What does the effective resistance of electrical circuits have to do with quantum algorithms?

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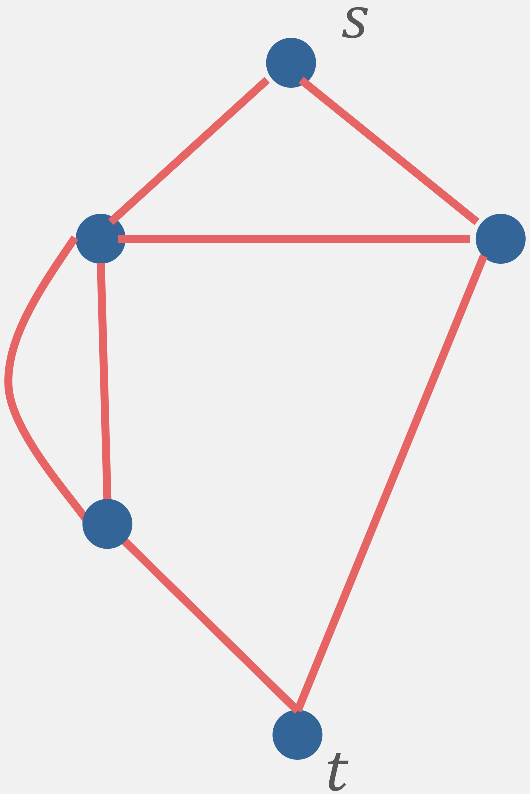
JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

(Simplest) Answer

$\boldsymbol{st-connectivity}$:
is there a path from s to t?



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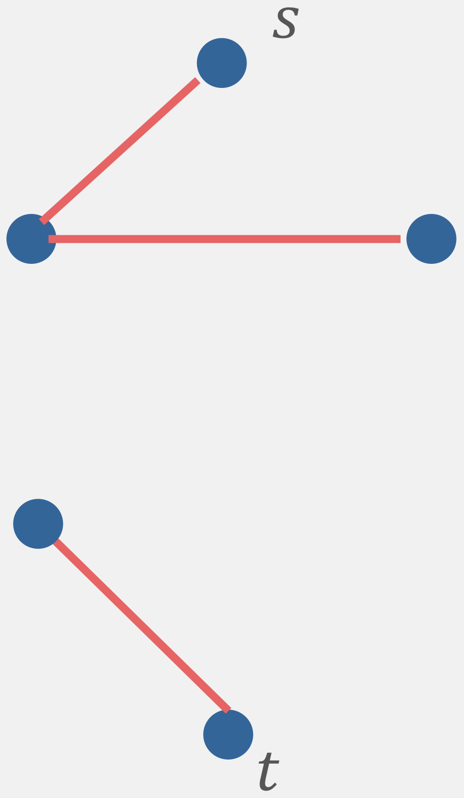
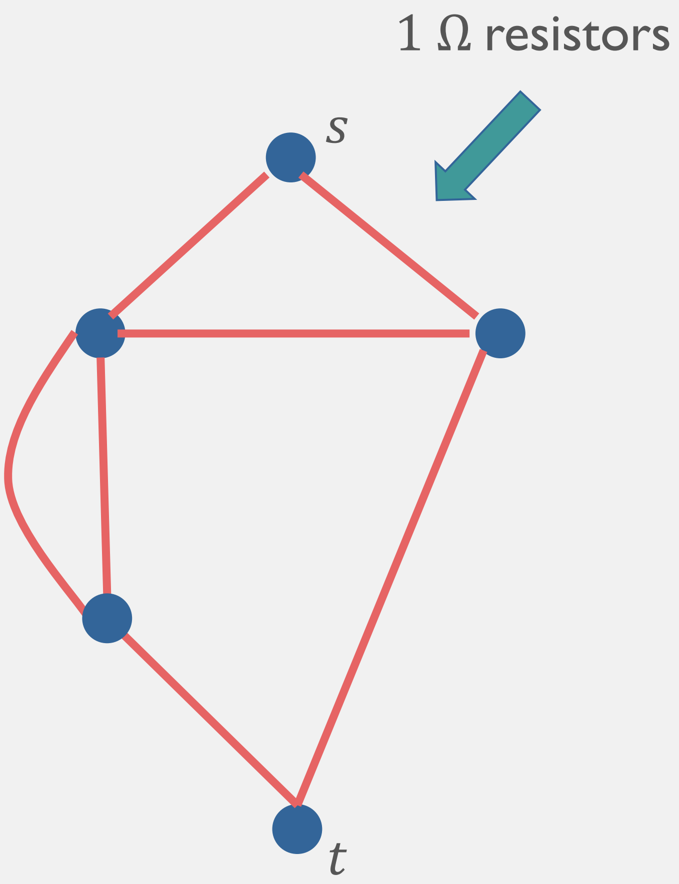


(Simplest) Answer

We can turn this into a circuit by attaching leads to s and t, and putting $1\ \Omega$ resistors wherever edges exist.



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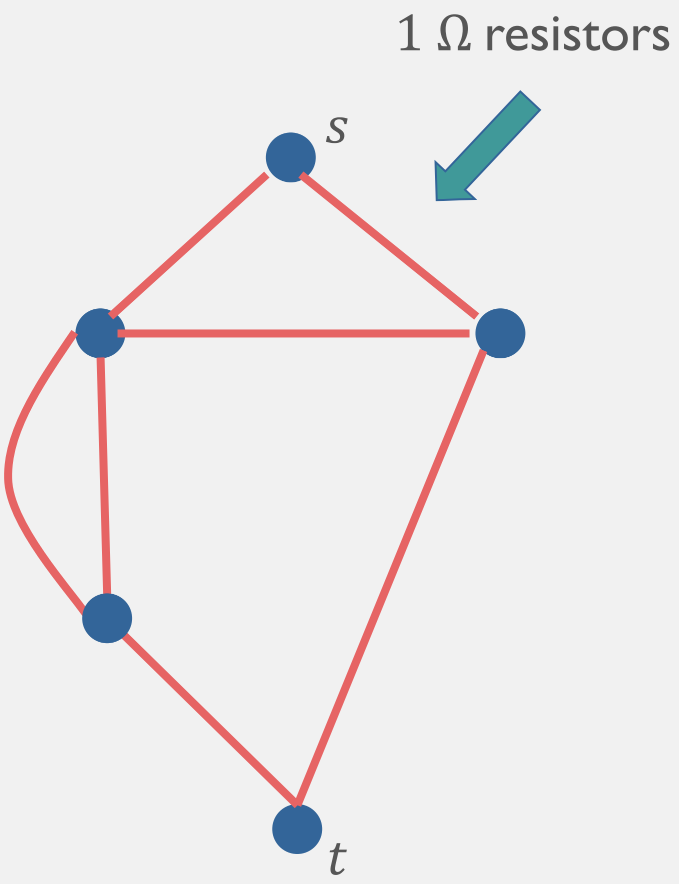


Speed of quantum algorithm for st-connectivity depends on effective resistance of this circuit! (Lower effective resistance -> quicker detection of path)

[Belovs, Reichardt '12]

