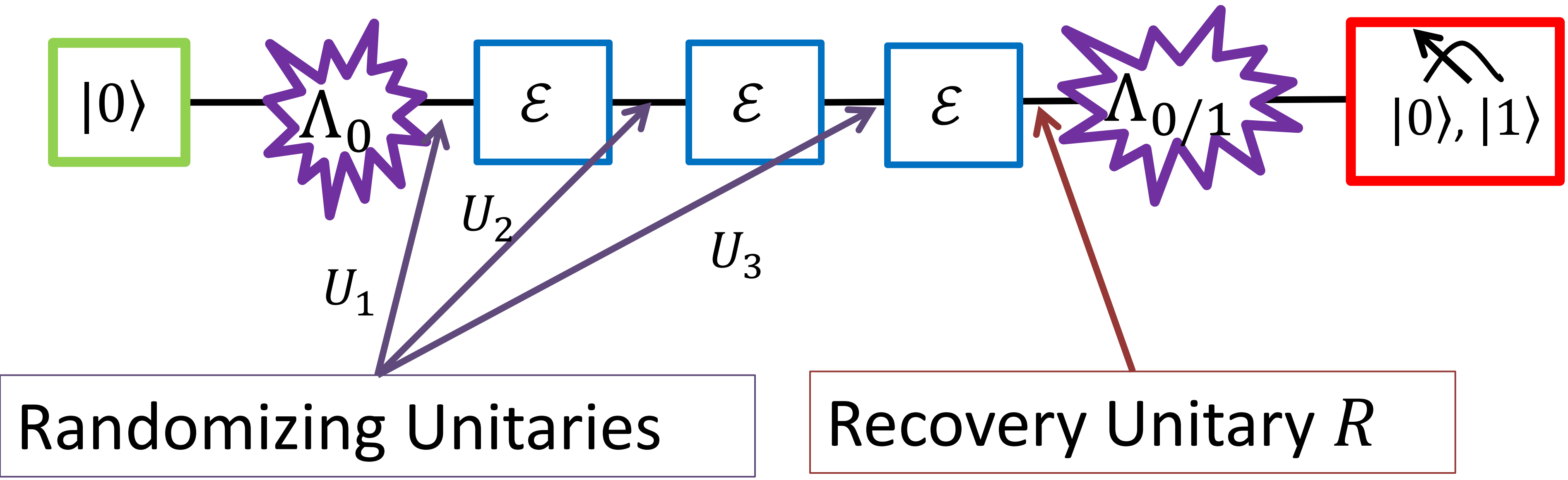


If eigenstate of $\mathcal{E}$, will only see how $\mathcal{E}$ acts on *this* state

