Robust Characterization of Quantum Processes

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Why don’t we have a working quantum computer?

Too Many Errors
Can Improve Operations with Better Characterization of Errors

“Depolarizing error”

“Extra rotation around z-axis”

Improvement to Computer

Cooling

Magnetic Shielding
Can Improve Error Correcting Codes with Better Characterization of Errors

“Non local, correlated error”

Improvement to Error Correcting Code
Standard Techniques Have Problems

Need nearly perfect state preparation, measurement and other operations. Otherwise systematic errors give inaccurate or even invalid results.

Not “robust”
Robust Techniques

• **Gate Set Tomography Procedures** [Stark ‘13, Blume-Kohout et al. ’13, Merkel et al. ‘12]
  
  – Characterizes many processes at once

• **Randomized Benchmarking (RB)** [Emerson et al. ‘05, Knill et al. ‘08, Magesan et al. ‘11, ‘12]
  
  – Can only characterize 1 parameter of 1 type of process.
  
  – Can efficiently test performance of a universal gate set.
Outline

• **Background:**
  – Issues with standard process characterization
  – Randomized benchmarking framework, challenges of current implementation

• **Our Results:**
  – Robust characterization of unital part of any process
  – Efficient bound on average fidelity of universal gate set.
Quantum Process (Map)

• Completely positive trace preserving (CPTP) map = any process that takes valid quantum states to valid quantum states.
• E.g. unitary, depolarizing process, dephasing process, amplitude damping process
• $n$ qubits, $O(16^n)$ free parameters
Problem with Standard Process Tomography

\[ |0\rangle, |1\rangle \]
Problem with Standard Process Tomographyography

\[ |0\rangle \xrightarrow{\Lambda_0} |\rangle \xrightarrow{\epsilon} |0/1\rangle \xrightarrow{\Lambda_{0/1}} |0, 1\rangle \]

\[ |+\rangle \xrightarrow{\Lambda_+} |\rangle \xrightarrow{\epsilon} |+/-\rangle \xrightarrow{\Lambda_{+/\pm}} |+, -\rangle \]

\[ \neq \]
Repeated Application

\[
|0\rangle \xrightarrow{\Lambda_0} |\epsilon\rangle \xrightarrow{\epsilon} |0\rangle, |1\rangle \\
|0\rangle \xrightarrow{\Lambda_0} |\epsilon\rangle \xrightarrow{\epsilon} |\epsilon\rangle \xrightarrow{\Lambda_0/1} |\epsilon\rangle \\
|0\rangle \xrightarrow{\Lambda_0} |\epsilon\rangle \xrightarrow{\epsilon} |\epsilon\rangle \xrightarrow{\epsilon} |\epsilon\rangle \xrightarrow{\Lambda_0/1} |\epsilon\rangle \xrightarrow{\Lambda_0/1} |\epsilon\rangle \xrightarrow{\Lambda_0/1} |0\rangle, |1\rangle \\
\vdots
\]
Repeated Application

If eigenstate of $\mathcal{E}$, will only see how $\mathcal{E}$ acts on this state
Randomized Benchmarking

|0⟩ → \Lambda_0 → Ε → \Lambda_0/1
|0⟩ → \Lambda_0 → Ε → \Lambda_0/1
|0⟩ → \Lambda_0 → Ε → \Lambda_0/1

Randomizing Unitaries

Recovery Unitaries
Randomized Benchmarking

Value of Measurement Observable

Simulated Randomized Benchmarking Experiment

Decay constant depends on one parameter of $\mathcal{E}$
Randomized Benchmarking

Randomizing Unitaries Have Errors!