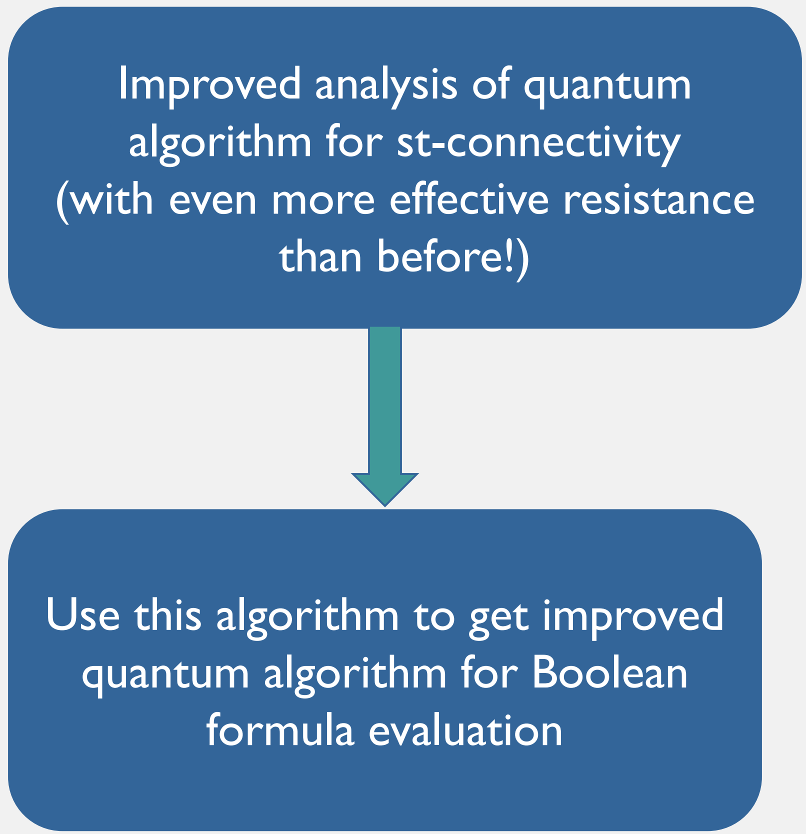
Applications of st-Connectivity

- Important (social) network problem
- Problem is a useful subroutine for many problems
  - Is there a length-k path? [Belovs, Reichardt '12]
  - Is a graph a forest? [Cade, Montanaro, Belovs '16]
  - Is a graph bipartite? [Cade, Montanaro, Belovs '16]

Applications of st-Connectivity

- Important (social) network problem
- Problem is a useful subroutine for many problems
  - Is there a length-k path?
  - Is a graph a forest?
  - Is a graph bipartite?
  - Boolean formula evaluation ← NEW

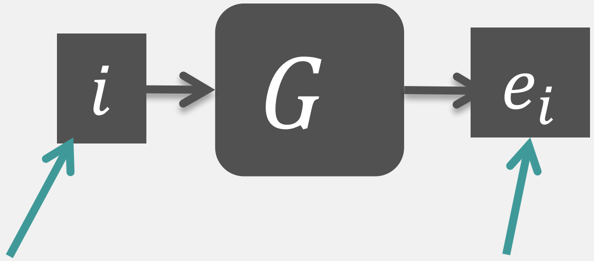
Our results:


Improved analysis of quantum algorithm for st-connectivity (with even more effective resistance than before!)
Use this algorithm to get improved quantum algorithm for Boolean formula evaluation

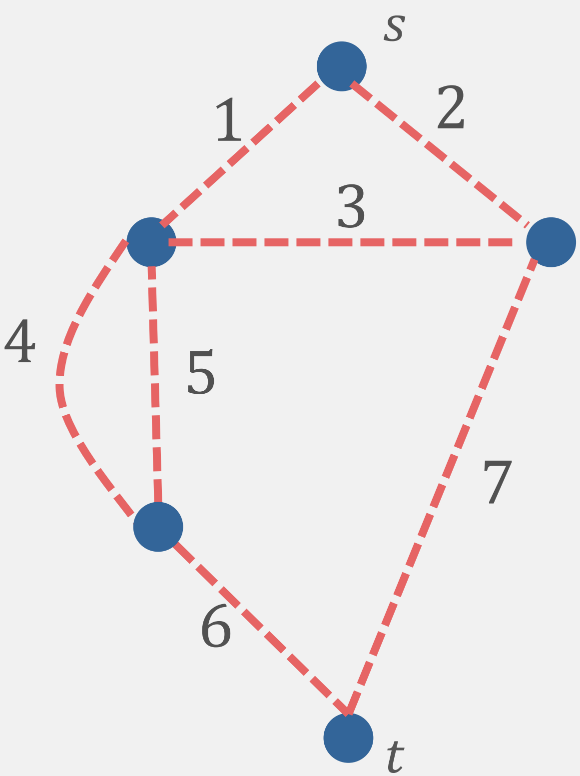
Outline

- Previous algorithm for st-connectivity
- Improved analysis for planar graphs
- Application to Boolean formulas

Black Box Algorithm

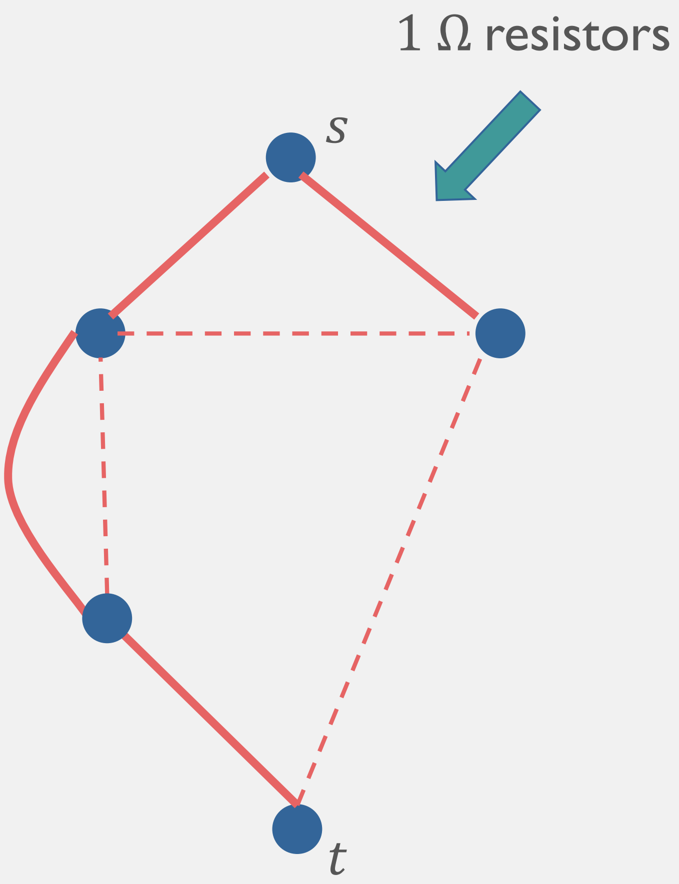


- $e_i = 1$ if i^{th} edge is there
- $e_i = 0$ if edge is not there



Previous Quantum Algorithm

$R(G)$ is the effective resistance of the circuit created by attaching a voltage between s and t, and $1\ \Omega$ resistors at all edges.