Repeated Application



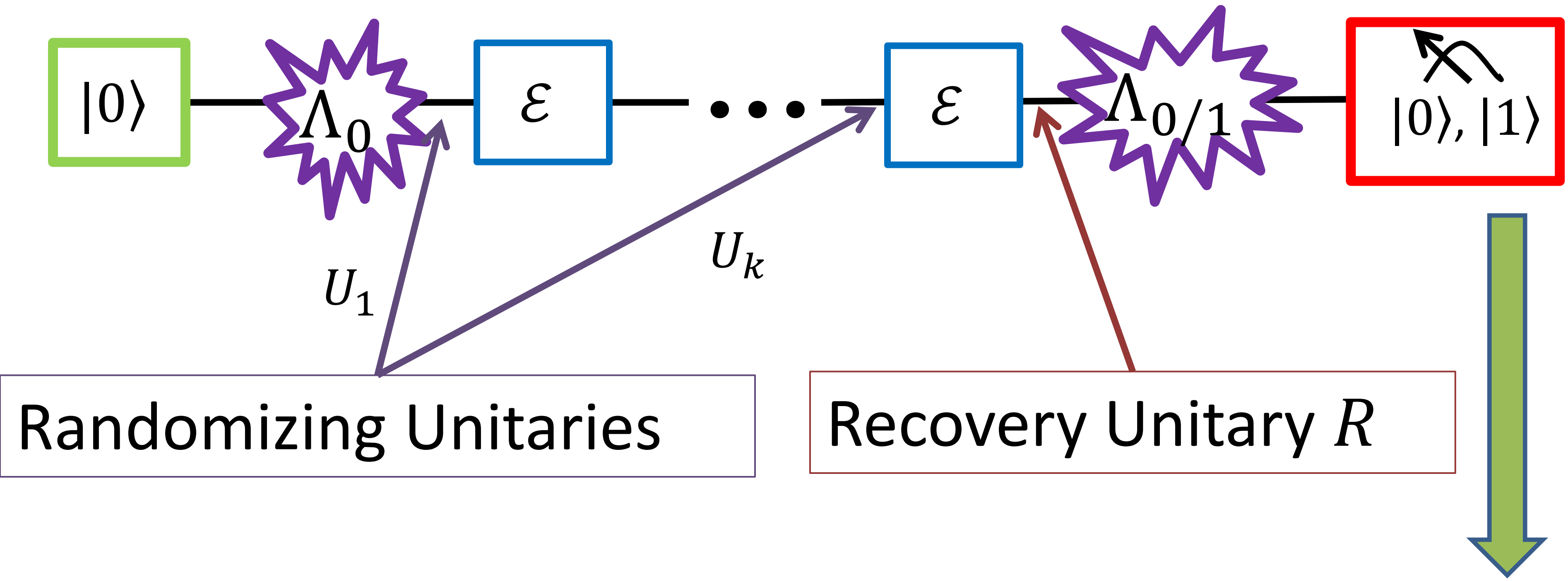
Recovery Operation



Pick a unitary V to compare $\mathcal{E}$ with. Then

$$R = (U_1\, V\, U_2\, \mathrm{V}\, U_3\, V)^\dagger$$

Randomizing Operations

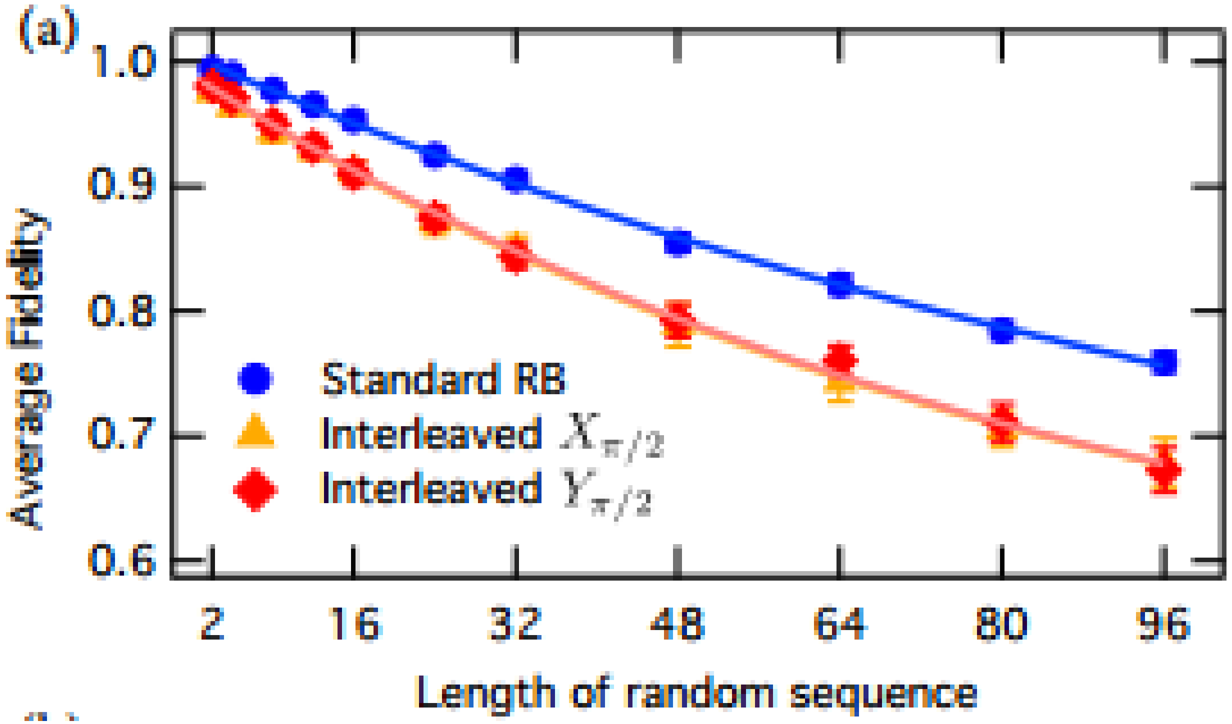


Prob. of outcome 0: $Ap^k + B$

A, B depend on $\Lambda_0, \Lambda_{0/1}$

p depends on overlap of $\mathcal{E}$ with V

Randomizing Operations



Magesan et al. '12

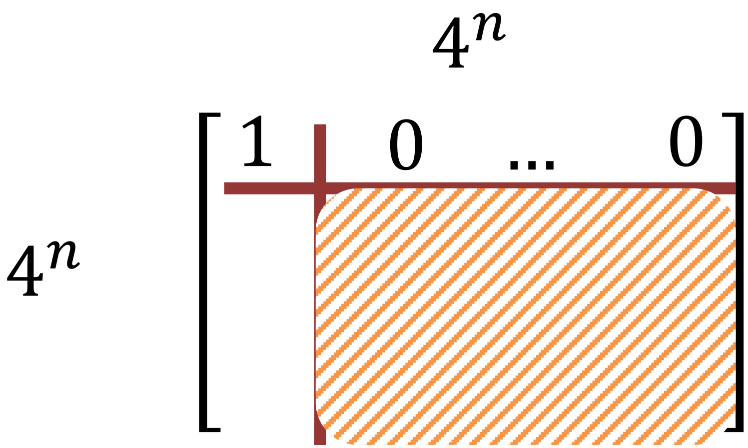
Prob. of outcome 0: $Ap^k + B$

A, B depend on $\Lambda_0, \Lambda_{0/1}$

p depends on overlap of $\mathcal{E}$ with V

“Overlap of $\mathcal{E}$ with V?"

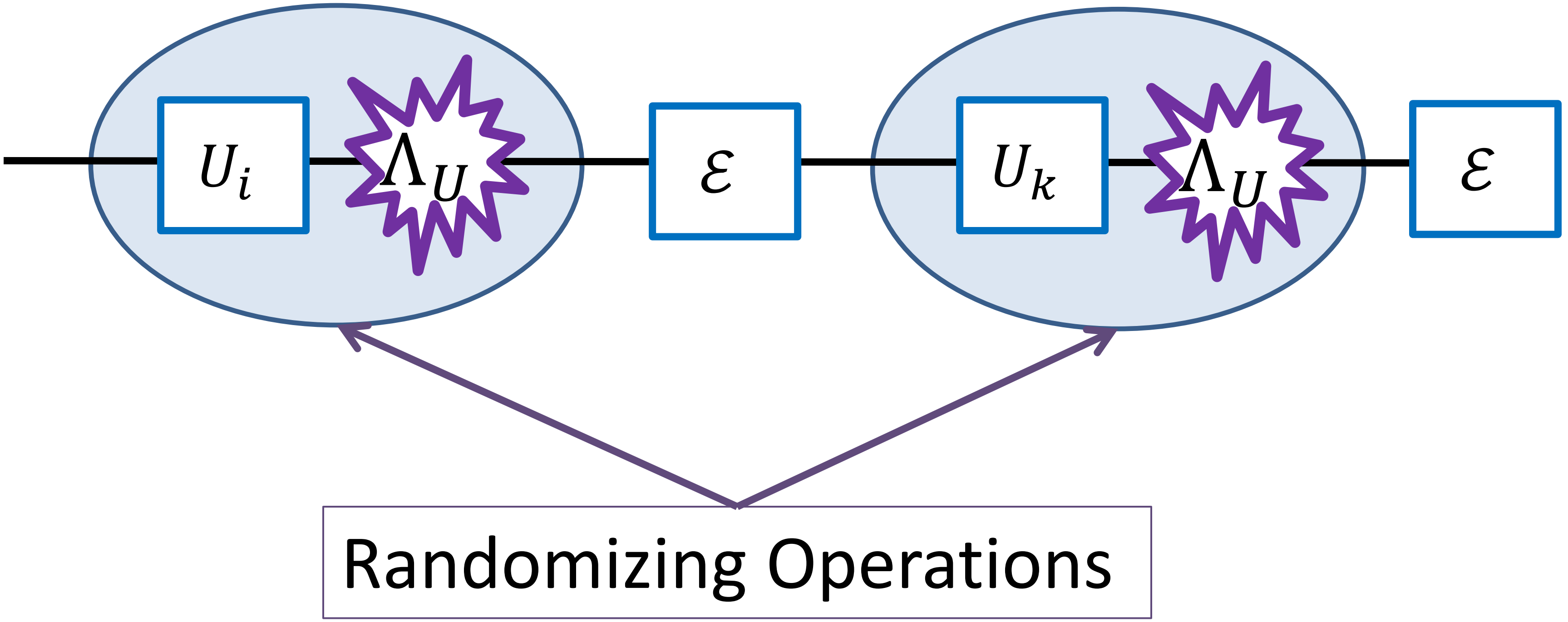
CPTP map: $16^n - 4^n$ parameters for n qubit map



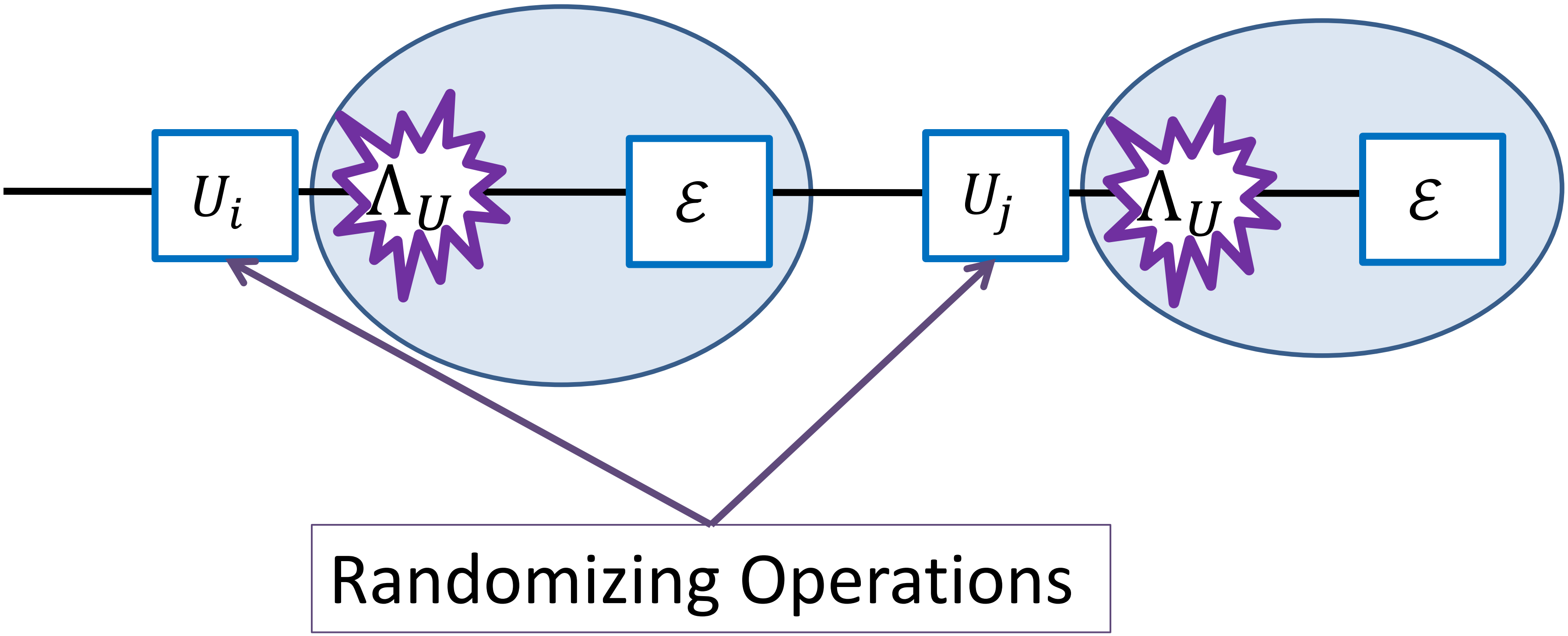
Choose many different $V's$ and measure overlaps. Each overlap gives a new parameter of this matrix!

