Quantum vs Classical Proofs

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P vs NP vs BQP
P vs NP vs BQP

Classical

Efficiently Solvable

P

Quantum

BQP

Efficiently Checkable

NP

BQP
P vs NP vs BQP

Classical

- **P**: Efficiently Solvable

Quantum

- **BQP**: Efficiently Solvable

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P vs NP vs BQP

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QCMA

QMA
P vs NP vs BQP

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QCMA

QMA

Computational power of quantum state vs classical state?
Outline

1. QMA and QCMA (what? why?)
2. Our approach to differentiating them
Informal Definitions

• QMA (Quantum Merlin Arthur)

Arthur
“I have a question – is the answer yes or no?”

e.g. Does this local Hamiltonian have a low energy state?
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“I have a question – is the answer yes or no?”

Merlin
“The answer is yes. Here is a quantum state (proof) to convince you.”

e.g. Does this local Hamiltonian have a low energy state?

\[ |\phi\rangle \in (\mathbb{C}^2)^\otimes n \]

\( n \sim \text{size of problem} \)
Informal Definitions

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“I have a question – is the answer yes or no?”

e.g. Does this local Hamiltonian have a low energy state?

“I don’t trust Merlin, but I can use $|\phi\rangle$ as input to my quantum computer to verify he is telling the truth.”
Informal Definitions

- QMA (Quantum Merlin Arthur)

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“I have a question – is the answer yes or no?”

e.g. Does this local Hamiltonian have a low energy state?

QMA:
Class of problems where if answer is
- YES, ∃ q. state that convinces Arthur with high probability
- NO, ∄ a q. state that convinces Arthur with high probability
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“The answer is yes. Here is a classical state (proof) to convince you.”

e.g. Does this local Hamiltonian have a low energy state?

\[ s \in \{0,1\}^n \]

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- QCMA (Quantum Classical Merlin Arthur)

Arthur
“I have a question – is the answer yes or no?”

e.g. Does this local Hamiltonian have a low energy state?

QCMA: Class of problems where if answer is
- YES, \( \exists \) c. state Merlin can send that convinces Arthur with high probability
- NO, \( \not\exists \) a c. state that convinces Arthur with high probability
Why Important

“Does this local Hamiltonian have a low energy state?”:

In QMA [Kitaev ‘02]
The quantum proof is just the low energy state if it exists.
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In QMA [Kitaev ‘02] The quantum proof is just the low energy state if it exists.

“Does this local Hamiltonian have a low energy state?”:

Not known if in QCMA Would imply there is a classical description of low energy states of local Hamiltonians.
Why Important

QMA vs QCMA
What is the relative computational power of quantum and classical states?
Our Goal

Show QCMA is less powerful than QMA.
(i.e. there are problems that you can verify with a quantum proof that you can’t verify with a classical proof.)
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But proving this directly is HARD.
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(i.e. there are problems that you can verify with a quantum proof that you can’t verify with a classical proof.)

But proving this directly is HARD.

Instead, will try to show QCMA$^O$ is less powerful than QMA$^O$.
• (With an oracle)
• Less impressive, but still interesting.
Oracle

In addition to the quantum computer, Arthur has a black box unitary operation $O$. 
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**In-place Quantum Oracle:**

Let $f : \{1, 2, \ldots, M\} \rightarrow \{1, 2, \ldots, M\}$ be a bijective function.

Standard basis states (in-place oracle permutes states)

$|x\rangle \rightarrow O_f \rightarrow |f(x)\rangle$
In addition to the quantum computer, Arthur has a black box unitary operation $O$.

**In-place Quantum Oracle:**

Let $f:\{1,2,\ldots,M\} \to \{1,2,\ldots,M\}$ be a bijective function.

- Standard basis states (in-place oracle permutes states)

$$|x\rangle \to O_f \to |f(x)\rangle$$

- Has classical counterpart (encodes classical function)

Previous result by Aaronson and Kuperberg (’07) proved separation with an oracle without a classical analog.
Outline

1. QMA and QCMA (what? why?)
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Our Yes-No Question

Intuition: Want a problem where quantum proof is a superposition of an exponentially large number of states.
Our Yes-No Question

Setup:
- Given oracle $O_f$ with $f: [N^2] \rightarrow [N^2]$
- Let $S_f = \{i: f(i) \in [N]\}$ = “preimage subset”
- Is $S_f$ mostly even? (Promised either mostly even or mostly odd)
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This problem is in QMA$^\circ$ (with an in-place oracle $O_f$)
Our Yes-No Question

Setup:
• Given oracle $O_f$ with $f: [N^2] \rightarrow [N^2]$
• Let $S_f = \{i: f(i) \in [N]\} =$ “preimage subset”
• Is $S_f$ mostly even? (Promised either mostly even or mostly odd)

If “Yes”
• Merlin provides superposition of preimage subset states
• Arthur either
  • Measures in standard basis, gets even outcome with high probability.
  • Applies $O_f$ and measures whether he got the superposition of the first $N$ standard basis states. Succeeds with probability 1.
Our Yes-No Question

Setup:
- Given oracle $O_f$ with $f: [N^2] \to [N^2]$
- Let $S_f = \{i: f(i) \in [N]\}$ = “preimage subset”
- Is $S_f$ mostly even? (Promised either mostly even or mostly odd)

If “No”:
- Merlin sends any state (on $n = \log (N^2)$ qubits)
- Arthur either
  - Measures in standard basis, gets even outcome with probability $p_1$.
  - Applies $O_f$ and measures whether he got the superposition of the first $N$ standard basis states. Succeeds with probability $p_2$.
- We show $p_1$ and $p_2$ can’t both be large.
In-Place Oracle Problem

Approach to proving problem is not in QCMA^O

• A short classical proof can’t contain enough information to convince Arthur about properties of a nearly structureless exponentially large subset.
In-Place Oracle Problem

Approach to proving problem is not in QCMA°
• Use **Adversary Method** to show can’t efficiently distinguish YES from NO instances..
In-Place Oracle Problem

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• Merlin’s proof complicates Adversary Method…
In-Place Oracle Problem

Approach to proving problem is not in QCMA°
• Use Adversary Method to show can’t efficiently distinguish YES from NO instances.
• Merlin’s proof complicates Adversary Method…
• Use Pigeon Hole Principle to show one proof corresponds to a large number of permutations – by restricting to only those permutations we can ignore proof and use the Adversary Method.
In-Place Oracle Problem

Approach to proving problem is not in QCMA°

- Use **Adversary Method** to show can’t efficiently distinguish YES from NO instances.
- Merlin’s proof complicates Adversary Method…
- Use **Pigeon Hole Principle** to show one proof corresponds to a large number of permutations – by restricting to only those permutations we can ignore proof and use the Adversary Method.
- Adapt Adversary Method to in-place and probabilistic oracles.
Other applications

We prove an oracle separation between QCMA and AM.

Our approach works in general for proving subset-based oracle problems, (including standard oracle problems), are not in QCMA.
Summary and Open Problems

• A quantum proof can be more powerful than a classical proof.
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• A quantum proof can be more powerful than a classical proof.
  • Intuition: a quantum proof can contain information about an exponentially large set via superposition, while a classical prof can’t.
  • Grilo, Kerenidis, Sikora ’15: QMA proof can always be a subset state
Summary and Open Problems

• Remove probabilistic oracle? (Less Hard – artifact of proof techniques)
• Separation without an oracle? (Extremely Hard)
• QCMA<QMA using a standard oracle? (Hard)
• Find an oracle problem where standard oracle is exponentially better than in-place (opposite is known) (Less Hard)