Previous Quantum Algorithm

st-connectivity algorithm time/queries ~

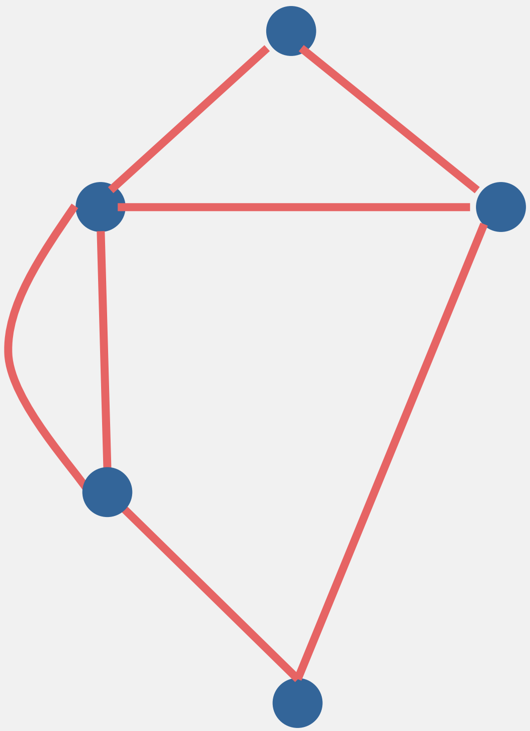
$$\sqrt{\max_{G:connected} R(G)}\ \sqrt{\max_{G:not\ connected} |G|}$$

# of edges in graph G

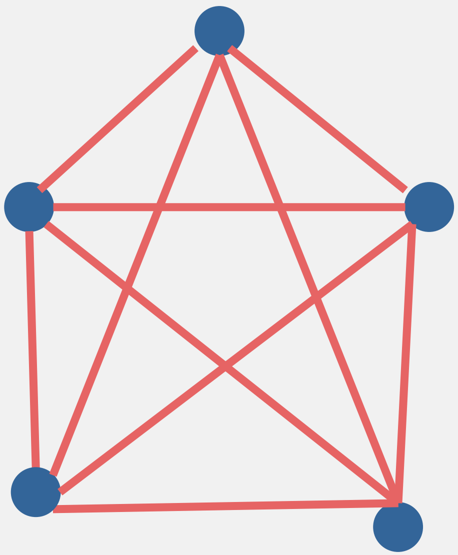
[Belovs, Reichardt '12]

Planar Graph

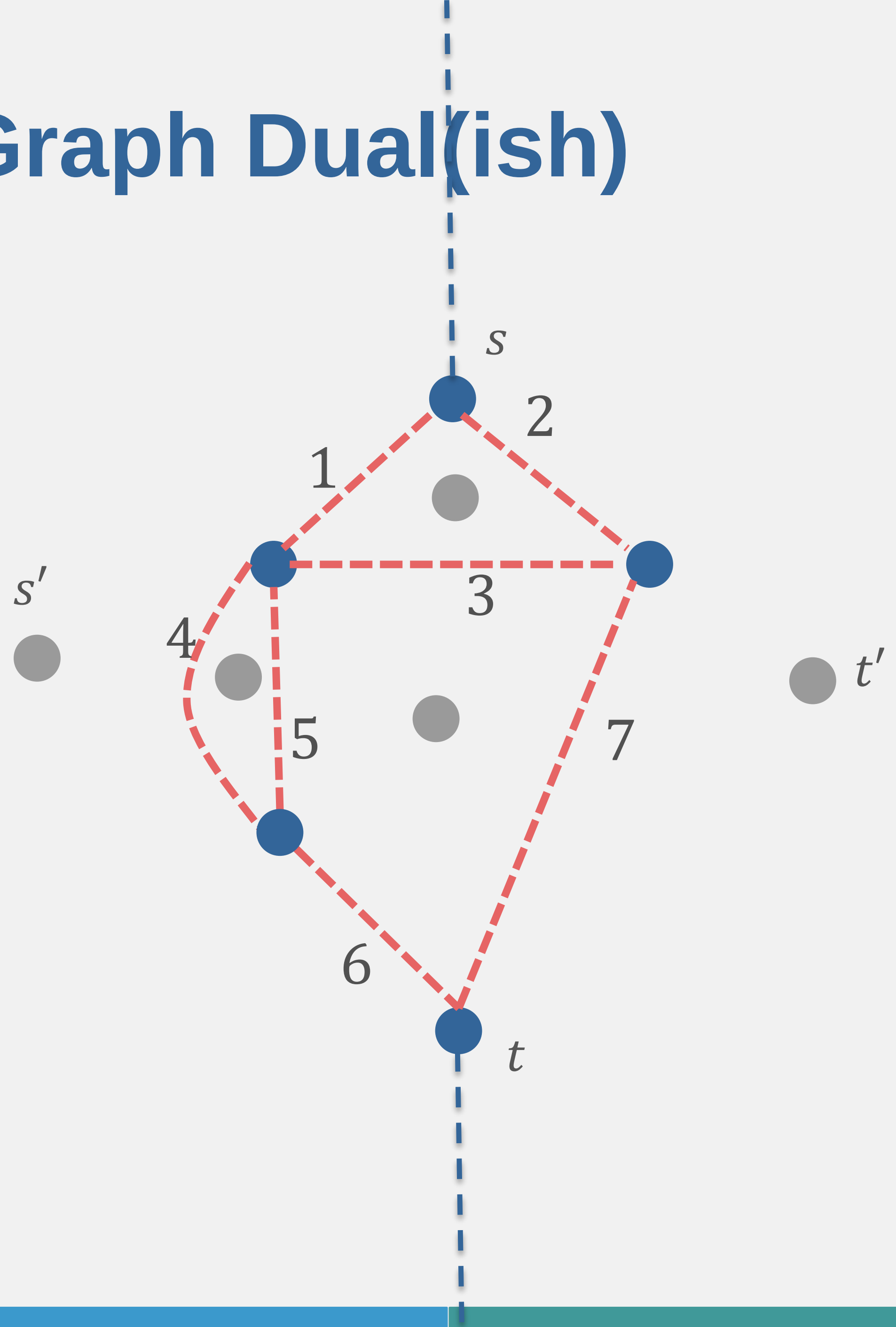
Planar



Not Planar



Planar Graph Dual(ish)