We learn: $16^n - 2 * 4^n + 1$ parameters

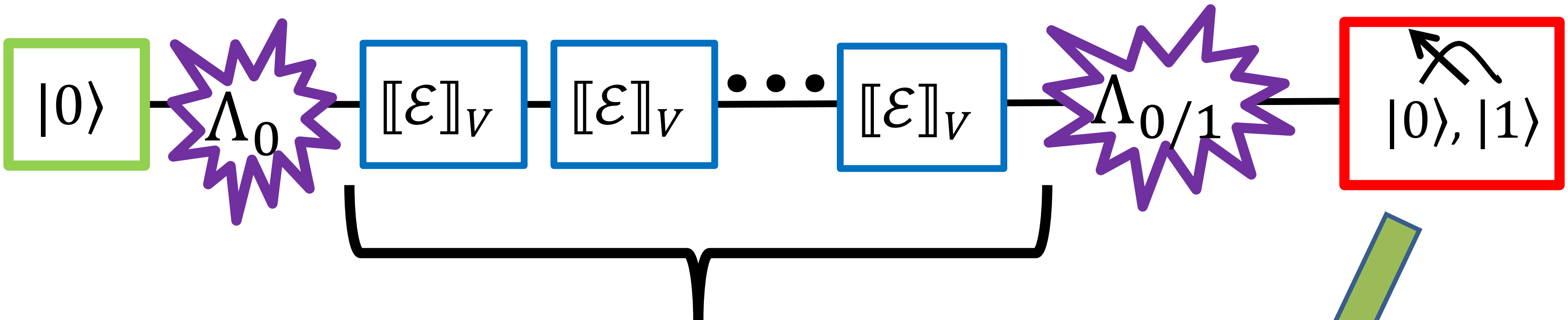
Errors!


U_i
Λ_U
$\mathcal{E}$
U_k
Λ_U
$\mathcal{E}$
Randomizing Operations

Errors!


U_i
Λ_U
$\mathcal{E}$
U_j
Λ_U
$\mathcal{E}$
Randomizing Operations

Randomizing Operations with Error



k applications of $\mathcal{E}$ with randomization

$Ap^k + B$

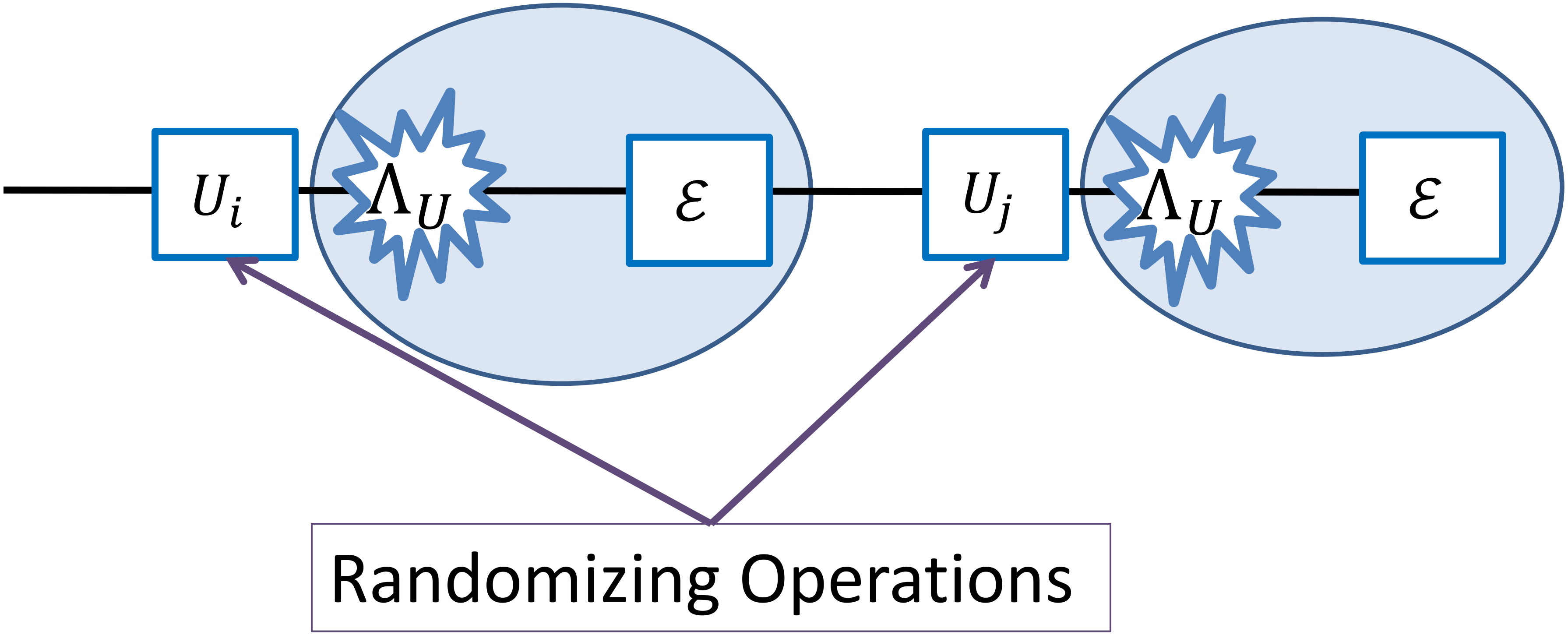
A, B depend on $\Lambda_0, \Lambda_{0/1}$