Recovery Unitaries
Two Issues with RB

1. How can we extract more than just 1 parameter?
2. How can we deal with errors on the randomizing operations?
Randomizing Operation: Clifford Twirl

\[
\frac{1}{|C_i|} \sum_{C_i \text{ in Cliffords}} C_i^\dagger \circ \mathcal{E} \circ C_i (\rho) = (1 - q)\rho + q \frac{\mathbb{I}}{d}
\]

Result is depolarizing channel (very simple process) that depends on only one parameter of \( \mathcal{E} \):

Average fidelity of \( \mathcal{E} \) to the identity

Average fidelity of \( \mathcal{E} \) = \[
\int d |\psi\rangle \langle \psi| \mathcal{E}(|\psi\rangle\langle \psi|)|\psi\rangle
\]
Randomizing Operation: Clifford Twirl

To implement (approximately), repeat many times, each time randomly choosing $C_i$, and average results

Everything inside the Clifford twirl gets simplified to a depolarizing channel
Randomizing Operation: Clifford Twirl

\[ 
|0\rangle \xrightarrow{\Lambda_0} \mathcal{C}_{i\dagger} \circ \mathcal{C}_{j} \xrightarrow{\xi} \mathcal{C}_{i\dagger} \circ \mathcal{C}_{j} \xrightarrow{\Lambda_{0/1}} |0\rangle, |1\rangle \]

Randomizing Operations

Recovery Unitaries
Randomizing Operations

Decay constant depends on 1 parameter of \( \mathcal{E} \): Average fidelity of \( \mathcal{E} \) to the identity.
1. Extracting More Information

Twirl simplifies too much!
- no twirl
- stick additional information inside twirl
1. Extracting More Information

$C_x$ is fixed – not random. The same $C_x$ is applied in each twirl.
1. Extracting More Information

Decay constant depends on 1 parameter of $\mathcal{E}$:

**Average Fidelity of $\mathcal{E}$ to $C_x^\dagger$** (can have fast decays)
1. Extracting More Information

CPTP map: $16^n - 4^n$ parameters for $n$-qubit map

To compose two maps, just multiply matrices!

- Vectors $V$ span a subspace $S$
- Learn inner product between $V$ and unknown vector $u$
- Can learn projection of $u$ onto $S$
- Cliffords span unital part
- Learn inner product between Cliffords and $\mathcal{E}$
- Learn projection of $\mathcal{E}$ onto unital subspace
2. Dealing with Errors

\[ C_i \xrightarrow{\Lambda_C} \varepsilon \xrightarrow{C_x \circ C_i^\dagger \circ C_j} \varepsilon \xrightarrow{\Lambda_C} C_x \circ C_j^\dagger \circ \]
2. Dealing with Errors
2. Dealing with Errors

almost complete characterization of $\Lambda_C$

\[
\begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

+ 

almost complete characterization of $\Lambda_C \circ \mathcal{E}$

\[
\begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

= 

almost complete characterization of $\mathcal{E}$

\[
\begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

All without the systematic errors of previous procedures!
Experimental Implementation
Negative Witness Test [Moroder et al. ‘13]

• To be a valid quantum process, must be trace preserving and completely positive

• Complete positivity = in Choi representation, all eigenvalues must be positive

• Negative witness test:
  – Look at value of smallest eigenvalues of reconstructed map in Choi representation.
  – If negative, BAD!
Negative Witness Test for Hadamard
Efficient Fidelity Estimate

\[
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0
\end{bmatrix}
\]

Requires an exponential number of measurement settings with different \( C_x \)

Instead, only want to check that your operations are good enough.

Want to check implementation of Clifford Gates and T gates = universal gate set
Efficient Fidelity Estimate

Average fidelity to any unitary $\mathcal{U}$ of
- $O(\log n)$ T gates
- $O(\text{poly } n)$ Cliffords only need to repeat for $O(\text{poly } n)$ different $C_x$.

If $\Lambda_C$ is close to Identity, can closely bound the average fidelity of $\mathcal{E}$ to $\mathcal{U}$.