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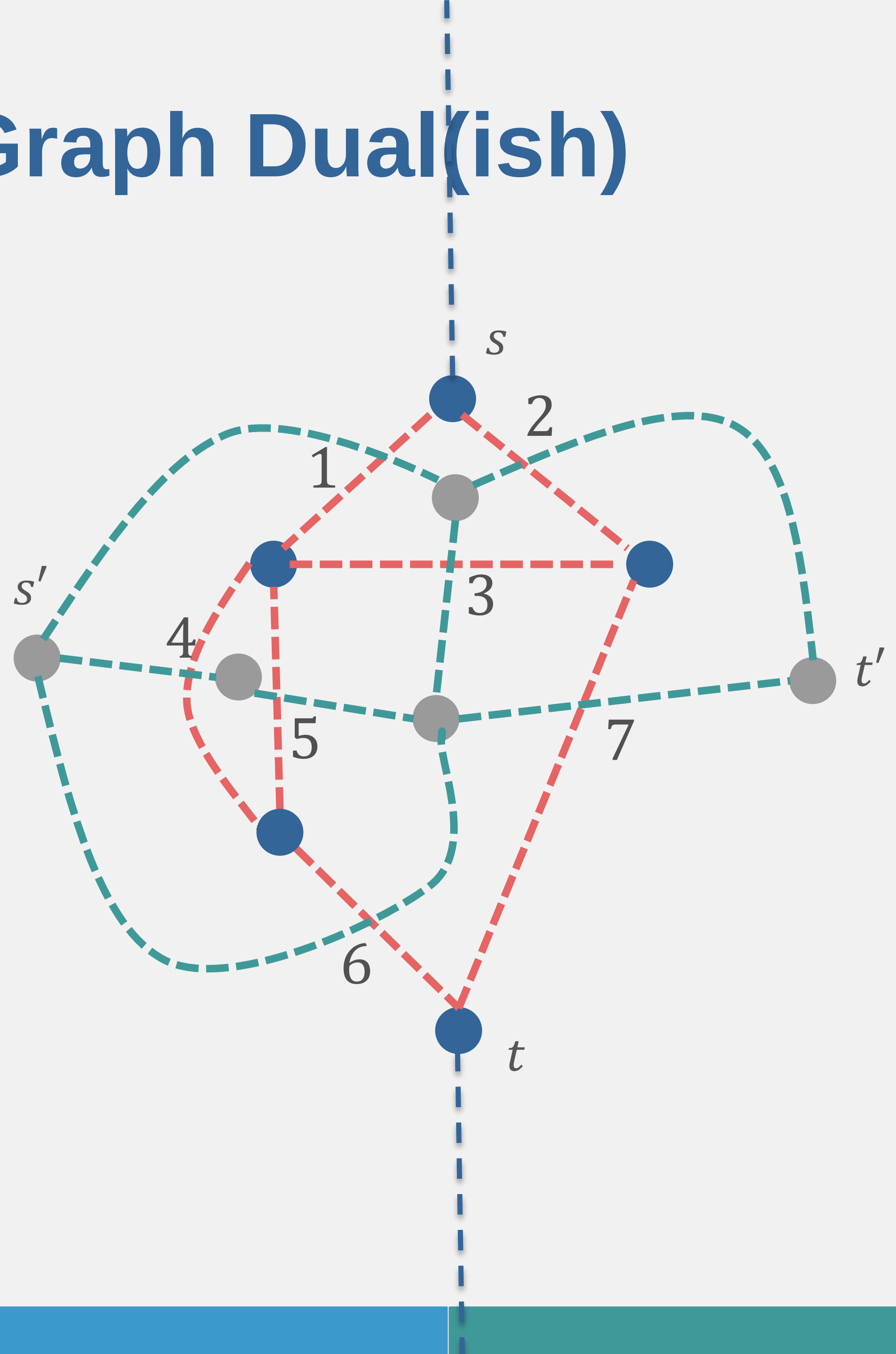
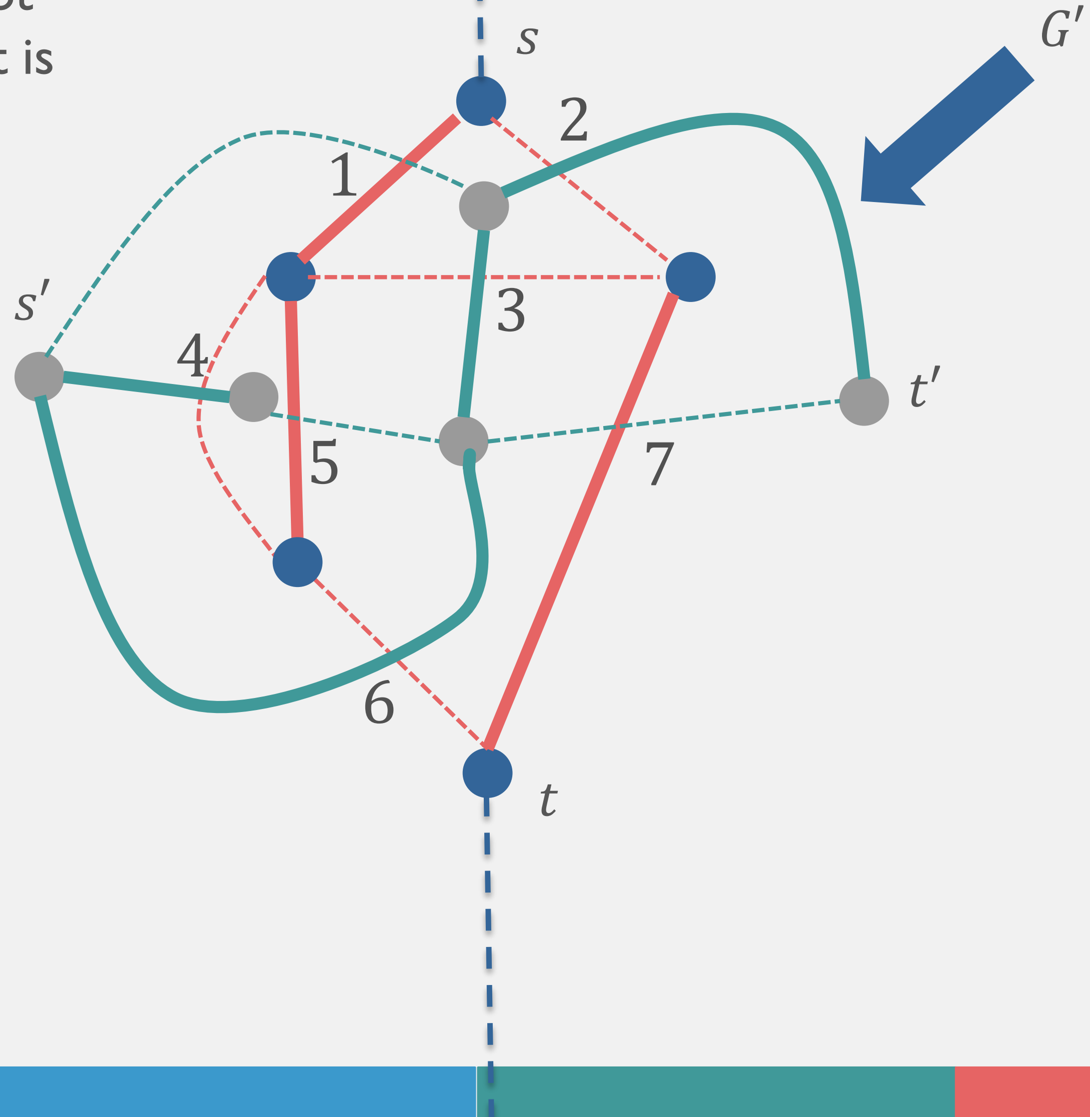


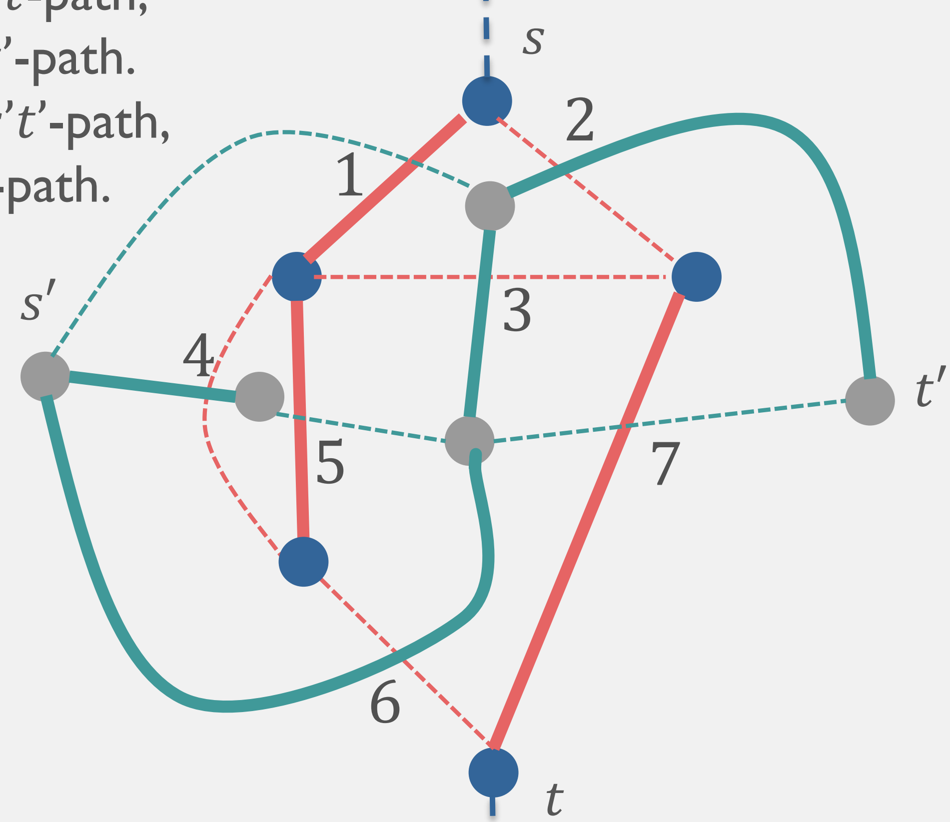
Planar Graph Dual(ish)

- If an edge is not present in G, it is present in G'



Planar Graph Dual(ish)

- If there is an st-path, there is no $s't'$-path.
- If there is an $s't'$-path, there is no st-path.