p depends on overlap of $\mathcal{E} \circ \Lambda_U$ with U

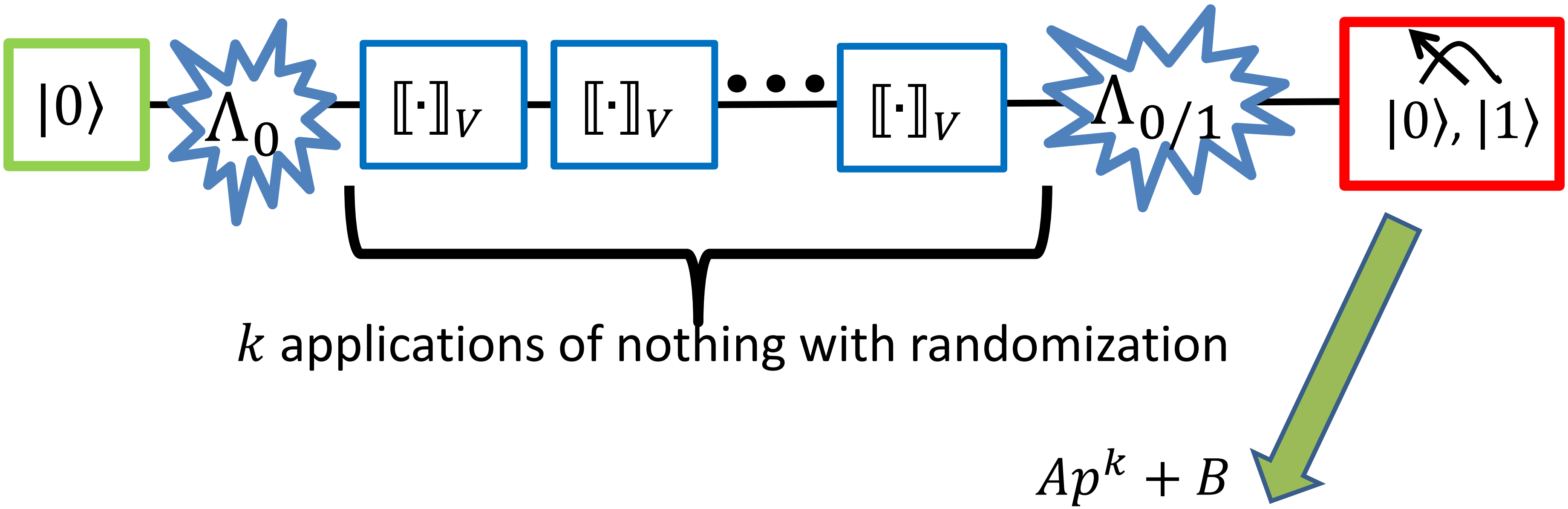
Trading One Error For Another?

- Originally had state preparation and measurement errors complicating things
- Now have randomizing errors complicating things
- BUT – we can characterize randomizing errors!

Errors!


U_i
Λ_U
$\mathcal{E}$
U_j
Λ_U
$\mathcal{E}$
Randomizing Operations

Randomizing Operations with Error



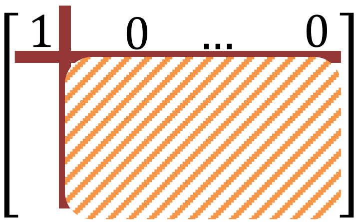
$Ap^k + B$

A, B depend on $\Lambda_0, \Lambda_{0/1}$

p depends on overlap of Λ_U with V

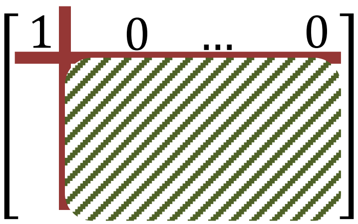
Main Result

almost complete characterization of Λ_U



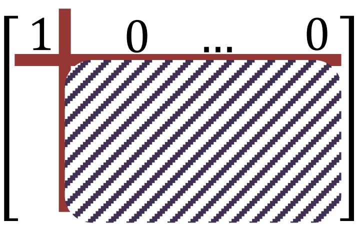
$+$

almost complete characterization of $\mathcal{E} \circ \Lambda_U$



$=$

almost complete characterization of $\mathcal{E}$



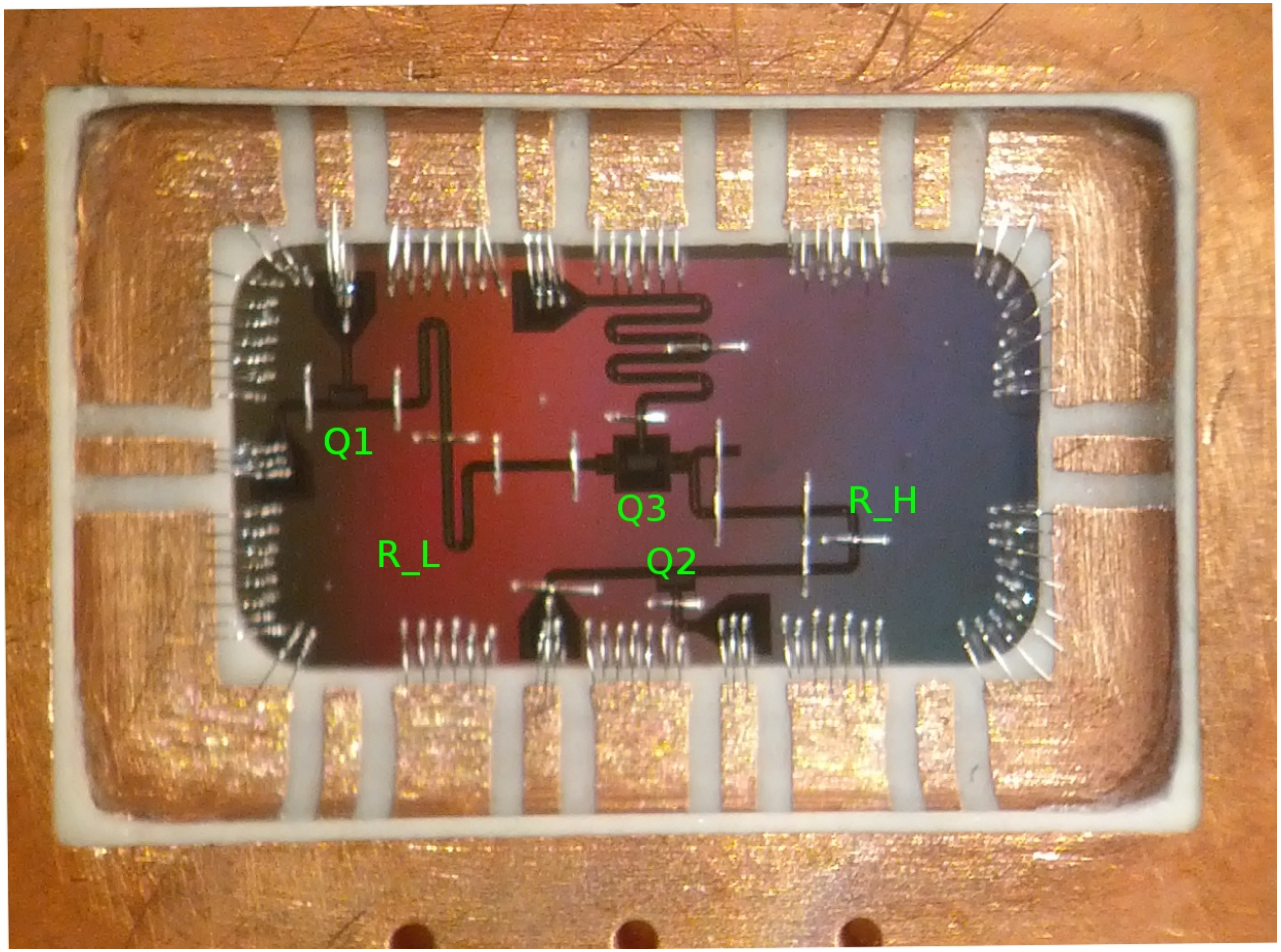
All without the systematic errors of previous procedures!

Experimental Procedure

- Repeat the following $\sim 16^n$ times
  - Pick a V
    - Run experiments of different lengths interleaving $\mathcal{E}$ with random unitaries to compare $\mathcal{E} \circ \Lambda_U$ to V.
    - Run same experiments interleaving “null operation” with random unitaries to compare Λ_U to V.
- Use this data to reconstruct $\mathcal{E} \circ \Lambda_U$ and Λ_U
- Reconstruct $\mathcal{E}$

Process can take days…

Experimental Implementation



Questions?