Can test a universal gate set!
Conclusions and Open Questions

• Can robustly measure unital part of any quantum process
• Can efficiently and robustly test fidelity of universal quantum gate set operations.
• Experimentally implemented with superconducting qubit system at BBN

• What about the non-unital part?
• Can we extract other information efficiently and robustly (compressed sensing?)
• How does RB compare to Gate Set Tomography methods?
Efficient Fidelity Estimate

Average Fidelity \((\mathcal{E}, U) \sim \text{tr}\left[\begin{bmatrix} \vdots & \mathcal{E} & \vdots \\ \vdots & \cdots & \vdots \\ \vdots & \cdots & \vdots \end{bmatrix}\begin{bmatrix} \vdots & \cdot & \cdots & \cdot & \cdot & U & \cdots & \cdot & \cdot & \cdots \end{bmatrix}\right]\)

\[ \sum_x a_x \begin{bmatrix} \vdots & \cdot & \cdots & \cdot & \cdot & C_x & \cdots & \cdot & \cdot & \cdots \end{bmatrix} \]

Unitaries composed of Cliffords and \(O(\log n)\) T gates can be written as a linear combination of \(O(\text{poly } n)\) Cliffords. Only need to measure \(O(\text{poly } n)\) traces, each of which can be done efficiently.
Efficient Fidelity Estimate

Average Fidelity \( (\mathcal{E}, U) \sim \text{tr} \left[ \begin{array}{ccc}
\Lambda_C \circ \mathcal{E} & \cdots & \\
\vdots & \ddots & \vdots \\
\cdots & \cdots & 
\end{array} \right] \left[ \begin{array}{ccc}
U & \cdots & \\
\vdots & \ddots & \vdots \\
\cdots & \cdots & 
\end{array} \right] \]

\[
\sum_x a_x \left[ \begin{array}{ccc}
C_x & \cdots & \\
\vdots & \ddots & \vdots \\
\cdots & \cdots & 
\end{array} \right]
\]

Since we haven’t characterized \( \Lambda_C \), we can’t get rid of its effect. However, we can measure its average fidelity to the identity, and if it is close to the identity, we can bound its effect.
Efficient Fidelity Estimate

To get average fidelity to any unitary $\mathcal{U}$ of $O(\log n)$ T gates and $O(\text{poly } n)$ Cliffords, only need to repeat for $O(\text{poly } n)$ overlaps $C_x$.

If $\Lambda_C$ is close to Identity, can closely bound the average fidelity of $\mathcal{E}$ to $\mathcal{U}$.

Can test a universal gate set!
What do we measure?

We measure

$$\text{tr}
\begin{bmatrix}
\vdots 
\Lambda_C \circ \mathcal{E} \\
\vdots 
\vdots 
\vdots
\end{bmatrix}
\begin{bmatrix}
\vdots 
C_x \\
\vdots 
\vdots 
\vdots
\end{bmatrix}$$
What can we measure?

CPTP map: $16^n - 4^n$ parameters for $n$ qubit map

Choose many different $C_x$'s and measure trace. By measuring enough traces can learn unital part

We learn: $16^n - 2 \times 4^n + 1$ parameters
What do we characterize?

Need to repeat for $16^n$ different $C_x$

(Pauli-Liouville Representation)
2. Dealing with Errors

\[ C_i \xrightarrow{\Lambda_C} \varepsilon \xrightarrow{C_x \circ C_i^\dagger \circ C_j} \Lambda_C \xrightarrow{\varepsilon} C_x \circ C_j^\dagger \circ \]

Randomizing Operation
= Twirl
Can we do better with Randomized Benchmarking?

• Can we robustly characterize many parameters of any operation?

   Can characterize almost all parameters of any quantum map.

   We show Cliffords span unital part of quantum maps. By learning average fidelity of $\mathcal{E}$ to many $C_x$'s, can learn projection of $\mathcal{E}$ into unital subspace.

• What information can we obtain robustly and efficiently?

   Can test performance of a universal gate set.