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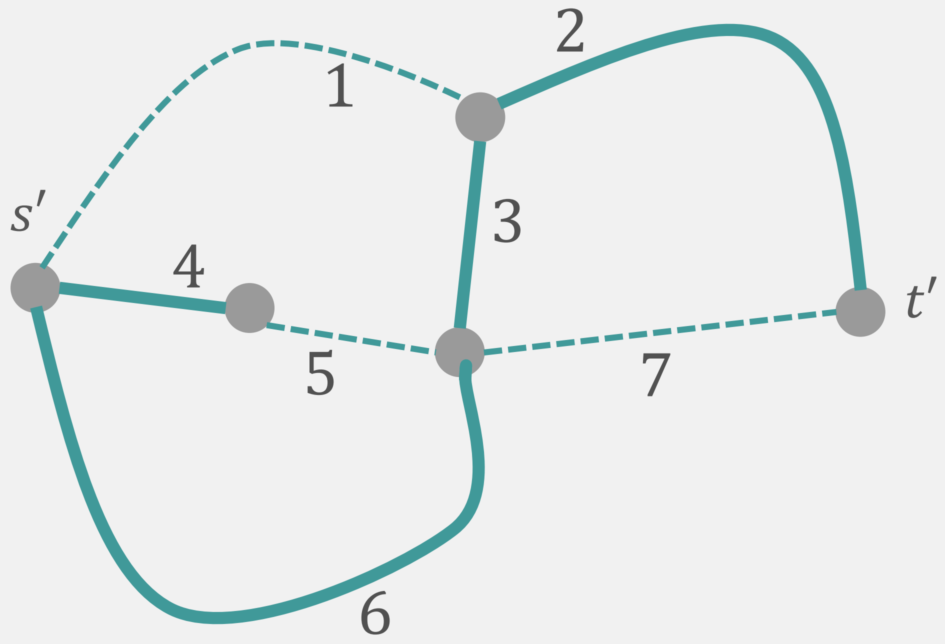
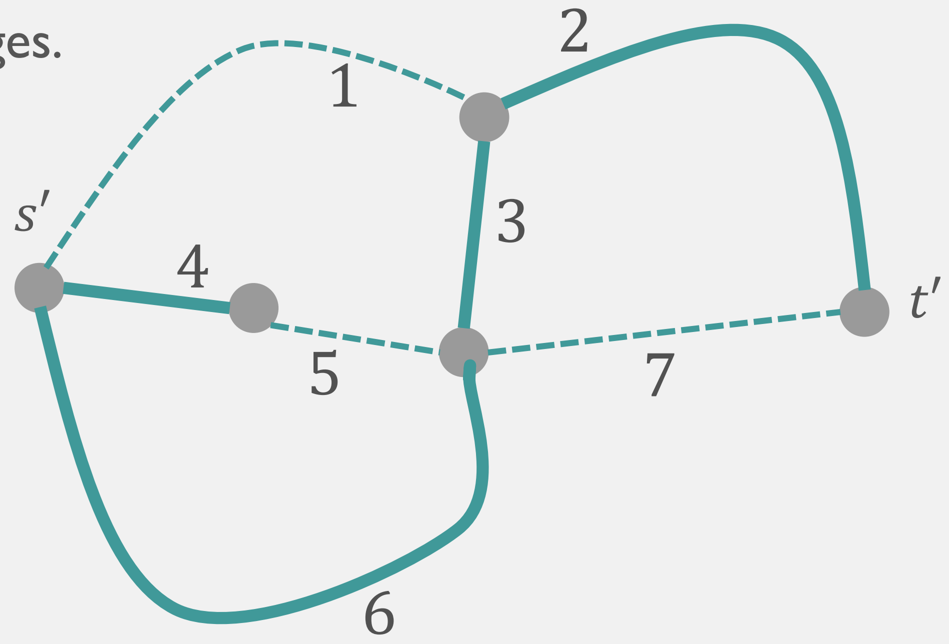


Planar Graph Dual(ish)

$R(G')$ is the effective resistance of the circuit created by attaching a voltage between s' and t', and $1\ \Omega$ resistors at all edges.