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Quantum Optics:

Journal of Modern Optics

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop20

Approaching Helstrom limits to optical pulse-position demodulation using single photon detection and optical feedback

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[b] National Institute of Information and Communications Technology, 4-2-1 Nukui-kitamachi,

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Quantum Optics:

PHYSICAL REVIEW A **80**, 052310 (2009)

Gaussian-state quantum-illumination receivers for target detection

Saikat Guha[1] and Baris I. Erkmen[2]
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[2]*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA*
(Received 23 July 2009; published 10 November 2009)

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Super-Conducting Qubits:

Phys. Rev. Lett. 104, 163601 (2010) [4 pages]

Direct Observation of Coherent Population Trapping in a Superconducting Artificial Atom

Abstract | References | Citing Articles (20)

Download: PDF (680 kB) Buy this article **Export:** BibTeX or EndNote (RIS)

William R. Kelly[*], Zachary Dutton[†], John Schlafer, Bhaskar Mookerji, and Thomas A. Ohki[‡]
Raytheon BBN Technologies, Cambridge, Massachusetts 02138, USA

Jeffrey S. Kline and David P. Pappas
National Institute of Standards and Technology, Boulder, Colorado 80305, USA

Characterizing Quantum Operations

Shelby Kimmel

Joint work with Marcus Silva,
Raytheon BBN Technologies

Trading 1 Error for Another?

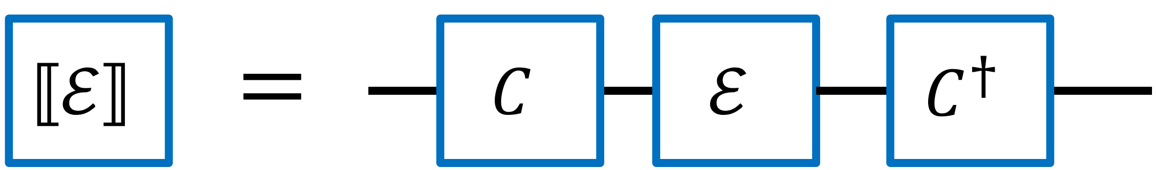
- Compare $\mathcal{E} \circ \Lambda_C$ to U for lots of U's
  - Learn $(15^n + 1)$ of $(16^n - 4^n)$ parameters of $\mathcal{E} \circ \Lambda_C$
- Do same but remove $\mathcal{E}$: Compare Λ_C to U for lots of U's
  - Learn $(15^n + 1)$ of $(16^n - 4^n)$ parameters of Λ_C

Clifford Twirl

- **Cliffords** are a set of unitaries
  - Apply a random Clifford ~ apply a random unitary
  - Can efficiently simulate classically

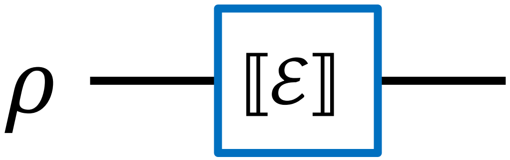
Clifford Twirl of $\mathcal{E}$:

Choose a random Clifford C



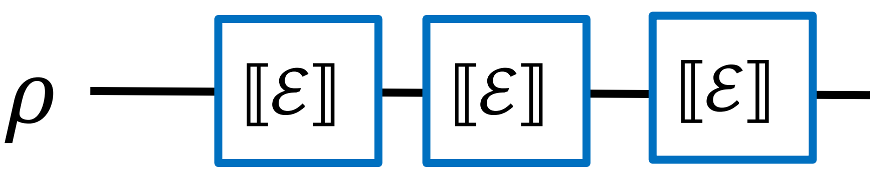
Clifford Twirl

Outcome is same for any initial state:



