Quantum Algorithms

Shelby Kimmel
Department of Computer Science
Middlebury College
The Quantum Question

• If I have a quantum computer, could I solve this difficult problem in my field faster/more accurately?
The Quantum Question

- If I have a quantum computer, could I solve this difficult problem in my field faster/more accurately?

The Answer:
- We don’t know yet (for many problems)
Quantum Algorithm Landscape

Large Advantage Over Best
Known Classical Algorithm
Quantum Algorithm Landscape

Large Advantage Over Best Known Classical Algorithm
Quantum Algorithm Landscape

Large Advantage Over Best Known Classical Algorithm

Small/No Advantage Over Best Classical Algorithms

- SEARCH
- PARITY
- COMPLETE GRAPH
- CONNECTIVITY
Quantum Algorithm Landscape

Large Advantage Over Best Known Classical Algorithm

Small/No Advantage Over Best Classical Algorithms

Quantum Advantage Unknown

Lots…
Quantum Algorithms: Promise and Perspective

1. Temper expectations:

2. Build excitement:
Quantum Algorithms: Promise and Perspective

1. **Temper expectations:** quantum computers are sometimes, but not always, helpful

2. **Build excitement:**
Quantum Algorithms: Promise and Perspective

1. **Temper expectations:** quantum computers are sometimes, but not always, helpful

2. **Build excitement:** quantum computing devices provide an unprecedented tool for designing and studying quantum algorithms
Quantum Advantage

Quantum computers seem really helpful for some problems but not others.

WHY?

- Superposition
- Interference
Metaphor for quantum computer

- Quantum computation is like interaction of waves, islands, and shore
Metaphor for quantum computer

- Quantum computation is like interaction of waves, islands, and shore
- I can control islands and shoreline
Metaphor for quantum computer

- Quantum computation is like interaction of waves, islands, and shore
- I can control islands and shoreline
- Output of computation is location of a large wave
Quantum Advantage

Quantum computers seem really helpful for some problems but not others.

WHY?

• Superposition

• Interference
Quantum Advantage

- Superposition – “can be in many states at once”
Quantum Advantage

- Superposition: waves explore many paths through the environment, hit all the points on the shore
Quantum Advantage

What gives quantum computers their power? Why is this power helpful for some problems, but not helpful for others?

- Superposition – “can be in many states at once”
- Interference
Quantum Advantage

- Interference
Quantum Advantage

- Interference: waves travel from different paths. If in sync when get to shore, get big wave, if out of sync, get no wave.
Quantum Advantage

What gives quantum computers their power? Why is this power helpful for some problems, but not helpful for others?

- Superposition – “can be in many states at once”
- Interference – “cancel the bad, enforce the good”
Metaphor for quantum computer

- Quantum computation is like interaction of waves, islands, and shore
- I can control islands and shoreline
- Output of computation is location of a large wave
Quantum Advantage

• Some problems have structure that helps build up interference fast:

What is the period of a periodic function?

Period
Quantum Advantage

- Some problems have structure that helps build up interference fast:

Period Finding

Ocean
Quantum Advantage

- Some problems have structure that helps build up interference fast:

  Period Finding

  Ocean

  Quantum computers can find the period of a function exponentially faster than regular computers

  Used to break cryptosystems
Quantum Advantage

- Other problems have very little structure, need more time to build up interference

Parity: Even or odd number of 1’s

011010101110 (seven 1’s → odd parity)
011010001110 (six 1’s → even parity)
Quantum Advantage

- Other problems have very little structure, need more time to build up interference
Quantum Advantage

- Other problems have very little structure, need more time to build up interference

Parity:
No quantum advantage
Quantum Advantage for $st$-connectivity

$st$ – connectivity: is there a path from $s$ to $t$?
Quantum Advantage for $st$-connectivity

$s t$ – connectivity:
is there a path from $s$ to $t$?
Quantum Advantage for $st$-connectivity

Less charge build-up (smaller effective capacitance) $\rightarrow$ easier for quantum computer to solve

More current flow (smaller effective resistance) $\rightarrow$ easier for quantum computer to solve
Quantum Algorithms: Promise and Perspective