Improved Quantum Algorithm for st-connectivity

Planar graph[†] st-connectivity algorithm time/queries =

$$\sqrt{\max_{G:connected} R(G)} \sqrt{\max_{G:not\ connected} R(G')}$$

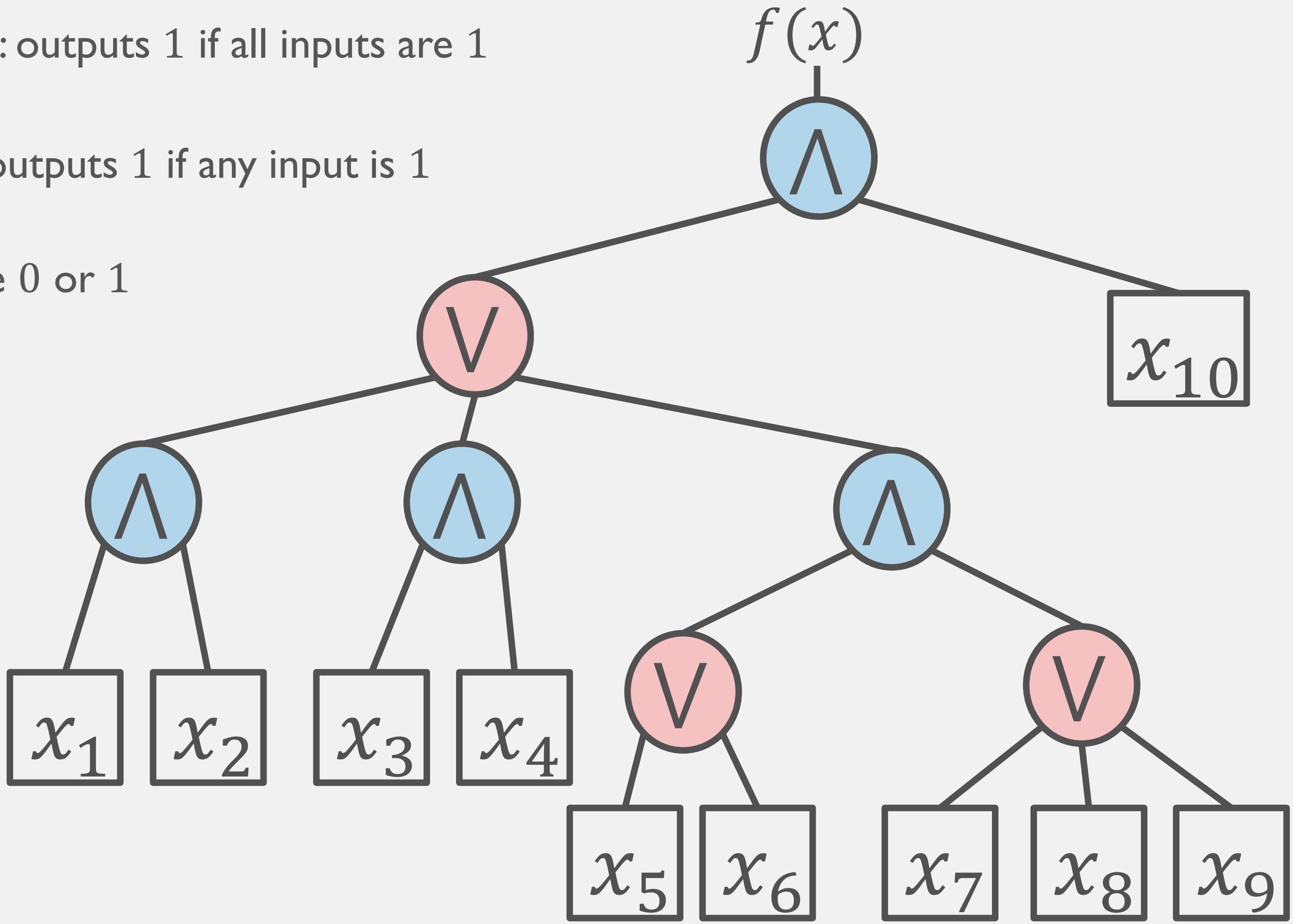
† with s, t on same face

Application to Boolean Formulas

∧ AND: outputs 1 if all inputs are 1

∨ OR: outputs 1 if any input is 1

x_1 Value 0 or 1



Application to Boolean Formulas

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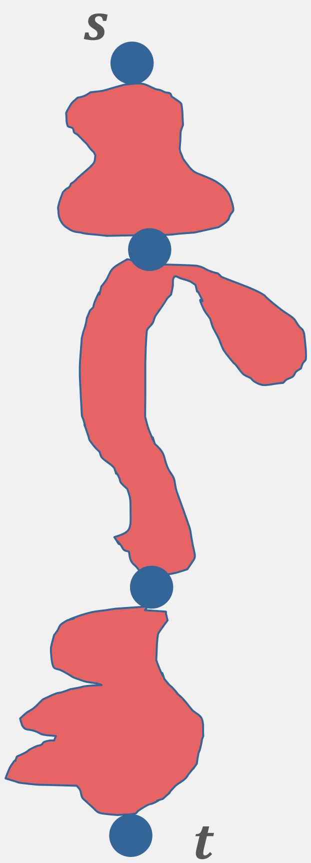

s and t are connected if all subgraphs are connected

Application to Boolean Formulas

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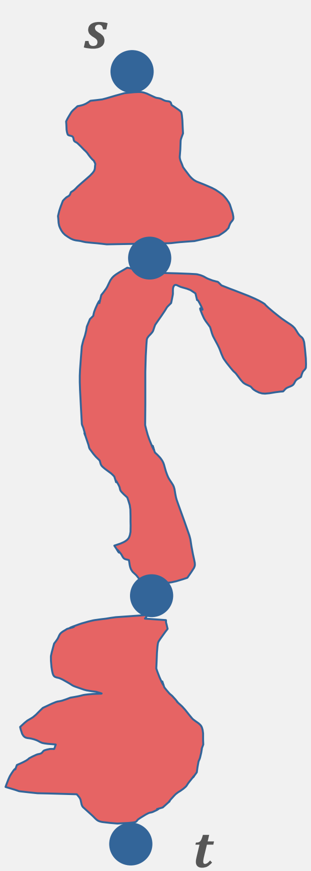

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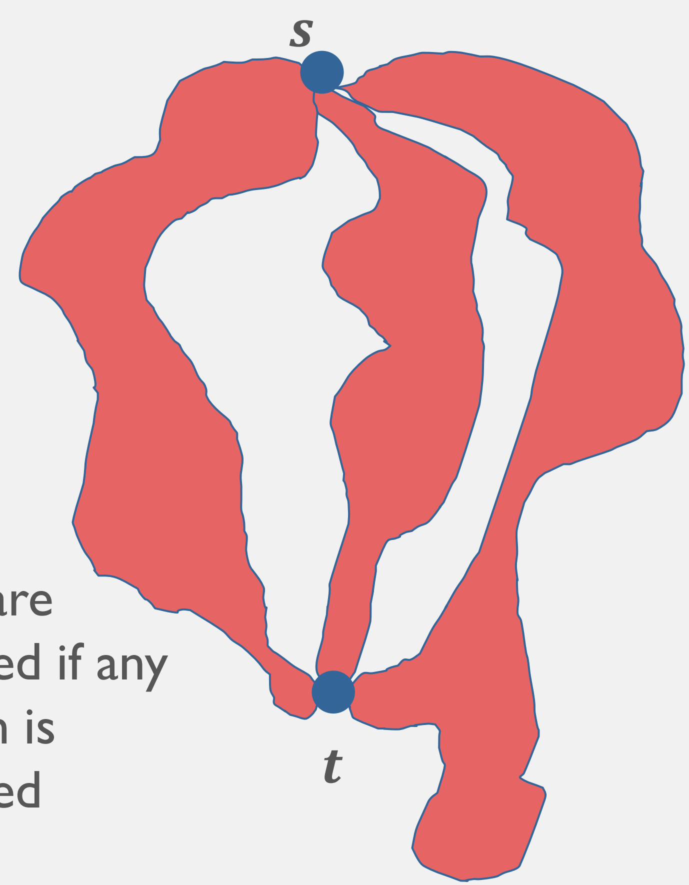

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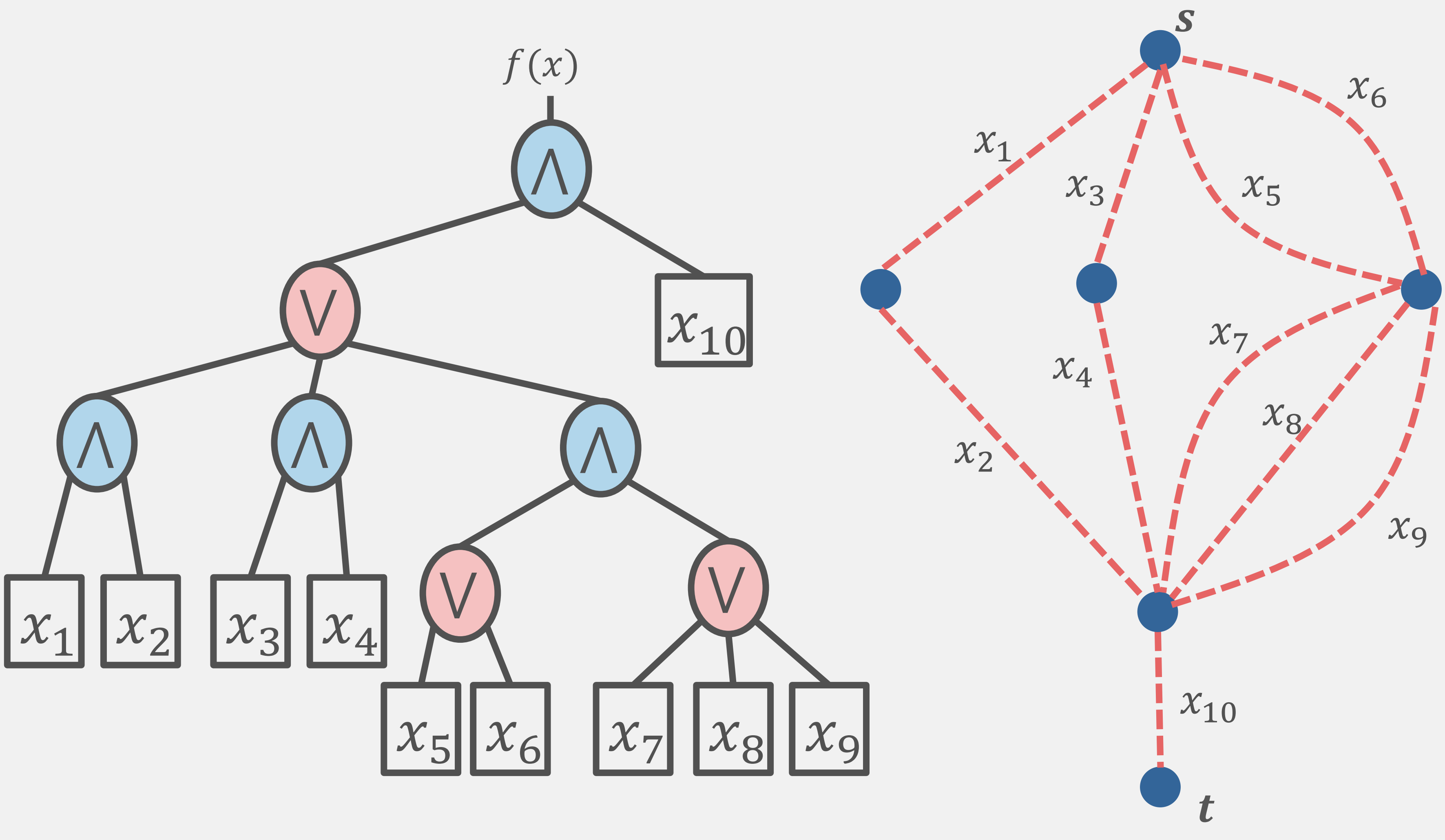
Application to Boolean Formulas