$$p\rho + (1-p)\varrho_{\mathbb{I}}$$

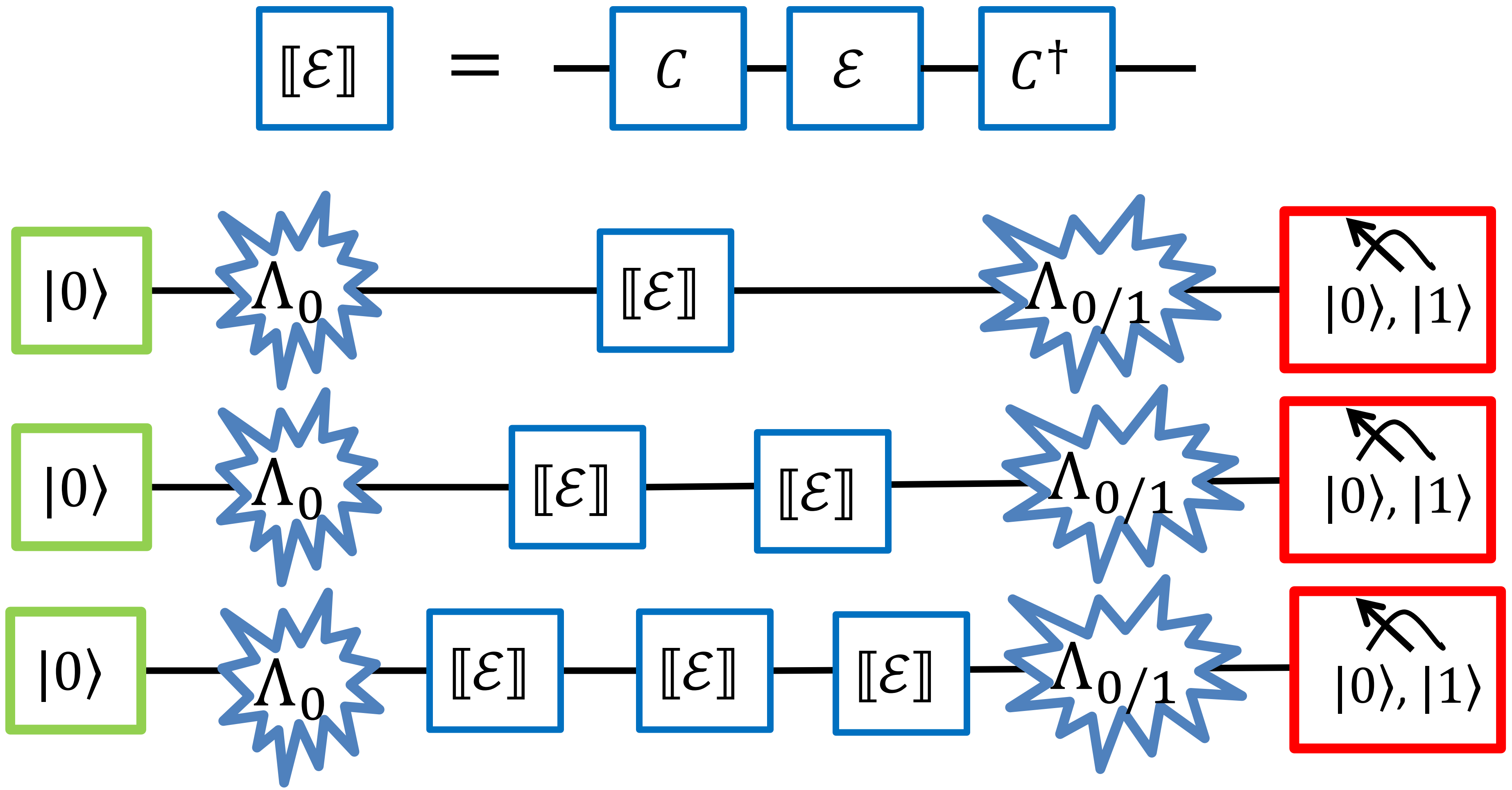
Composes Easily



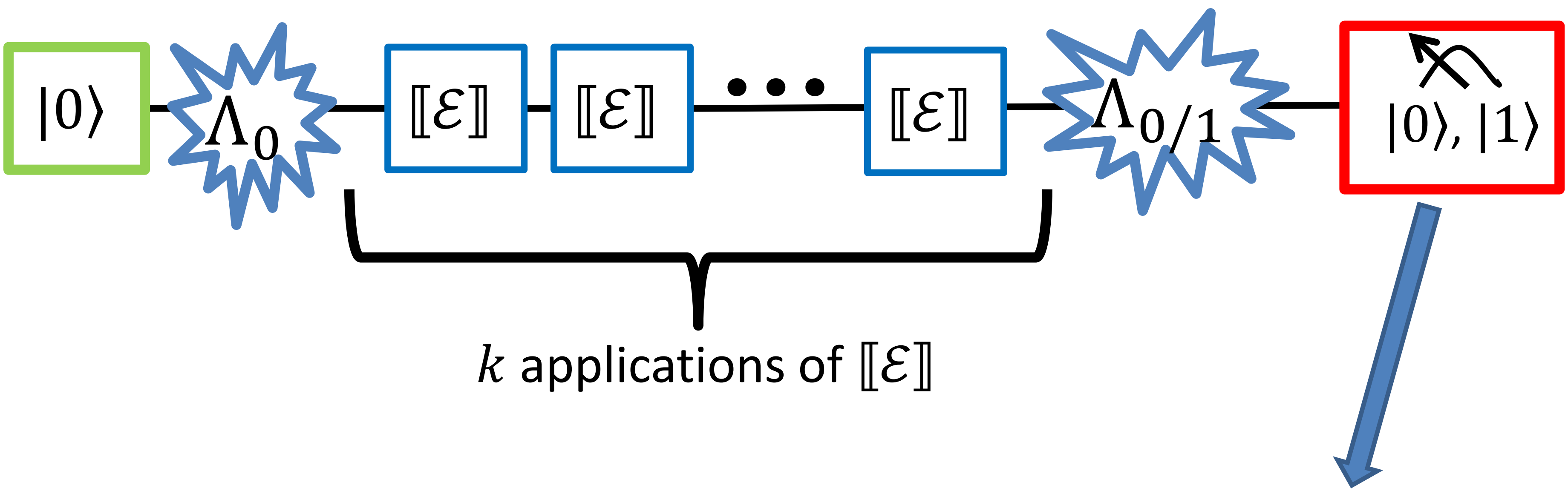
$$p^3\rho + (1-p^3)\varrho_{\mathbb{I}}$$

Repeated Applications

Choose a random Clifford C



Repeated Applications



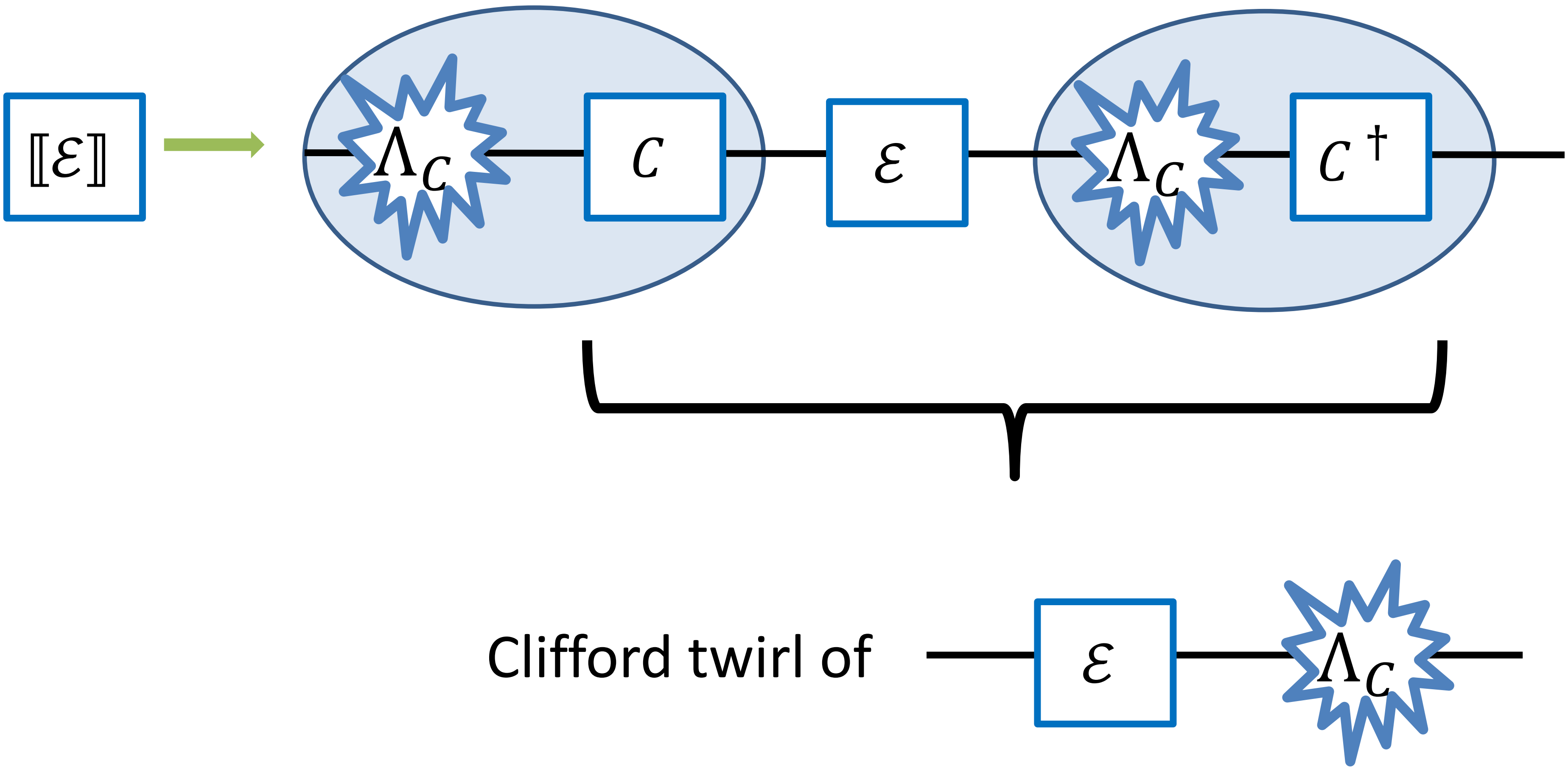
Measurement Outcomes:

$$Ap^k + B$$

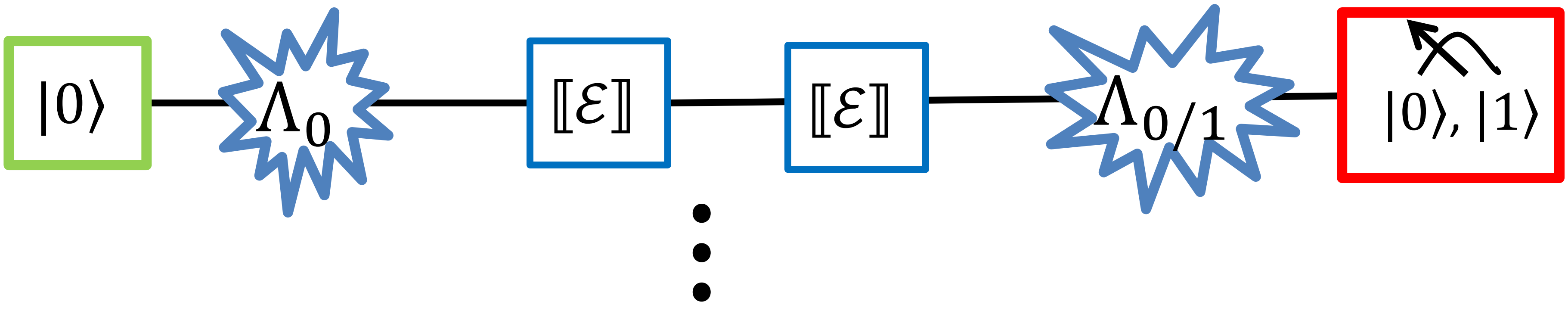
A, B depend on $\Lambda_0, \Lambda_{0/1}$

depends on closeness of $\mathcal{E}$ to U

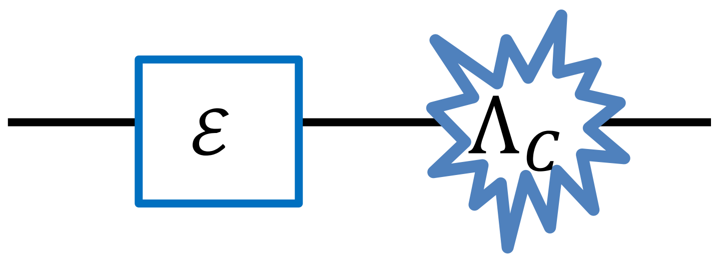
What about Errors!