1. **Temper expectations:** quantum computers are sometimes, but not always, helpful

2. **Build excitement:** quantum computing devices provide an unprecedented tool for designing and studying quantum algorithms
Designing a Quantum Algorithm

Quantum Algorithm Designer’s Toolbox

- Quantum Walk
- Span Program
- QAOA
- Phase Estimation
- Adiabatic Evolution
- Quantum Annealing
Designing a Quantum Algorithm

Quantum Algorithm Designer’s Toolbox

- Quantum Walk
- QAOA
- Span Program
- Phase Estimation
- Adiabatic Evolution
- Quantum Annealing

Unless the problem has very nice/simple structure, analyzing correctness and performance is very difficult
Designing a **Quantum Algorithm**

**Quantum Algorithm Designer’s Toolbox**

- Quantum Walk
- Span Program
- QAOA
- Phase Estimation
- Adiabatic Evolution
- Quantum Annealing

With advent of quantum computing devices, can test!
Challenges

**Quantum Algorithm Designer’s Toolbox**

- Quantum Walk
- Phase Estimation
- Adiabatic Evolution
- QAOA
- Span Program
- Quantum Annealing
Then there exists a quantum algorithm that estimates

Then there is a bounded error quantum algorithm that decides

**Theorem 16**

Since

A span program for

**Definition 15**

relaxed.

similar to the positive and negative witness sizes, but with the conditions

Then we refer to the following span program as

Ref. [20]

\[ N \]

such that for all

Ref. [16]

\[ \{ 0, 1 \} \]

defines the

Let

\[ \{ 0, 1 \} \]

is an undirected graph. We assume

\[ \{ 0, 1 \} \]

is an

\[ G \]

and elementary gates.

\[ x \]

\[ 1 \]

then

\[ x \]

\[ x \]

\[ \text{Span program algorithm:} \]

\[ \forall i \in [N], b \in \{0, 1\} : H_{i,b} = \text{span}\{|e\rangle : e \in \vec{E}_{i,b}\} \]

\[ U = \text{span}\{|v\rangle : v \in V(G)\} \]

\[ \tau = |s\rangle - |t\rangle \]

\[ \forall e = (u,v,\ell) \in \vec{E}(G) : A|u,v,\ell\rangle = \sqrt{c(u,v,\ell)}(|u\rangle - |v\rangle) \]

```
p.defgate("RACL", oracl)
p.defgate("ZEROPHS", zeroPhase)

#initialize
p.inst([H(i) for i in range(n)])

#run Grover iterate
for _ in range(int(m)):
    p.inst(["RACL"+s])
    p.inst([H(i) for i in range(n)])
    p.inst(["ZEROPHS"+s])
    p.inst([H(i) for i in range(n)])
```
Challenges

\[ \forall i \in [N], b \in \{0, 1\} : H_{i,b} = \text{span}\{|e\} : e \in \overline{E}_{i,b} \}
\[ U = \text{span}\{|v\} : v \in V(G)\}
\[ \tau = |s\rangle - |t\rangle \]
\[ \forall e = (u, v, \ell) \in \overline{E}(G) : A[u, v, \ell] = \sqrt{c(u, v, \ell)}(|u\rangle - |v\rangle) \]

Quantum Machine Code
(sequence of simple quantum operations)
Challenges

\[
\forall i \in [N], b \in \{0, 1\} : H_{i,b} = \text{span}\{ |e\rangle : e \in \overrightarrow{E}_{i,b} \}
\]

\[
U = \text{span}\{ |v\rangle : v \in V(G) \}
\]

\[
\tau = |s\rangle - |t\rangle
\]

\[
\forall e = (u,v,\ell) \in \overrightarrow{E}(G) : A|u,v,\ell\rangle = \sqrt{c(u,v,\ell)}(|u\rangle - |v\rangle)
\]

Quantum Machine Code
(sequence of simple quantum operations)
(tailored for particular implementation)
Outlook

1. Temper your expectations...

2. Potential of quantum algorithms is just beginning to be explored
Questions?

Theoretical collaborators: Stacey Jeffery, Michael Jarret, Alvaro Piedrafita