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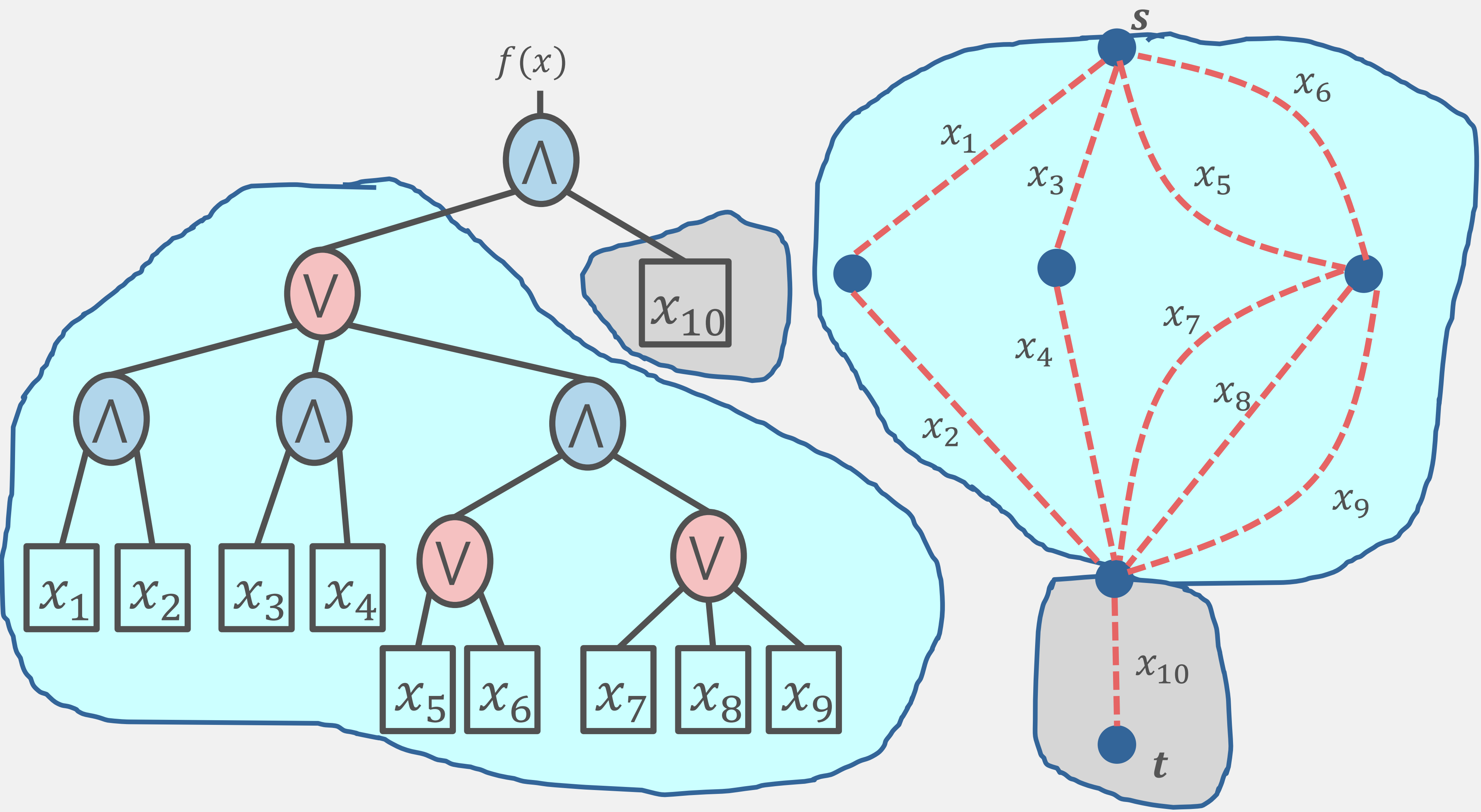