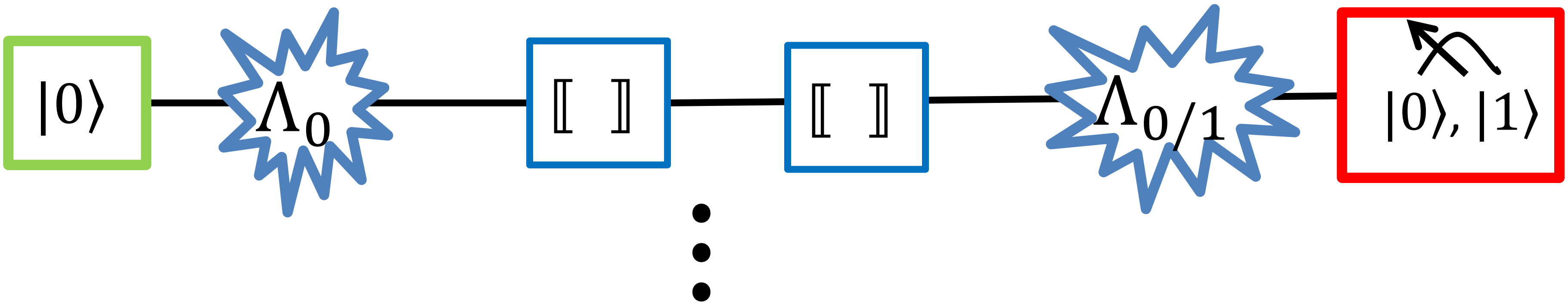
Procedure Step 1



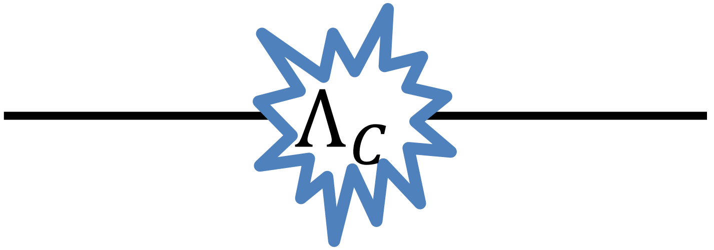
Learn p (probability of error) for



Procedure Step 2

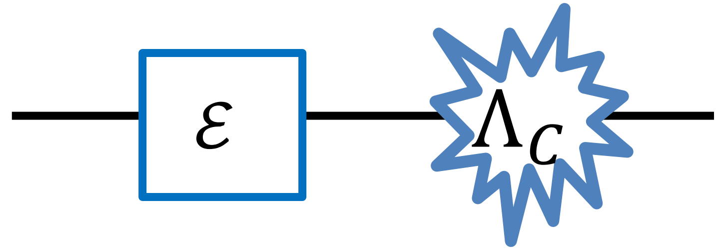


Learn p (probability of error) for



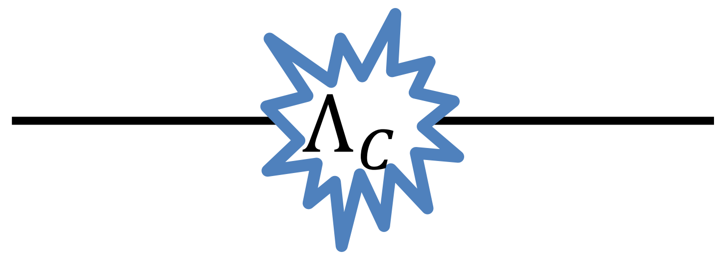
Procedure Step 3

Learn p (probability of error) for



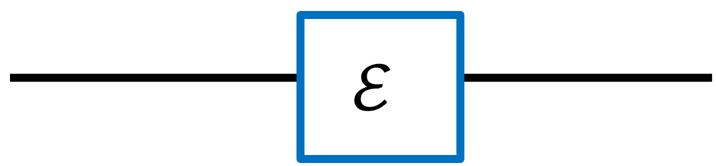
+

Learn p (probability of error) for



=

Bound on p for

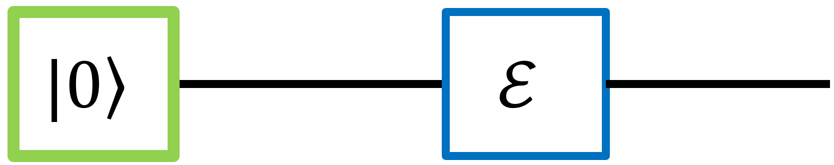


Need More Information!

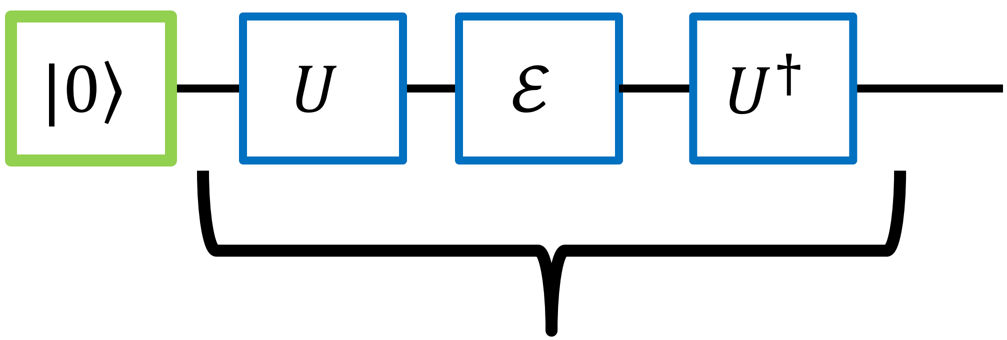
Only learn 1 parameter: p

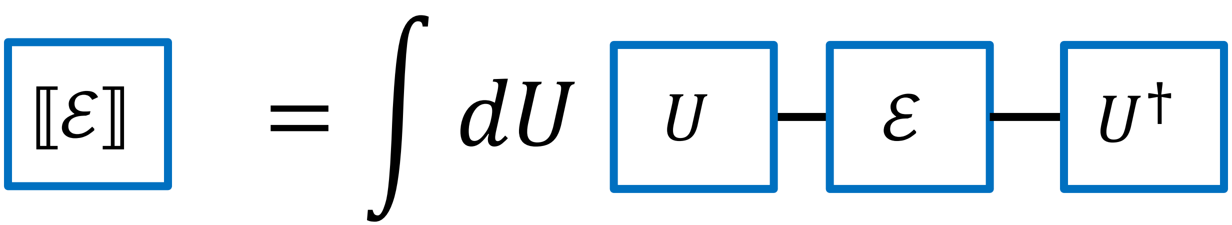
Need $16^n - 4^n$ parameters to get full process