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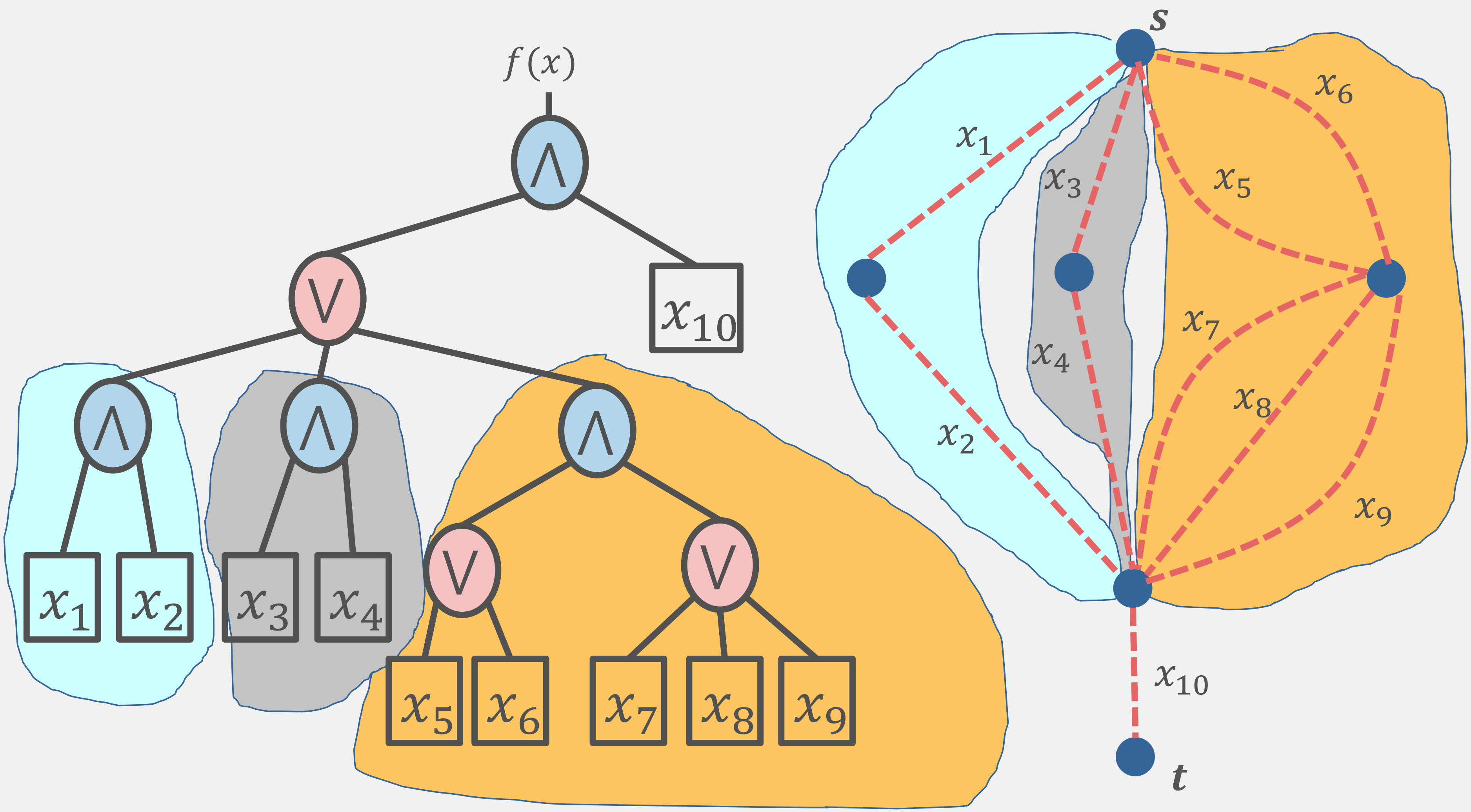
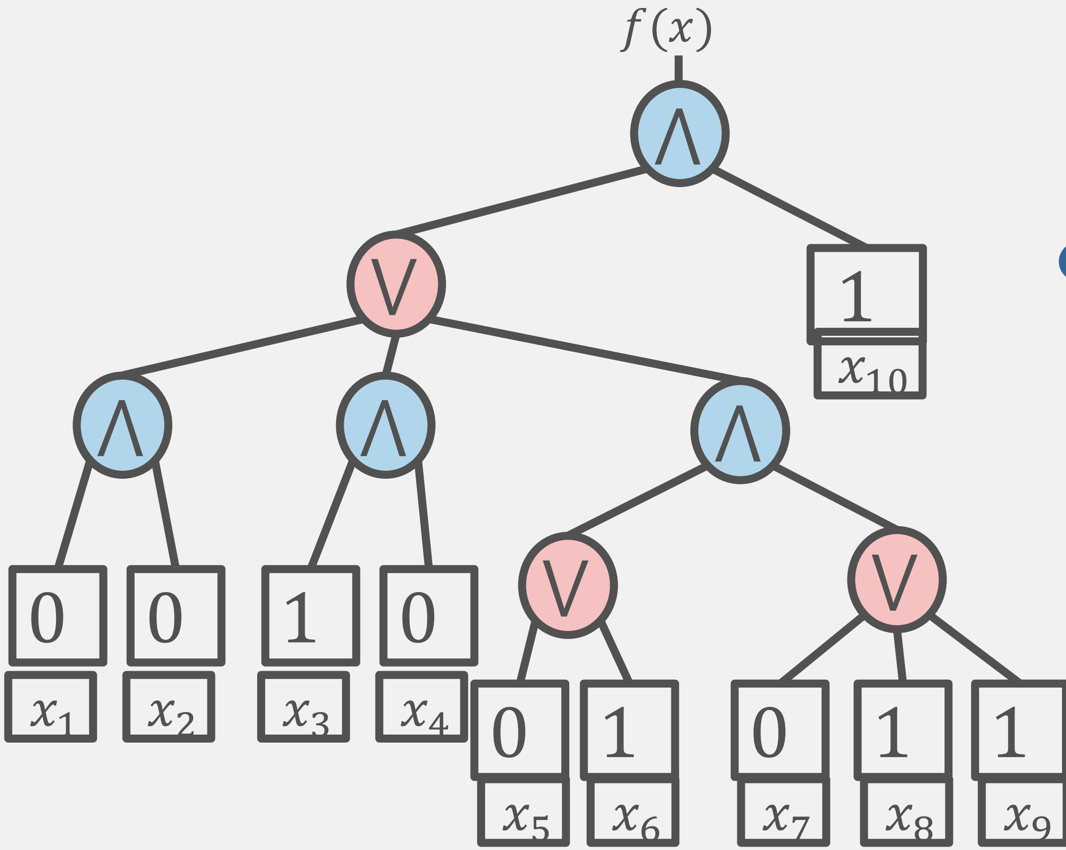
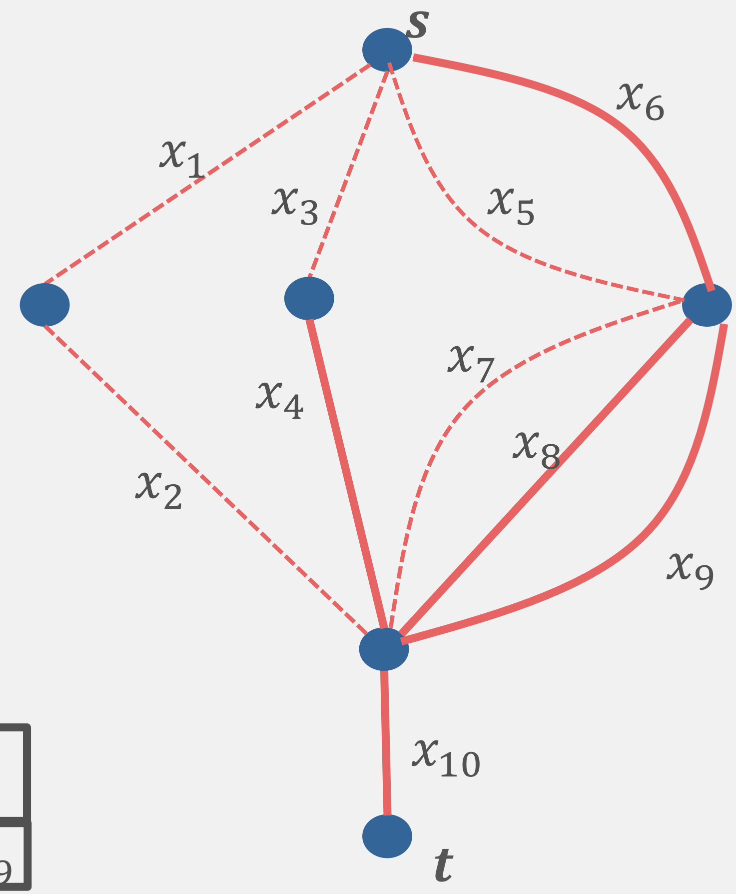


Application to Boolean Formulas





- If we put edges where $x_i = 1$, s and t are connected iff $f(x) = 1$!

Application to Boolean Formulas

- The graph associated with a formula will always be planar, with s, t on external face.
- Can use our st-connectivity algorithm! Time required depends on the effective resistance of circuit of corresponding graph.

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- The graph associated with a formula will always be planar, with s, t on external face.
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- This algorithm is pretty good! (for read-once formulas)
  - Gives a simple proof of $O(\sqrt{N})$ bound on N input formulas
  - Improves scope of superpolynomial quantum-classical separation of [Zhan et al '14]

Open Questions

- When is our algorithm optimal for Boolean formulas?
- Can we extend these ideas to non-planar graphs?
- Are there other problems that reduce to st-connectivity?
- What is the classical time/query complexity of st-connectivity? Can we relate it to effective resistance?
- I've answered how quantum algorithms and effective resistance are connected, but what about why?

Partial results:
arXiv:1511.02235