Twirling

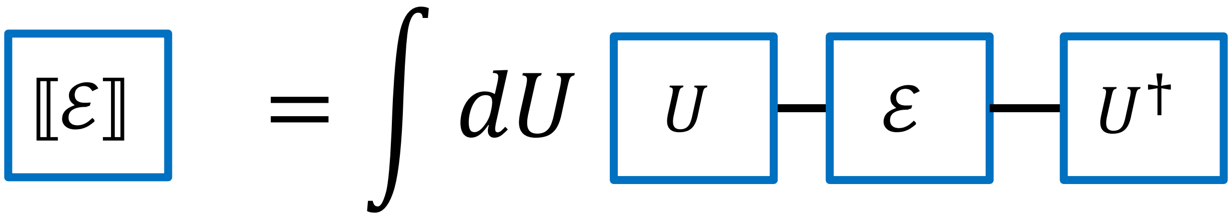


What is action on an average state? Choose a random unitary U:

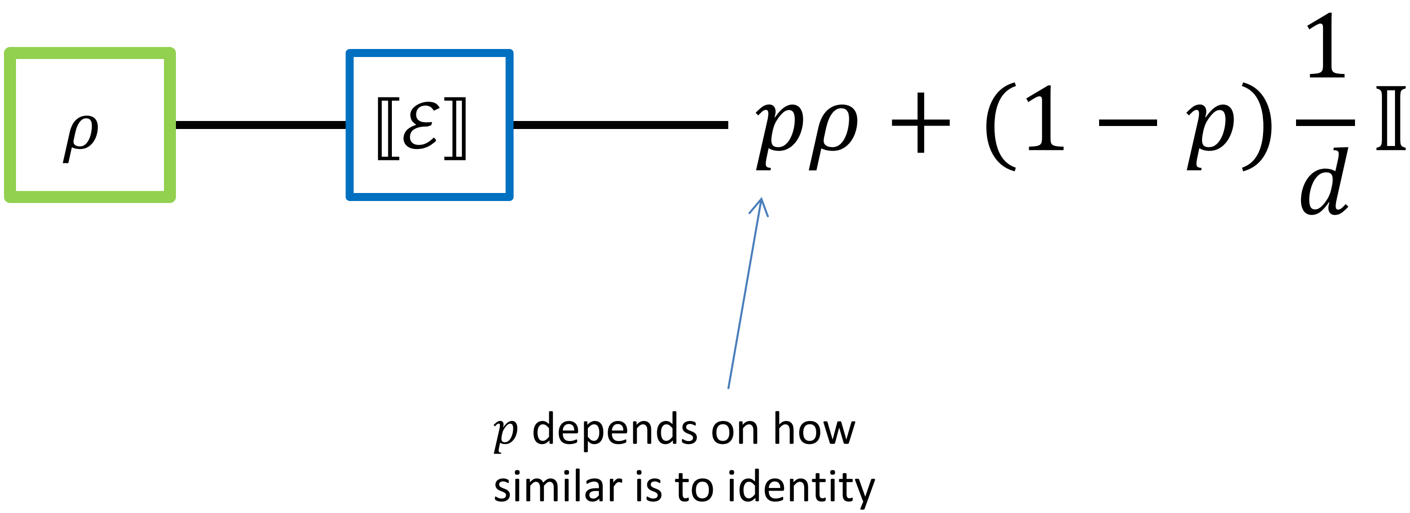




Twirling 2



$[\![\mathcal{E}]\!]$
$= \int dU$
U
$\mathcal{E}$
$U^\dagger$



ρ
$[\![\mathcal{E}]\!]$
$p\rho + (1-p)\frac{1}{d}\mathbb{I}$
p depends on how similar is to identity

Randomized Benchmarking (imagine no Error)

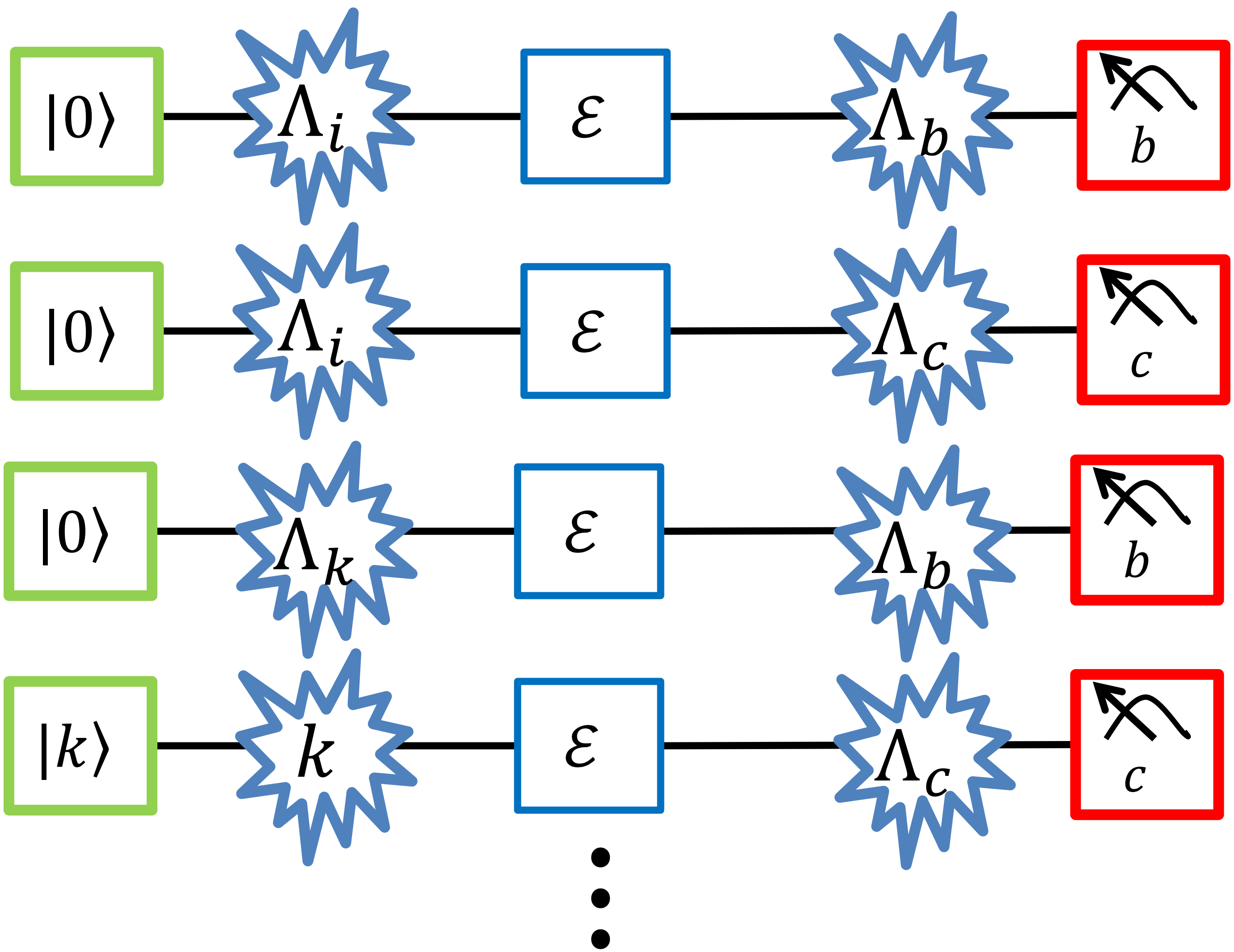
Inserting Clifford

Unital Block

What about Error

Experimental Implementation

The Problem



$|0\rangle$
Λ_i
$\mathcal{E}$
Λ_b
b
$|0\rangle$
Λ_i
$\mathcal{E}$
Λ_c
c
$|0\rangle$
Λ_k
$\mathcal{E}$
Λ_b
b
$|k\rangle$
k
$\mathcal{E}$
Λ_c
c